Universidad Autónoma de Nuevo León

Facultad de Ingeniería Mecánica y Eléctrica

División de Estudios de Posgrado

Study of Mixed Integer Programming Models for the Concrete Delivery Problem

por

Oscar Alejandro Hernández López

en opción al grado de

Maestría en Ciencias

de la Ingeniería con Orientación en Sistemas

San Nicolás de los Garza, Nuevo León julio 2020

Universidad Autónoma de Nuevo León

Facultad de Ingeniería Mecánica y Eléctrica

División de Estudios de Posgrado

Study of Mixed Integer Programming Models for the Concrete Delivery Problem

por

Oscar Alejandro Hernández López

en opción al grado de

Maestría en Ciencias

de la Ingeniería con Orientación en Sistemas

San Nicolás de los Garza, Nuevo León julio 2020

UANL

UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA

SUBDIRECCIÓN DE ESTUDIOS DE POSGRADO

Los miembros del Comité de Tesis recomendamos que la Tesis Study of Mixed Integer Programming Models for the Concrete Delivery Problem, realizada por el alumno Oscar Alejandro Hernández López, con número de matrícula 1985273 , sea aceptada para su defensa como opción al grado de Maestría en Ciencias de la Ingeniería con orientación en Sistemas.

El Comité de Tesis

Dr. Vincent André Lionel Boyer
Director

Dra. Sara Verónica Rodríguez Sánchez
Revisor

Dr. Norberto Alejandro Hernández Leandro
Revisor

Vo. Bo.

Dr. Simón Martínez Martínez
Subdirector de Estudios de Posgrado

SUBDIRECCIÓN DE ESTUDIOS DE POSGRADO
UANL
FIME
014

San Nicolás de los Garza, Nuevo León, 6 de julio de 2020

To my parents: for their example and eternal love.

To my grandmother (†): for the way she raised me since I was a little boy.

Contents

List of Figures

List of Tables

Acknowledgements

First of all, I am grateful to my parents who sacrifice a lot for me every day and for their unconditional love.

Thanks to my family, for their support and caring all this time I have been away. All of you are my motivation.

Special thanks to my supervisor Dr.Vincent Boyer, for his guidance, patience, and all the advices, and accepting to be my mentor in this project.

Thanks to the members of the committee for their valuable comments, which allow us to improve this work.

I am grateful to all the professors who shared their knowledge in every class.

Thanks to all my classmates and friends from PISIS, for the help in the homework, for walking alongside me in this path, and all the experiences we shared in these two years.

Finally, thanks to CONACyT for the scholarship, which allowed me to study in the Graduate Program in Systems Engineering at Facultad de Ingeniería Mecánica y Eléctrica (FIME) at Universidad Autónoma de Nuevo León.

Abstract

Oscar Alejandro Hernández López.

Candidato para el grado de Máster en Ingeniería
con especialidad en Ingeniería de Sistemas.

Universidad Autónoma de Nuevo León.
Facultad de Ingeniería Mecánica y Eléctrica.

Título del estudio:

STUDY OF MIXED INTEGER PROGRAMMING MODELS FOR THE CONCRETE DELIVERY PROBLEM

Número de páginas: 58.

OBJECTIVES AND METHODS OF STUDY: The main objective of this research is the study of Mixed Integer Programming (MIP) formulations for the Concrete Delivery Problem (CDP).

Contributions and Conclusions: The main contribution of this thesis is two new compact MIP models. These formulations, based on a graph representation, reduce the number of used variables and restrictions involved in the problem and allow us to solve to optimality more instances than the mathematical models that have been presented in the literature for this problem.

Firma del asesor: ______________________

Dr. Vincent André Lionel Boyer

Chapter 1

Introduction

Concrete is a basic building material used in the construction of commercial and industrial buildings, bridges, roadways, sidewalks, houses, dams, and other structures , for this reason, it has an important role in the construction industry.

Production and dispatch of concrete is a key factor to assure the presence of this product whenever is needed. The coordination of these activities in a network, to guarantee timely delivery to customers is one of the most relevant aspects of the supply chain management (SCM) of concrete. The aim is to achieve the best synchronization in the activities of the actors, in order to reduce operative cost and improve customer service. Furthermore, from the logistic viewpoint, SCM involves a set of complex and interdependent combinatorial problems (i.e. scheduling, vehicle routing, assignments, etc.)

