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A mixed integer programming model for a continuous move transportation problem with service constraints (Un método de programación mixta entera para un problema de transportación de movimiento continuo con restricciones de servicio)

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Abstract. We consider a Pickup and Delivery Vehicle Routing Problem (PDP) commonly encountered in real-world logistics operations. The problem involves a set of practical complications that have received little attention in the vehicle routing literature. In this problem, there are multiple vehicle types available to cover a set of pickup and delivery requests, each of which has pickup time windows and delivery time windows. Transportation orders and vehicle types must satisfy a set of compatibility constraints that specify which orders cannot be covered by which vehicle types. In addition we include some dock service capacity constraints as is required on common real world operations. This problem requires to be attended on large scale instances (orders ≥ 500), (vehicles ≥ 150). As a generalization of the traveling salesman problem, clearly this problem is NP-hard. The exact algorithms are too slow for large scale instances. The PDP-TWDS is both a packing problem (assign order to vehicles), and a routing problem (find the best route for each vehicle). We propose to solve the problem in three stages. The first stage constructs initials solutions at aggregate level relaxing some constraints on the original problem. The other two stages imposes time windows and dock service constraints. Our results are favorable finding good quality solutions in relatively short computational times.

Palabras claves. Algoritmos genéticos, logística de ruteo, metahurística, programación, ventana de horario

Resumen. En la solución de problemas combinatorios, es importante evaluar el costo-beneficio entre la obtención de soluciones de alta calidad en detrimento de los recursos computacionales requeridos. El problema planteado es para el ruteo de un vehículo con entrega y recolección de producto y con restricciones de ventana de horario. En la práctica, dicho problema requiere ser atendido con instancias de gran escala (nodos ≥100). Existe un fuerte porcentaje de ventanas de horario activas (≥90%) y con factores de amplitud ≥75%. El problema es NP-hard y por tal motivo la aplicación de un método de solución exacta para resolverlo en la práctica, está limitado por el tiempo requerido para la actividad de ruteo. Se propone un algoritmo genético especializado, el cual ofrece soluciones de buena calidad (% de optimalidad aceptables) y en tiempos de ejecución computacional que hacen útil su aplicación en la práctica de la logística. Para comprobar la eficacia de la propuesta algorítmica se desarrolla un diseño experimental el cual hará uso de las soluciones óptimas obtenidas mediante un algoritmo de ramificación y corte sin límite de tiempo. Los resultados son favorables.

1. Introduction

Multiple Vehicle Pickup and Delivery Problem with Time Windows and Dock Service Constraints (PDP-TWDS) is an important problem in logistics and transportation management. The PDP-TWDS is a variant of the well-known Vehicle Routing Problem with Time Windows (VRP-TW). Particularly, our real-world application deals with the schedule of a transportation operation on a network with several plants and distribution centers. Vehicle routing plays a central role in logistics management. A wide variety of vehicle routing problems have been studied in the literature. Different vehicle routing problems address different practical situations but focus on a common and a simple problem, the efficient use of a fleet of vehicles that must pick up and/or deliver a set of customer orders within a time window framework. We need to identify which transportation orders should be covered by each vehicle and at what times so as to minimize the total cost subject to a variety of constraints.

As is defined, in a general PDP problem a set of routes must be generated in order to satisfy a set of transportation requests at a total minimum cost (or a similar objective function) and subject to a set of constraints. Each transportation request (i.e. a transportation order) specifies a volume of product, a site of origin and a destination site. Each request must be transported by only one vehicle. However we consider that some trans-shipments can occur across a route sequence from one node to the next. For all this operation, a previous defined fleet of vehicles is available. These vehicles are spread throughout a set of specific depot sites. This fleet of vehicles may consist of different vehicle-types, each with a unique set of transportation relevant characteristics. Indeed, in a PDP-TW problem, time windows constraints are usually added to the transportation request. This is specifying a time interval for pickup and/or delivery operation at the origin or destination site. Our business application considers that the available vehicle fleet is represented on a node basis. In other words, at the beginning of the planning stage, each plant o distribution center provides the expected number of available trucks per type and at a specific starting time hour. This information defines the consolidated transportation capacity. Because of carrier requirements contract, we start and finish a route at the origin depot. Indeed, contract payment used in practice by the industry fix the transportation price on the basis that a route starts and finishes at the first pickup site. In our PDP context, each transportation request has a single time window. This is an earliest pickup time at origin and a latest delivery time at destination.

From a given set of transportation orders we have an origin and a destination (O-D). Usually transport planners determine first the best route for each O-D pair, and later assign trucks to these predetermined routes. The problem of determining the best assignment of trucks to O-D routes is typically referred as an assignment problem where trucks are assigned to routes or transportation lanes such that all transportation orders are covered and transportation costs are minimized. It is easy to verify that with each head haul move of the truck, goods are transported from its origin to its destination and revenue is generated. However, without goods, the truck moves an empty haul, in which only costs are incurred and no revenue is generated. Attempt to secure a transportation order from a destination location back to the location where the truck originates results on an unsuccessful practice. This is because the truck will run an empty haul. These empty hauls represent a serious problem for transportation operations, as well as the country's economic system. This is clearly true because an empty haul does not generate any economic value.