The distribution of concrete is a complex problem in logistics due that this is a perishable product given that it can be in the concrete mixer for a certain amount of time before it loses quality and hardens. Another factor is that a maximum time lag must be considered to assure the correct bonding of the concrete, which is the maximum delay between two consecutive deliveries to the same customer. Besides, failures to deliver concrete on time can result in construction delays or loss of the product if the time threshold for concrete hardening has exceeded. Also, in some cases, when too many truck mixers simultaneously queued up to be unloaded on-site, it can result in wasted time for the operation.

This work focuses on the Concrete Delivery Problem (CDP). The CDP consists in satisfy a set of orders to customers by a fleet of vehicles within a time window.

1.1 Objective

The main objective of this work is to study mathematical formulations for the CDP. It is desired to present them based on the information available and gather in the literature about this problem. A particularly objective is to propose Mixed Integer Programming (MIP) models for the CDP. These models will be tested on a set of instances from the literature to evaluate their performance.

1.2 Scope

This work contributes to the field of rich vehicle problems and their solution procedure. It considers aspects such as time windows, resource capacity, among others; nevertheless is one of a few works that deal with maximum time-lag constraints, which state the delay between two successive operations is bounded by a maximum value.

1.3 Hypothesis

A reformulation of the problem will allow us to solve to optimality more instances than the mathematical models that have been presented in the literature. With a more compact formulation, it is expected that the models find more optimal solutions under the same conditions than others.

1.4 Metodology

In order to develop this work the next methodology will be followed:

1. Literature review related to the CDP.

2. Study of Mathematical formulations of the CDP.

3. Study of different reformulations of the problem.

4. Computational Test with instances from the literature.

5. Analysis of the results.

Chapter 2

Literature Review

2.1 The Vehicle Routing Problem

The CDP is a variant of the well-known Vehicle Routing Problem (VRP). The VRP is defined by a set of vehicles which have to deliver goods to a group of customers; the problem consists in the design of a set of vehicle routes where the customers should be visited exactly once by one vehicle and also these routes must start and end at a depot [18]. The objective is to minimize the total traveling cost which can be achieved by reducing the total traveled distance and/or the number of required vehicles.

Dantzig and Ramser introduced the VRP in [7], and from this seminal work, many variations have been studied inspired by real world problems. These studies tackles new constraints such as vehicle capacity [21, 23, 32], time windows [8, 34], split deliveries [2, 9], backhauls [12, 37], multiple depots [19, 29], stochastic demands [4, 20] among others.

The VRP and its variants are NP-hard combinatorial optimization problems. Hence heuristics are a practical approach to find a solution to the problem, in contrast with exact algorithms that can solve only small instances within a reasonable computational time [6]. According to Golden *et al.* [13] since the 1990s there have been more researches focused on metaheuristics to solve these problems for their efficiency in finding high-quality solutions.

2.2 The Concrete Delivery Problem

Kinable *et al.* [17] states the CDP has a certain similarity with the Capacitated Vehicle Routing Problem with Time Windows and Split Deliveries. Asbach *et al.* [3] also shows that it can be seen as a type of VRP and introduces new combinatorial challenges because of the characteristic of the material itself. Aspects of the production and delivery of concrete can be found in [36].

2.2.1 Exact Solution Methods

To solve this problem, both exact methods and heuristic algorithms have been used. Hertz *et al.* [14] formulated the problem as a Mixed Integer Linear Programming problem, in a two-phase solution method. In the first phase, vehicles are assigned in a set of deliveries to each vehicle of the fleet and the second phase determines the sequence of the deliveries in order to build the vehicle routes. The second phase reaches the optimal solution with the cost of high computing time. Here the authors treat those phases as two subproblems, formulating them as integer linear programming problems, and then combine both phases in a single integer linear program as well.

Lin *et al.* [22] proposed a Mixed Integer Programming (MIP) model. In this paper, the problem is formulated as a job shop problem [1]. Each delivery represents a job, carried out by the trucks that correspond to the machines. Another characteristic of the model is its multiobjective nature, that is to say, the minimization of lateness in orders, the minimization of the vehicle usage, and balancing the utilization rate of trucks.

Zhang y Zeng [38] also define a MIP model based on a network flow model representing each possible delivery to customers, each possible reload of vehicles at the depot, and the starting and end points as a node. Here the objective is to minimize the total transportation costs.

Kinable *et al.* [17] provide a fundamental version of CDP as well as a MIP and a Constraint Programming (CP) model. They modeled the problem as a directed weighted graph which combines two models: the Capacitated Vehicle Routing Problem with Time Windows and Split Deliveries, and the Parallel Machine Scheduling Problem with Time Windows and Maximum Time Lags. With this MIP model, the authors propose a core problem to promote further investigation and provide a set of test instances. Hence, we base our study on the CDP as defined by these authors.

2.2.2 Heuristic Approaches

In order to respond to the dynamic factors in deliveries of concrete [10] i.e, uncertainties in transportation times, the demand of the customers, the emergence of new customer demands, vehicle and depot malfunctions, weather variations, traffic conditions, etc. fast algorithms are demanded so they can be used for constant revisions. Here heuristics can work as an important decision tool.

Liu *et al.* [25] developed a heuristic that integrates the Ready Mixed Concrete production scheduling with dispatching of vehicles. Here the model deals with three kinds of vehicles (trucks, pumps, and mixers) each of them with a different function in the process. In order to plan the sequence of visited construction sites, the authors follow a set of priority rules. The first one is to schedule the visits according to the smallest starting time of the time window associated with the unvisited sites and the second rule is based on the smallest ending time of the time window among unvisited sites. The rules for truck dispatching consist in selecting the available truck which capacity is the nearest to the unsatisfied demand of the site, in case all trucks are busy, the earliest available is selected. In order to select the mixer, the first rule is to pick the one with the highest mix rate, and second, the selection is done according to the lowest production cost per cubic meter.

Matsatsinis [27] also presents a heuristic algorithm to schedule the routes of pumps to construction sites. This algorithm consists of an initial assignment of pumps that have been established as a starting point depending on the pump, the customer, and the departure time from the depot. For each route, the earliest working time is determined according to the pumps availability and customer readiness and depending on the execution ability each route is designed as valid or non-valid for the cycle. Then from the valid routes, the one with the minimum working time is chosen. If in the current solution there is an unserved customer by a pump the last routing of the first cycle and first routing of the second cycle are canceled. However, when a route is canceled, all the valid combinations of the pump-route assignment is checked in order to find a feasible one. The algorithm is executed sequentially until all the orders are satisfied.

Kinable *et al.* [17] also propose a constructive procedure, which schedules the visits to customers, following a best-fit policy. The heuristic algorithm schedules customers one-by-one, according to the starting time of the visits and the vehicle capacity. It iterates over all customers and fixes the starting time of the visits according to the travel time, the maximum time lag to the previous visit, and the customers' deadline and assigns it to the earliest available vehicle. For this strategy 3 ordered selection steps are followed. First, the earliest available vehicle is selected. Second, the vehicle has to minimize the surplus amount of concrete, respect to the demand of the client, that it will deliver. Last, in the case of a draw, the vehicle is selected according to the higher capacity.

As a result, different solutions can be found for different permutation of customers when this procedure is executed several times, hence the authors propose a Steepest Descent heuristic as a local search procedure to modify the customer's vector. The criteria of the initial ordering are by the customer's earliest deadline, the highest demand, and the earliest release time of the customer date. All possible shifts of a position of a customer within the sequence are considered (full neighborhood search) at each iteration. With this Steepest Descend heuristic, the authors required notable less time to solve instances in comparison to other solution methods applied.

Also, a local search approach is used by Asbach *et al.* [3], which starts with an incumbent solution and a neighborhood operator which destroys partially the solution and repairs it using a black-box solver. Here the authors consider the canceled demand due to insufficient resources as a key factor in the quality of a solution.

Evolutionary Algorithms

Within the heuristic approaches in the literature Genetic Algorithms (GA) have been widely used to solve the CDP, although the representation of the problem has several differences. Naso *et al.* [30] used the GA to perform the assignment of demand-to-production center, and the production sequencing at each center, while the remaining part of the scheduling problem is handled by constructive heuristic algorithms.