We can verify that the least efficient route that can be planned by a dispatcher is the one of simple trips where the vehicle travels loaded from the origin to the delivery site and then returns empty. On this case, half of the hauling distance is traveled empty. Even more if a dispatcher tries to avoid simple trips, the actual structure of transportation flows that he is responsible for does not always permit it. In this situation, pooling these transportation requests with those of another dispatcher may avoid simple trips planning by replacing the empty return of a simple trip with a transportation request of another dispatcher. Indeed, the new structure of transportation flows generated by the collaboration of two or more dispatchers will allow transportation cost-savings. The empty part of the overall route is smaller when two trips are pooled together compared when making them independently. It is estimated that at least 46% of truck movements in México country are empty haul moves. This means millions of kilometers of empty haul moves and also millions of liters of fuel lost per year. This is a major economic loss for the country, especially in the current situation where fuel prices have skyrocketed. The Department of Land Transportation note that over 160,000 tons of pollution is released to the environment directly as a result of empty haul moves. Thus, empty hauls are a serious problem which needs immediate attention.

Time constrained sequencing and routing problems arise in many practical applications. Typically, computational difficult for those type of problems has been measured in terms of its size. However the difficult for PDP-TWDS depends strongly on the structure of the time windows that are defined around the nodes and vehicles as well. Indeed, multiple vehicles environment generates some dock service capacity constraints. Both the PDP and PDP-TW are generalizations of the classical Vehicle Routing Problem (VRP) and are thus NP-hard. As a result, the development of solution methods for these problems has focused on heuristics J.-F. Cordeau, G. Laporte, and M.W.P. Savelsbergh (2007). Due to the fact that the PDP is NP-hard problem, combined with the reality that practical PDPs are very large, having hundreds of requests to serve, there is no much hope for finding an optimal model that will work acceptably fast in practice. However, when the problem is sufficiently constrained, it is possible to obtain good solutions within reasonable computation time. We propose a Hybrid Mixed Integer Programming (MIP) approach to this complex problem which is focused on finding good solutions in reasonably short time. The paper is organized as follows. In Section 2 we introduce the problem definition and its associated complications. In Section 3 we briefly sketch some related problems and previous work. In Section 4 we proceed to introduce some notation that will be used throughout the paper and we model the problem as well. Section 5 contains a description of some empirical results we found on our implementation and some concluding remarks are given.

2. Problem Definition

The PDP is a generalization of the VRP, which is a generalization of the TSP, the well-known hard combinatorial optimization problem. Considering also that the problem in practice is, usually, of a large-scale, it is obvious why the problem is a challenge. The general pickup and delivery problem (GPDP) is a problem of finding a set of optimal routes, for a fleet of vehicles, in order to serve a set of transportation requests. Each vehicle from the fleet of vehicles has a given capacity, a start location, and an end location. Each transportation request is specified by a load to be transported, an origin, and a destination location. In other words, the pickup and delivery problem deals with the construction of optimal routes in order to visit all pickup and delivery locations and satisfy precedence and pairing constraints. From here we can move on to include some others considerations. That is, the problem deals with a number of transportation orders that are to be served by a fleet of vehicles while a number of constraints must be observed. Each vehicle has a limited capacity (the capacity constraint). Each vehicle starts and ends at a specified depot. A request must be picked up from a pickup location to be delivered to a corresponding delivery location. In addition, every request must be served within a predetermined time window (TW) interval (the time window constraint). A vehicle may serve multiple transportation orders as long as time windows and other capacity constraints are satisfied. A solution to the problem should assign requests to vehicles and find a route for each vehicle, such that the total service cost is minimized and all problem constraints (precedence, capacity, time windows and dock service) are adhered with. The total volume of product to delivery on some nodes may exceed the capacity of all types of truck. Thus a site within the same route could be visited more than once. In addition, in the classical PDP, when a delivery has been made, no pickup is allowed until the truck is empty. However in our problem's case, when a delivery has been made, we allow pickup even if the truck is not completely empty. This makes routing much more complex than classical PDP. The problem can be outlined in: (1) objective function and (2) operation constraints.

I. Objective Function:

The goal of our model is to determine the optimum route for a multiple vehicles dedicated for a given physical distribution operation. A route is defined as the arrival sequence of a vehicle (i.e. trailer) which has to attend to a set of nodes or warehouses waiting for service. This service can be defined as a delivery or pickup of any kind of item (i.e. product). In a typical operation we arrive to a node, make a delivery for product A and then afterwards pickup for product B that is required on another point that is ahead on the route sequence. On any case, the vehicle departs from an origin node (i.e. a distribution center) and then returns to the same node at the end of the route. An optimal route is obtained when we achieve the minimal cost (or distance or time) in order to attend all the customer nodes waiting for service.

II. Operation Constraints:

1. We have a set of M different vehicles that are considered as the available fleet in order to perform a transportation process. For each vehicle entity one only origin node is defined. Several origin nodes are defined on the network where vehicles start from. At the start of the day, each vehicle leaves from the origin node. Then each vehicle attends to a set of geographically scattered nodes (i.e. customers). At the end of the route, each vehicle returns to its origin point.
2. Each vehicle has a finite load capacity. Vehicle Capacity is modeled as the quantity of boxes, pallets or weight that the vehicle can load taking in mind the space constraints as well. Indeed, vehicle capacity is defined at a SKU level in such a way we can cubic a capacity requirement to transport any given load mixture. This is any set of different volumes per SKU to make a full load. Capacity constraints guarantee that load of items on a vehicle should be less than the vehicle capacity. Log trailer is a set of 12 to 16 individual compartments depending on the truck-type, each with loading & unloading access by the sides. This design is not constrained by the nested precedence constraints we found on the general freight PDP in which loading and unloading access is restricted by the truck trailer rear door.
3. We have a set of N orders to be transported from origin nodes (i.e. plants) to destination nodes (i.e. distribution centers). Each order K member of set N consists of a pickup at some location (node i) and a delivery at some other location (node j) in the underlying transportation network. Precedence constraints must be considered which imply that a vehicle should visit the pickup location before the delivery location of a transportation order. Each order K member of set N is a specific mix of products (i.e. different SKU's) which has a weight or space requirement. According to the sequence of the route, all the time we must observe the load capacity of the vehicle.
4. Certain compatibility constraints must be satisfied in real-world distribution operations because of physical restrictions. For each vehicle entity we

define a specific set of nodes where the vehicle can operate. In other words, a vehicle cannot arrive to nor departs from any node not included on that defined set. Something similar is defined at a transportation lane level in order to constraint the use of a vehicle not included on a set of previous defined arcs.

5. The quantity of time (i.e. hours) required to accomplish the deliver and the pickup service in each node depends on the type of vehicle. This consideration is true because the type of vehicle is close related to the volume of product that is delivered or pickup at any given node.
6. Each node has a particular time window for service. Because a location (e.g., plant, warehouse, retail store) has specific working periods, the pickup or delivery of an order at a location can only take place during its working period. A time window is defined by an open & close time that should be considered for make a deliver or pickup on the node. Time windows constraints make sure that a service has to be given between the earliest arrival time and the latest arrival time.
7. The same constraint about time windows applies at a vehicle level. This means that any given vehicle cannot operate before its open window neither after its close window. In addition, an order itself may be associated with a specific time interval within pickup or deliver operation must be done. The wide of the time window at each node or vehicle is equal to the difference between the close time and the open time for service. Indeed, each time window has different wide depending on the characteristics of the node or the vehicle as is corresponds.
8. According to the sequence of the route, we will obtain arrivals and departures times for each vehicle across the nodes on the network. However, we define for each node a specific quantity of docks for service. Indeed, this capacity service at each node is not constant because is constrained depending the hour of the day. Our approach to deal with this dock service capacity is to constraint the quantity of vehicles can arrive at each node and at each hour of the day. As can be verified, dock service capacity imposes new time windows constraints which emerge according the traffic of vehicles waiting for service at a node at any hour.
9. We have a cost matrix that defines the time or distance required to go from each node to all others around a distribution network. Moreover, transportation cost for each arc (i,j) depends on the type of vehicle.

The problem is to find a sequence of the nodes, starting at the depot node 0 at time 0 and ending at the same node 0, with minimal cost such that for every node $i \in V$ the arrival time at node $i \in V$ lies within the given time window. In our case not waiting times are allowed. The PDP-TWDS is NP-hard since the PDP is NP-hard (Desrosiers, Dumas, Solomon, & Soumis, 1995). Indeed, it is strongly NP-complete to find a feasible solution for the PDP-TWSD. Furthermore, (Tsitsiklis, 1992) showed that the symmetric version TSP-TW with general time windows is strongly NP-complete, even if the underlying graph G is a path and all processing times equal 0.

3. Previous Research.

There are well known and extensively studied routing problems which are special cases of the General-PDP. The Dial a Ride Problem (DARP) is a routing problem in which the loads to be transported represent people. Therefore, we usually speak of clients or customers instead of transportation requests and all load sizes are equal to one. The Vehicle Routing Problem (VRP) is a routing problem in which either all the origins or all the destinations are located at the same depot. The research of time constrained pickup and delivery problems emerged in the last 15-20 years. Researchers have developed a variety of heuristics and optimization methods. The development of optimization methods started in the early 1980s and lasted almost a decade. Heuristics for solving real-life pickup and delivery problems began to appear in the literature in the 1970s. The majority of published work on General-PDP is on dial-a-ride problems (DARP). In contrast to this, very little work has been done on pickup and delivery of packages and goods with time windows constraints (PDP-TW).

In regard to routing applications, we found that the variant with less research work corresponds to physical product distribution (Mitrovic 1998). We have the basic model named Traveling Salesman Problem with Time Windows constraints (TSP-TW). Christofides in 1976 describe a branch-and-bound algorithm in which the lower bound computation is performed via a state space relaxation in a dynamic programming scheme. Problem instances were solved up to 50 nodes with "moderately tight" time windows. Dumas et al. (1995) present a dynamic programming algorithm for the TSP-TW. They were able to solve problems of up to 200 nodes with "fairly wide" time windows. We refer now about the work presented by Ascheuer et al. (2001) for the TSP-TW. They tested instances up to 233 nodes. For an instance of 69 nodes was required 5.95 minutes of computational time. In general, all larger instances required more than 5 hours of computational time to converge in a feasible solution. The experimental results with the TSP-TW made by Ascheuer et al. proved that this problem is particularly difficult to resolve for instances with more than 50% of active nodes with time window constraints.

We move our research now from the typical TSP-TW to a more sophisticated problem named as Vehicle Routing Problem (VRP). The most widely studied extensions of the VRP are the capacitated vehicle routing problem (C-VRP) and the vehicle routing problem with time windows (VRP-TW). The basic model C-VRP assumes that all the vehicles are homogeneous with the same capacity and located initially at the same node (i.e. depot) and customers have no specific service time windows (i.e. can be covered at any time). A more complex model is the VRP-TW. On VRP-TW customers have time windows within which they must be covered. Solomon (1984) developed 87 test instances for the VRP-TW. Indeed, the largest instance he solved was about 100 nodes. Until year 1999 there were 17 instances that still remained without being solved. In that year in Rice University, were solved 10 of these instances (Cook & Rich 1999). VRP with multiple pickup and delivery locations have been studied by Savelsbergh (1998).