In this paper, the chromosome encoding contains the number of demands (requests), which are the chromosome elements. It is performed in two parts: the first one defines the depot to which request is assigned, where each gene is an integer between 1 and the number of depots. So in this part, it is decided if the order i is produced at the depot d. The second part indicates the order in which the requests will be considered in the scheduling, with integers representing the number of the request. Then a heuristic procedure is used to decode the chromosome which assigns iteratively the orders to nearby clients. Computational results were obtained using data provided by a concrete supplier company. In order to test this algorithm, the authors compare it with four different assignment criteria, suggested by the experts, which lead to different schedules. In every cases, the total cost of the proposed GA could be diminished in ranges of 22-48% compared to the other four policies.

Liu *et al.* [24] presented a MIP model to formulate the problem, and a GA is proposed to solve the integrated scheduling model. In this GA the chromosomes contain three parts: the first part is characterized by the sequence of clients IDs served by the plant, the second part is the sequence of the accumulative number of vehicles to the client, and the third part is the permutation of vehicles IDs dispatched to the corresponding client of the first part. In order to evaluate the performance of the proposed algorithm, the authors applied different combinations of priority rules for production and vehicles and recorded the respective traveling costs. In all cases, the GA outperforms costs of these combinations in ranges of 4.7-6.7%.

Maghrebi *et al.* [26] also used a GA for the solution of a CDP. The proposed structure of the solution consists of two parts. The first part consists of the depot allocation where a sequence of integers indicates the depot where a client will be served from. The second part concerns the truck allocation and is characterized by integers k that will serve the client i. In order to evaluate their algorithm, they compare it with a random solution over a large number of iterations and adjusting the other genetic operators. The author reports average improvements in their GA by reducing operation costs in 39,28% respect to their generated random solution.

Mayteekrieangkrai y Wongthatsanekorn [28] uses a bee algorithm (BA) to optimize the scheduling of trucks from a single plant to multiple sized customers in a large search space using uncertain factors. The solution structure has a length defined by the total number of trucks to be dispatched and shows its dispatching sequence. For example, an instance of three construction sites requiring three, four, and five trucks, would have a solution length of twelve. Then an array of random numbers is generated alongside its corresponding construction site ID, representing each bit of the dispatching sequence. For the decoding process, this array is ordered in ascending order indicating the sequence in which each site ID will be visited. This BA is compared to a GA and showed that the proposed approach outperforms the GA. The authors resume their results with 12 instances with an average of 91,94% for the BA and 55% for the GA respect to the optimal values.

Variable Neighbourhood Search

A Variable Neighbourhood Search (VNS) [11] has been applied by Payr y Schmid [31]. Vehicles are assigned at random at first, as a biased solution. With a shaking operation, an order of the vehicles is replaced according to the random solution. Then the neighborhoods are constructed by increasing the number of disrupted orders and by a replacement strategy of the vehicles not being used. The local search includes moving and swapping vehicles between orders. This approach was applied in a real-world scenario within a period of 12 days with an average improvement in the operation costs of 23.63%.

A VNS is also proposed by Schmid *et al.* [33]. In this case, after an initial feasible solution is found using this metaheuristic, it is improved using a Very Large Neighbourhood Search. A solution consists of a sequence of trucks per order. These sequences are modified in the shaking process and improved in iterative steps. The authors tested their approaches in real case scenario of a concrete company, highlighting the strength of the VLNS in medium-sized real-world test instances, with values of 12% of average gap respect to the computed lower bounds.

Article	Vehicle Fleet	Objective	Solution method	Time Windows	Outsourcing
Matsatsinis [27]	Heterogenous	Minimize the distribution cost.	Heuristic	Soft	Not considered
Asbach *et al.* [3]	Heterogenous	Minimizes simultaneously the total sum of travel costs of edges used by any vehicle and penalty costs for customers whose demand is not fully satisfied.	Heuristic, Local Search	Hard	Not considered
Naso *et al.* [30]	Homogenous	Minimize distribution cost	Metaheuristic (Genetic Algorithm)	Soft	Considered
Payr y Schmid [31]	Heterogenous	Minimize distribution cost	Metaheuristic (Variable Neighbourhood Search)	Soft	Considered
Schmid *et al.* [33]	Heterogenous	Minimize total cost, and penalties for delays between any two consecutive unloading perations	Metaheuristic (Variable Neighbourhood Search)	Soft	Not considered
Lin *et al.* [22]	Homogenous	Minimize the total lateness of RMC	Goal programming	Hard	Not considered
Hertz *et al.* [14]	Heterogenous	Minimize the used vehicles	Integer Linear programming	Soft	Not considered
Maghrebi *et al.* [26]	Homogenous	Minimize travel distance	Metaheuristic (Genetic Algorithm)	Hard	Not considered
Liu *et al.* [24]	Heterogenous	Minimize the total cost of plants and construction sites	Metaheuristic (Genetic Algorithm)	Hard	Not considered
Zhang y Zeng [38]	Homogenous	Minimizing the operating cost of each vehicle.	Hybrid (Heuristic algorithm with Mixed Integer Programming)	Hard	Not considered
Kinable *et al.* [17]	Heterogenous	Maximize number of satisfied customers.	Constraint Programming, Heuristic	Hard	Not Considered