The most general model is the Pickup and Delivery problem with Time Windows Constraints (PDP-TW). PDP-TW is more difficult to solve than VRP–TW. This is true because, the first problem is a generalization of the second (Palmgren 2001). According with Savelsbergh (1995), we have a variant for one alone vehicle (SPDP-TW) and one another for multiple vehicles (MPDP-TW). The first case is considered a restrictive TSP-TW while the second variant is considered a restrictive VRP-TW. The PDP-TW is NP-hard since the VRP and PDP is NP-hard (Desrosiers, Dumas, Solomon, & Soumis, 1995). Indeed, it is strongly NP-complete to find a feasible solution for the PDP. Furthermore, Tsitsiklis (1992) showed that even the basic TSP-TW is strongly NP-complete. Our PDP-TWDS is less studied than the classical vehicle routing problems. Indeed, this problem is a generalization of the vehicle routing problem (VRP) and the pickup and delivery problem (PDP). The problem involves a set of practical features that are commonly seen in practice but have received little attention in the vehicle routing literature. Some complex features involved in the PDP-TWDS such as dock service capacity and compatibility constraints, have not been addressed in the vehicle routing literature. For PDP-TWDS extension we just add some constraints on dock capacity service at each node and at each hour of the day. Therefore, the PDP-TWDS is more general and more complex to solve than any existing VRP-TW or a single PDP model. Furthermore, no existing model has incorporated dock service capacity constraints explicitly.

The first optimization algorithm for the PDP-TW was a branch-and-price algorithm presented by Dumas, Desrosiers, & Soumis (1991). A column generation approach was proposed. Indeed, a set partitioning formulation is solved by a branch-and-price method in which columns of negative reduced cost are generated by a dynamic programming algorithm. The method has been successful in solving instances with tight capacity constraints and a small number of requests per route. They show that this approach is capable of solving some instances with up to 22 vehicles and 190 requests. Savelsbergh & Sol (1995) presented an integer programming formulation of the general pickup and delivery problem (GPDP) which considered several pickup and delivery locations of a transportation orders. Savelsbergh, & Sol (1998) proposed a branch-and-price algorithm for the PDP-TW using both a heuristic algorithm and a dynamic programming algorithm for the column generation problem. They applied a new branching scheme based on assignment rather than routing decisions. In the past two decades, a tremendous amount of research results on these models have been published. Recent books and survey papers include, among others, Laporte (1992), Desrosiers et al. (1995), Savelsbergh and Sol (1995).

Cordeau et al (2007), developed a branch-and-cut algorithm for the DARP, based on a three-index formulation with a polynomial number of constraints. It uses several families of valid inequalities that are either adaptations of existing inequalities for the TSP and the VRP. However, direct implementation of methods for solving DARP is not a solution for GPDP. The GPDP is mostly capacited and the time windows are wider. These differences seem to imply that the set of feasible solutions is larger in GPDP than in the problems where people are transported. More recently, a branch-and-cut algorithm for the capacitated multiple-vehicle PDP and PDP-TW was later described by Dessouky et.al (2006). Their formulation contains a polynomial number of constraints and uses two-index flow variables, but relies on extra variables to impose pairing and precedence constraints. Instances with up to 5 vehicles and 25 requests were solved optimally with this approach. By using appropriate inequalities, Ropke, Cordeau and Laporte (2007) introduced a new formulation for the PDP-TW which do not require the use of a vehicle index to impose pairing and precedence constraints. They report computational experiments on several sets of test instances and show that this approach is capable of solving some instances with up to 8 vehicles and 96 requests. In general, the best results found on literature are obtained by column generation methods. Instances of up to 880 requests and 53 vehicles can be solved with this method.

Many solution methods have appeared for vehicle routing problems. In general, heuristics can solve problems with larger scales in less computation times than optimization methods. For example, the recent progress in meta-heuristics such as Tabu Search, simulated annealing, and genetic algorithms can solve vehicle routing problems with wide time windows with nearly 500 transportation requests. However, as pointed out by Fisher (1995), heuristics usually lack robustness and their performance is very much problem dependent. Fisher states that “It’s not uncommon that a heuristic developed for a particular geographic region of a company’s operation will perform poorly in another region served by the same company.”

It is not easy to compare different approaches to the PDP-TW. Moreover, in most of the cases authors only use randomly generated data. It is not clear what their findings mean for "real-world instances" which is actually our case. The existing vehicle routing models are useful for various practical applications. However, many important practical issues have not been addressed in these models, as pointed out by Fisher (1995), "Real vehicle routing problems usually include complications beyond the basic model....". Given the enormous complexity of the PDP problems, it is not realistic to apply pure optimization methods. Instead, we focus on a strategy that can not only be as robust as optimization methods but also are capable of finding good solutions within acceptable computation time. Thus, we develop hybrid approach to integrate fast heuristics into an optimization framework of a cut generation method.

4. Proposed Model.

Our PDP model is focused on a continuous move strategy implementation. On this strategy attempts are made to match multiple truckload pickups and deliveries to one truck in sequential order such that the prior delivery is made before the next pickup in the sequence. The benefit of continuous moves derives from the overall reduction in empty haul distances.

Careful planning can ensure that the relocation of a truck from the prior delivery location to the next pickup location will minimize the overall empty haul distances for the entire network. So, we focus our attention on finding optimal routes for the continuous move problem, using a large-scale mathematical model. A continuous move (i.e. c-move) trip occurs when two or more truckload trips are sequentially combined. That is, if trips $T_{i1,j1}$ and $T_{i2,j2}$ are combined, then a c-move trip will require as follows:

1. Deliver goods from origin i1 to destination j1.
2. Make an empty haul move to a new origin i2.
3. Pick up goods from origin i2 and deliver them to a final destination j2 and
4. Return to the initial origin i1.