Table 2.1: Summary of the CDP revised models.

The term outsourcing is indicating if it considered hiring new trucks in case an increase in fleet capacity is needed due to unsatisfied customers. A hard time window is defined when deliveries to the customers cannot be performed outside the interval limited by the starting and ending times. On the other hand, the presence of a soft time window implies violation in the intervals is permitted but it has to be charged as a penalty in the objective function.

As shown in Table 2.1, mostly metaheuristic has been used to solve the CDP ant its variants.

Indeed the main advantages of metaheuristics are the reasonable computation times they spend in reaching feasible solutions which may draw near-optimal results. This benefits the response to industrial real-world scenarios that requires providing good solutions in relatively short times satisfying technical constraints.

This work focuses on the CDP defined by Kinable *et al.* [17]. Indeed, to the best of our knowledge, it is the variant of the problem that contains most-real world constraints (see Table 2.1). Besides the instances of this paper are available online and can be used to evaluate the performance of our models.

Chapter 3

Mathematical models

From the literature review, the model proposed by Kinable *et al.* [17] is the one that describes the problem in a more general way than the others. Nevertheless, it could be possible to incorporate new restrictions if needed. This model is based on Mixed Integer Programming and it is described in this chapter. For clarity, we will use the same notation of Kinable *et al.* [17].

In the CDP, each construction site (customer) $i \in C$ requests d_i amount of concrete. The material is transported by a heterogeneous set of trucks K, each one with a capacity of q_k, $k \in K$. The trucks start their trips at a source depot and at the end of a day they return to a sink depot which may or may not be the same as the start.

Each construction site has a time window $[a_i, b_i]$ associated, this is the time interval the concrete must be delivered. There is also a maximum time lag between consecutive deliveries to the same customer, and it can be defined as the maximum time a client can wait before its next delivery is performed. The parameters which define the CDP are shown in Table 3.1

3.1 Mixed Integer Programming Model for the CDP

3.1.1 Parameters of the CDP

Parameter	Description
P	Set of concrete production sites
C	Set of customers
K	Set of Vehicles
d_i	Requested amount of concrete by customer $i \in C$
q_k	Capacity of truck $k \in K$
p_k	Time required to empty the vehicle $k \in K$
$[a_i, b_i]$	Time window during which the amount of concrete may be delivered to customer $i \in C$
t_{ij}	Time to travel from i to j
γ	Maximum time lag between consecutive deliveries.
$0, n+1$	start and end depots of the trucks respectively
V	Vertex set $V = P \cup C \cup \{n+1\}$

Table 3.1: Parameters defining the CDP

The following model is proposed by Kinable *et al.* [17]. It was taken as a guideline in order to understand the problem. To model the CDP, the authors define for each customer $i \in C$ an ordered set, consisting of deliveries, $C_i = \{1, ..., n(i)\}$. Here $n(i) = \dfrac{d_i}{\underset{k \in K}{min}(q_k)}$ will determine the maximum number of deliveries to customer i. Also, c_i^j will denote delivery j for customer i. Each delivery $u \in C_i, i \in C$ has an associated time window $[a_u, b_u]$. Furthermore, $D = \underset{i \in C}{\cup} C_i$ constitutes the union of all deliveries.

The problem is modeled on a directed weighted graph $G(V, A)$, with the vertex set defined as $V = \{0\} \cup D \cup \{n+1\}$. Vertices 0 and $n+1$ are the initial and final depots respectively.