For each c-move trip, we compute its cost, which includes the summation of all costs including those associated with the empty hauls which the model seeks to minimize. Some assumptions are considered in our case. We only consider a daily operation, in which all trips are planned for one day of operation in order to enforce and simplify truck location requirements. In other words, all trucks starts the day at an origin i and must return to that origin by the end of the day. Another assumption excludes stochastic and dynamic considerations. This is justifiable as the model that we propose is meant as a planning tool, not as an operational tool.

It is apparent that the PDP-TWDS can be formulated as a dynamic program and that it can be attacked by various branch-and-bound and other enumerative techniques of mixed integer programming (MIP). We have chosen a hybrid (HMIP) approach with some heuristics included. In this case, the PDP-TWDS is formulated as a mixed integer linear program that is solved by a cutting plane algorithm. We model a linear relaxation of the original problem resulting in a master problem solved very efficiently by any MIP solver. The relaxation of the problem corresponds to the dock capacity service constraints imposed on each node and at each hour of the day. An integer feasible solution is obtained for time windows constraints on all nodes and all vehicles. An iteration procedure is performed to add dock capacity constraints as necessary. We found that our approach is capable of obtaining near-optimal solutions in acceptable computational times for real business instances with up to 50 nodes and 400 transportation orders. We present our model in 3 stages.

4.1 Relaxed Capacitated Vehicle Routing Problem (C-VRP) Model

Here we assume different vehicles capacities that are initially located at different nodes (i.e. depots). However customers have no specific service time windows constraints. In other words the service can be cover at any time. The objective is to find an optimal cost solution that completes all the transportation workload orders at aggregate level taking in mind vehicle cubic capacity constraints, vehicle compatibility constraints and 24-hours of operation per vehicle per day constraints. One of the main features of this relaxed C-VRP model is to identify an optimal assignment for the vehicles to cover the all the transportation orders. This means to identify if one vehicle m1 is going to be grouped (hooked) to another vehicle m2. The regular case is when we operate a single trailer with just one haul. However, in our model when we group a vehicle m1 with a vehicle m2, as a result we obtain physically one vehicle with a new summed capacity. This is a double trailer case, in other words, a vehicle operating with two hauls. We model as follows:

Sets and Parameters:

$N =$ *set of nodes on the network (i.e. plants, distribution centers or customers)*

$R =$ *set of transportation orders to satisfy. Includes product from i to j & empty bottles from j to i*

$M =$ *set of vehicles (trailers).*

$K =$ *set of SKUS. Including regular products and returnable empty bottles.*

$Pi =$ *subset of vehicles located at node i,*
$\forall i \in N, Pi \subseteq M$

$A_{ij} =$ *subset of compatible vehicles m to be used on arc (i,j)*
$\forall (i,j) \in N, m \in M, Aij \subseteq M$

$ST_{ij} =$ *transportation time (minutes) on arc (i,j) on single trailer*
$\forall (i,j) \in N$

$FT_{ij} =$ *transportation time (minutes) on arc (i,j) on double trailer*
$\forall (i,j) \in N$

$SC_{ij} =$ *transportation cost on arc (i,j) on single trailer*
$\forall (i,j) \in N$
$FC_{ij} =$ *transportation cost on arc (i,j) on double trailer*
$\forall (i,j) \in N$
$D_{ijk} =$ *planned demand (cases) from node i to j of SKU k,*
$\forall (i,j) \in N, (i,j,k) \in R$
$H_k =$ *quantity of cases of SKU k per cubic meter,*
$\forall k \in K$
$Q_m =$ *quantity of cubic meters on vehicle m,*
$m \in M$

Decision Variables:

- $X_{ij}^{\ m1,m2} \geq 0$, *integer* $\Rightarrow$ *# of trips from node i to j using vehicle (m1,m2),* $\forall (i,j) \in R, (m1,m2) \in Aij$
- $F_{ijk} \geq 0$, $\Rightarrow$ *quantity of cases to transport from node i to j of SKU k,* $\forall (i,j,k) \in R$
- $W_{m1,m2}$ *binary* $\Rightarrow$ *(1) if vehicle m1 is linked to vehicle m2, (0) otherwise,* $\forall (m1,m2) \in Pi$

The C-VRP relaxed can be formulated as the following mixed integer model:

$$(CVRP\ Relaxed)\ \ Minimize \sum_{i \in N} \sum_{j \in N} \left[\sum_{(m1=m2) \in M} X_{ij}^{m1,m2} \cdot SC_{ij} + \sum_{(m1 \neq m2) \in M} X_{ij}^{m1,m2} \cdot FC_{ij} \right]$$

Subject to:

$$\sum_{m2 \in M} W_{m1,m2} \leq 1, \quad \forall\, m1 \in M$$

$$\sum_{m1 \in M,\ m1 \neq m} (W_{m1,m} + W_{m,m1}) \leq 1, \quad \forall\, m \in M$$

$$\sum_{i,j \in N} (1 + ST_{ij}/60) \cdot X_{ij}^{m1,m2} \leq 24 \cdot W_{m1,m2}, \quad \forall\, (m1 = m2) \in M$$

$$\sum_{i,j \in N} (2 + FT_{ij}/60) \cdot X_{ij}^{m1,m2} \leq 24 \cdot W_{m1,m2}, \quad \forall\, (m1 \neq m2) \in M$$

$$\sum_{k \in K} \frac{F_{ijk}}{H_k} = \sum_{\forall\, (m1=m2) \in M} X_{ij}^{m1,m2} \cdot Q_{m1} + \sum_{\forall\, (m1 \neq m2) \in M} X_{ij}^{m1,m2} \cdot (Q_{m1} + Q_{m2})\,, \quad \forall\, (i,j) \in R$$

$$\sum_{i \in N} F_{ijk} - \sum_{h \in N} F_{jhk} \geq \sum_{i \in N} D_{ijk}, \quad \forall\, (j,k) \in R$$

$$\sum_{i \in N} F_{ijk} - \sum_{h \in N} F_{jhk} \leq 15 \sum_{i \in N} D_{ijk}, \quad \forall\, (j,k) \in R$$

$$\sum_{i \in N} X_{ij}^{m1,m2} = \sum_{i \in N} X_{ji}^{m1,m2}, \quad \forall\, j \in N, (m1,m2) \in Pj$$

4.2 Pickup and Delivery Problem with Time Window Constraints (PDP-TW) Model

As a result from the previous model we obtain the optimal assignment of the vehicles. That is, binary variable $W_{m1,m2}$ identify which vehicles is going to operate a single trailer (i.e. with just one haul) and which others will operate on double trailer (i.e. a vehicle operating with two hauls). From here to the end, all double trailers will be modeled as one only vehicle with a summed capacity. Indeed, we can verify on the previous model that integer variable $X_{ij}{}^{m1,m2}$ calculates the optimal quantity of trips required on each final vehicle and on each arc between origin nodes and destination nodes. Our next PDP-TW model is implemented in order to take advantage from the previous information. Thus, on this model we add time windows constraints. We model as follows:

<u>Sets and Parameters:</u>

$R =$	*set of transportation orders to satisfy from node i to j on vehicle m*
$L =$	*set of stops or sequences on a any given route (1..9)*

$X_{ij}{}^{m}$ = # of trips from node i to j using vehicle m,
$\forall$ (i,j,m) $\in$ R
IN_i = opening time at node i,
$\forall$ i $\in$ N
CN_i = closing time at node i,
$\forall$ i $\in$ N
IV_m = opening time of vehicle m,
$\forall$ m $\in$ M
CV_m = closing time of vehicle m,
$\forall$ m $\in$ M
TC_{ijm} = transportation cost on arc (i,j) on vehicle m
$\forall$ (i,j,m) $\in$ R
Z_{ijm} = transportation and loading time on arc (i,j) on vehicle m
$\forall$ (i,j,m) $\in$ R

Decision Variables:

- *$Y_{ij}{}^{ml}$ binary $\Rightarrow$ (1) if vehicle m is routed from node i to j on sequence l, (0) otherwise. $\forall$ (i,j,m) $\in$ R, l $\in$ L*
- *$T_{ij}{}^{ml} \geq 0 \Rightarrow$ arrival time at node j from node i on vehicle m at sequence l, $\forall$ (i,j,m) $\in$ R, l $\in$ L*

The PDP-TW can be formulated as the following mixed integer model:

$$(PDP.TW)\ \ Minimize \sum_{i\in N}\sum_{j\in N}\sum_{m\in M}\sum_{l\in L}\left[Y_{ij}^{ml}\cdot TC_{ijm}+T_{ij}^{ml}\right]$$

Subject to:

$$\sum_{l \in L} Y_{ij}^{ml} = X_{ij}^{m}, \quad \forall\, (i,j,m) \in R$$

$$\sum_{i \in N} \sum_{l \in L} Y_{ij}^{ml} = \sum_{i \in N} \sum_{l \in L} Y_{ji}^{ml}\ , \quad \forall\, j \in N, m \in Pj$$

$$T_{ij}^{ml} \geq IN_i \cdot Y_{ij}^{ml}, \qquad \forall\, (i,j,m) \in R, l \in L$$

$$T_{ij}^{ml} \leq CN_i \cdot Y_{ij}^{ml}, \qquad \forall\, (i,j,m) \in R, l \in L$$

$$T_{ij}^{ml} \geq IV_m \cdot Y_{ij}^{ml}, \qquad \forall\, (i,j,m) \in R, l \in L$$

$$T_{ij}^{ml} \leq CV_m \cdot Y_{ij}^{ml}, \qquad \forall\, (i,j,m) \in R, l \in L$$

$$\sum_{i \in N} Y_{ij}^{ml} \leq 1, \qquad \forall\, (j,m) \in R, l \in L$$

$$\sum_{i \neq j \in N, m \notin P_i} T_{ij}^{ml} + \sum_{i \neq j \in N} \frac{Z_{ijm}}{60} \cdot Y_{ij}^{ml} \leq \sum_{h \neq j \in N, m \notin P_h} T_{jh}^{ml} + \sum_{h \neq j \in N, m \in P_h} T_{jh}^{m,l+1}$$

$$\forall\, j \in N, m \in M, l = 1, m \notin Pj$$

$$\sum_{i \neq j \in N} T_{ij}^{ml} + \sum_{i \neq j \in N} \frac{Z_{ijm}}{60} \cdot Y_{ij}^{ml} \leq \sum_{h \neq j \in N, m \notin P_h} T_{jh}^{ml} + \sum_{h \neq j \in N, m \in P_h} T_{jh}^{m,\{l+1,1\} \in L}$$

$$\forall\, j \in N, m \in M, l \neq 1 \in L, m \in Pj$$

4.3 Pickup and Delivery Problem with TW and Dock Service Constraints (PDP-TWDS) Model

As a result from the previous PDP-TW model we obtain the optimal assignment of the vehicles considering vehicles capacity and time windows constraints as well. That is, binary variable $Y_{ij}{}^{ml}$ identify if a vehicle m is routed from node i to j on sequence l. This is the route sequence for each vehicle. At the same time, positive variable $T_{ij}{}^{ml}$, calculates the arrivals time at each node for all the vehicles. With this in mind, we can proceed now to apply dock service capacity constraints on our final model. Our previous model works as the master model. Then, the logic we apply here is to iteratively generate cuts in a Brach & Cut scheme. For that purpose we identify in the incumbent solution, at each arrival node and at each working hour, the subset of vehicles that are violating the dock service constraint. For that purpose we compare the quantity of vehicles that are being dispatched simultaneously at a given node and at a given hour versus the docks quantity that the node is capable to attend at a given hour. Then we add these cuts to the master model.

The generated cuts are kept in a pool of constraints that are managed separately of the rest of the cuts generated automatically by the B&C scheme. The procedure continues until is found the first optimal solution for the problem that does not violate the dock service capacity on all nodes and at each 24-hour planning day. We model as follows:

Sets and Parameters:

$S_{ih} =$ *quantity of docks at node i at working hour h,* $i \in N, h \in \{1..24\}$

$E =$ *set of cases where a vehicle is violating the dock service constraint at node i at hour h*

Decision Variables for vehicles violating dock service constraint at node i at hour h ($e \in E$):

$B^{+}_{e} \geq 0 \Rightarrow$ *quantity of time (hours) between vehicle α & vehicle β on case e, $e \in E$*

$B^{-}_{e} \geq 0 \Rightarrow$ *quantity of time (hours) between vehicle β & vehicle α on case e, $e \in E$*

U_e *binary $\Rightarrow$ (1) if vehicle α is served before vehicle β on case e, (0) otherwise, $e \in E$*

Subject to:

$$T_{ij}^{ml}(\alpha) - T_{ij}^{ml}(\beta) = B_e^{+} - B_e^{-}, \qquad \forall\, i \in N, \forall(\alpha_{jml}, \beta_{jml}) \in E$$

$$B_e^{+} + B_e^{-} \geq 1, \qquad \forall e \in E$$

$$B_e^{+} \leq 24 \cdot U_e, \qquad \forall e \in E$$

$$B_e^{-} \leq 24 \cdot (1 - U_e), \qquad \forall e \in E$$

As can be verified on the previous model, these constraints grow exponentially as the number of nodes and vehicles are large. Thus we add these constraints on an iterative scheme only as required.

5. Experimental Results and Conclusions

We present some computational results indicating the efficiency of our method for solving large scale instances (50 nodes and 500 transport orders). CPU configuration used for our implementation was Win X64, 2 Intel Cores at 1.4GHz. We implement our model on X-PRESS MIP Solver from FICO (Fair Isaac). Table 1 shows the optimal solutions that we found with our model as we can input different values on two parameters: (1) quantity of vehicles to be considered per arc and (2) quantity of arcs to be considered per vehicle. Basically these two parameters affect the network size to be considered by our model.

If our network is small we can obtain good solutions in short computational times. However, the trade off we have to pay with this strategy is that it is possible we have an over constrained solution space. By the other hand, on the last block of table 1 (i.e. 40, 40), our network size is larger and require more time to be solved. However we obtain as a result better solutions.

Table 1 Optimal solutions found. (1) # of vehicles per arc versus (2) # of arcs per vehicle

# Vehicles per Arc	20		30		40			40
# Arcs per Vehicle	20		30		30			40
LP Solution	221,470		219,011		218,348			218,348
Comput Time (Mins)	% Gap	Best IP Solution	% Gap	Best IP Solution	% Gap	Best IP Solution	% Gap	Best IP Solution
2	10.94%	254,329	8.29%	246,870	6.86%	244,775	+inf	NA
3	9.97%	251,664	6.92%	243,310	5.33%	240,881	8.57%	244,719
5	8.09%	246,555	6.71%	242,775	3.87%	237,317	8.10%	243,485
10	7.91%	246,065	5.72%	240,235	3.76%	237,103	5.57%	236,943

As we can verify on Table 1, we can obtain good solutions in short computational times. As long as we have more time we can improve our solutions. This is true for example when we run our model with a network formulated with up to 40 vehicles per each arc and 40 arcs per each vehicle. Our best solution is obtained in less than 10 minutes. From this solution we move to the next stage that corresponds to 2nd and 3rd model. It is important to consider that these two models were implemented on just one single program. That is, the 2nd model is the master model and the 3rd model runs iteratively adding the cuts to consider dock service constraints as necessary.

As follows, on table 2 we show some useful statistics indicating evidence about how constrained is the dock service capacity on each node. As long as we have more added cuts on a node, this is a clear indication about how many vehicles asking for service are violating the dock service capacity. This information is very interesting and useful for business reasons. This is true, because management can be advised to make some changes on general infrastructure (i.e. open more docks) in order to assure transportation service.

Table 2 Quantity of cuts added on each node

NODE	# Cuts Added
1	33
2	13
4	3
5	28
6	16
14	88
15	20
16	3
19	1
27	3
28	16
31	3
32	1

From practical business application standpoint, this operations research application was developed & implemented to optimize the transportation network between manufacturing plants and distribution centers. During the last years, the firm was interested in developing a better transportation & routing schedules. Indeed, this is the first OR application that has been implemented in the bottler company where we implement this model. It is important to point out that the overall results have been very positive. The firm's top management recognize that features included on the OR model implemented were truly outstanding due to a fine work at a technical level & a practical ease of use as well. The project was a major undertaking, requiring a great deal of thought and effort. The first plans for transportation routes suggested by the optimization model were implemented eight months ago. Throughout the ramp-up and launch of the project, these plans for distribution operation were analyzed and the company found to be an extremely viable idea. Sometime after, during the course of the project, has resulted in a significant increase in productivity and direct savings to the firm. We can list some of the benefits that the company has achieved within this project:

- An increase on effectiveness on the planning process required to set up an efficient transportation & route schedules. The typical fully-manual planning process time was reduced from 6 hours to less than an hour using the new OR application. This permitted to the company to fine its truck capacity by season on a dynamic basis. As a result the company achieves an optimal capacity to attend the demand on each territory and leveling by season.
- Identify an efficient set of activity measures to target & balance on each truck resource to schedule. This results on an optimal fleet of trucks, drivers & warehouse workers.
- Streamline truck capacity to align it to a new transportation strategy. The added throughput allows the firm to defer investments on trucks and infrastructure that were originally allocated. The save on investments for trucks was about 8% of the entire fleet.
- Identify & implement an optimal cost of service. This allowed the firm to set an optimal deliver frequency. This means less travel time between plants and depots and a 14% increase in volume delivered per route per day.

As was verified, our problem considers the schedule of several vehicles simultaneously. As a results some difficulties arises about dock service capacity issues. The problem instances that we found in the business environment are above 150 vehicles and more than 500 transportation orders to schedule and with a high presence of time windows & dock service constraints. Indeed, time windows constraints can be found on the nodes or on vehicles as well. MIP models when are used to solve instances as described, require a strong computational effort in time. This strategy usually compromises its practical implementation in business applications. We proposed on this work a model implementation that offer good quality solutions (i.e. optimality ≥ 90%) in short computational times (i.e. time ≤ 5 minutes). We implemented our model on a MIP formulation with a heuristic on the last stage in order to add dock service capacity constraints on an iterative scheme only as is required. Computational results for a real-world instance with up to 150 vehicles and 500 transportation orders are reported, showing the suitable of our model to provide good quality solutions. Given the current state of the art for the solution of vehicle routing problems with time windows, it seems fair to say that these are large instances.

With respect to the literature on routing and scheduling problems, it is interesting to observe that although PDP are as important as VRP, they have received far less attention. Apart from the vehicle capacity constraints and the intrinsic precedence constraints, time constraints arise in almost every practical pickup and delivery situation. Time constraints play an even more prominent role in PDP-TW. We can point out as follows:

1. Although the single vehicle VRP is NP-hard, it can be solved efficiently as long as the number of transportation requests is relatively small, which is the case in many practical situations. However, the main problem in solving multiple vehicles VRP (i.e. PDP) is the assignment of transportation requests to a set of several vehicles.
2. Moreover, if there are no time constraints (i.e. PDP), finding a feasible pickup and delivery plan is trivial: arbitrarily assign transportation requests to vehicles, arbitrarily order the transportation requests assigned to a vehicle and process each transportation request separately. The presence of time constraints (i.e. PDP-TW) complicates the problem considerably. The problem of finding a feasible pickup and delivery plan is NP-hard.
3. Assigning transportation requests to vehicles in the PDP-TW is much

more difficult than assigning transportation requests to vehicles in the VRP-TW. In VRP-TW, all the origins of transportation requests are located at the depot. Therefore, transportation requests with geographically close destinations are likely to be served by the same vehicle. In the PDP-TW, geographically close destinations may have origins that are geographically far apart and we cannot conclude that they are likely to be served by the same vehicle.

We reported part of the research results we implemented on Embotelladoras ARCA (Coca-Cola) aiming at the optimization of Manufacturing & Transportation operation. One of the individual optimization problems arising here is the task to schedule the operation on a transportation network with several plants and distribution centers. In this case we aim to make an optimization over a full fleet of tractors vehicles. In general, the performance of a method is difficult to compare. Clearly, the diversity of theoretical and practical problems is immense. Consequently, there are not too many papers working on the same problem. Constraints can be different, objective functions can be different. Another possible way to compare a method is in checking the problem size that can solve and the amount of computer time and space it needs. It is clear that future research should be done in order to statistically test our method. This issue will be overcome of the subsequent paper. However the results obtained so far, indicate that our model is robust to solve this hard problem, reaching good solutions in short computational times.

In this paper, we considered a particular PDP application that is frequently encountered in the real-world logistics operations. Our PDP-TWDS problem incorporated a diversity of practical complexities. Among those are a heterogeneous vehicle fleet with different travel times, travel costs and capacity, order/vehicle compatibility constraints, and different start and end locations for vehicles. Instead of assuming that each vehicle becomes available at a one only central depot, we modeled as each vehicle is given a start location where it becomes available at a specific time of the day. Particularly, on our PDP-TWDS extension we add some constraints for dock capacity service at each node and at each hour of the day. The current situation in freight transportation reflects the need for improved efficiency, as the traffic volume increases much faster than the road network grows. Moreover, along with the increasing use of geographical information systems, companies seek to improve their transportation networks in order to tap the full potential of possible cost reduction. Over the last decades extensive research has been dedicated to modeling aspects as well as optimization methods in the field of vehicle routing. Still, there are areas and sub-problems, yet, to be researched.

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