The arc set A is defined as:

- The initial and final depots have outgoing and incoming edges respectively to/from all other vertices.
- A delivery c_h^i has a directed edge to a delivery node c_j^i if h < j, $i \in C$, $h, j \in C_i$.
- There is a directed edge from c_u^i to c_v^j, $i \neq j$ except if, c_v^j needs to be scheduled earlier than c_u^i

The arc costs are:

- $c_{0,c_j^i} = \underset{p \in P}{min}\ t_{0,p} + t_{p,i}\ \forall\ c_j^i \in D.$
- $c_{c_u^i,c_v^j} = \underset{p \in P}{min}\ t_{i,p} + t_{p,j}\ \forall\ c_u^i, c_v^j \in D, c_u^i \neq c_v^j.$
- $c_{c_j^i,n+1} = t_{i,n+1}.$
- $c_{0,n+1} = 0.$

δ^- and δ^+ are the incoming and outgoing neighborhood sets respectively.

Besides, the authors use the following decision variables:

- y_i is a binary variable, indicating whether customer $i \in C$ is serviced.
- x_{ijk} is a binary variable, indicating whether the vehicle $k \in K$, travels from i to j, $i, j \in V$.
- C^i record the time that a delivery $i \in D$ is completed.
- C^{n+1} records the total makespan.

The model proposed by Kinable *et al.* [17] is then defined as follows::

$$max \sum_{i \in C} d_i y_i$$

$$\sum_{j \in \delta^+(0)} x_{0jk} = \sum_{i \in \delta^-(n+1)} x_{i,n+1,k} = 1 \qquad \forall k \in K \tag{3.1}$$

$$\sum_{j \in \delta^-(i)} x_{jik} = \sum_{j \in \delta^+(i)} x_{i,j,k} \qquad \forall i \in D, k \in K \tag{3.2}$$

$$S(i,1) \leq 1 \qquad \forall i \in D \tag{3.3}$$

$$S(j+1,1) \leq S(j,1) \qquad \forall i \in C, j \in \{1, ..., n(i)-1\} \tag{3.4}$$

$$\sum_{j \in C_i} S(j, q_k) \geq d_i y_i \qquad \forall i \in C \tag{3.5}$$

$$C^i - M(1 - x_{ijk}) \leq C^j - p_k - c_{ij} \qquad \forall (i,j) \in A, i \neq 0, k \in K \tag{3.6}$$

$$C^i - M(1 - x_{ijk}) \leq C^j - c_{ij} \qquad \forall (0,j) \in A, k \in K \tag{3.7}$$

$$C^i - S(i, p_k) \geq a_i \qquad \forall i \in D \tag{3.8}$$

$$C^{j+1} - S(j+1, p_k) - C^j \leq \gamma \qquad \forall i \in C, j \in \{1, ..., n(i)-1\} \tag{3.9}$$

$$C^{j+1} \geq C^j + S(j, p_k) \qquad \forall i \in C, j \in \{1, ..., n(i)-1\} \tag{3.10}$$

$$a_i \leq C^i \leq b_i \qquad \forall i \in V \tag{3.11}$$

$$x_{ijk} \in \{0,1\} \qquad \forall (i,j) \in A, k \in K \tag{3.12}$$

$$y_i \in \{0,1\} \qquad \forall i \in C \tag{3.13}$$

Where S(i,α) = $\sum_{k \in K} \sum_{j \in \delta^+(i)} \alpha x_{ijk}$ for all $i \in D$.

Constraints (3.1) specifies the starting and ending location of a tour. Constraints (3.2) assures flow preservation and Constraint (3.3) indicates a node should be visited at most once. In addition, Constraints (3.4) and (3.9) establishes the maximum time lag between two consecutive deliveries. Constraints (3.4) indicates precedence relationship between consecutive visits to a customer. Constraints (3.5) guarantee that the customer should be satisfied if visited. Constraints (3.6-3.11) assure time consistency and time windows satisfaction.

3.2 The Compact Model

A new formulation of the problem is proposed as follows, seeking to reduce the number of variables involved in the problem. The main idea of this new model is to discretize the flow by vehicle type instead of by vehicle as in the model Kinable *et al.* [17]. Hence, we define the set of types of vehicles $T = \{1, 2, ..., t\}$ where a group of vehicles with the same unloading time and capacity fall in the same type.

Here x_{ijt} is a binary variable, indicating whether the vehicle type t, travels from i to j, $i, j \in V$.

g_t is the number of vehicles of type t, $t \in T$

With this new set and P, C and V defined as before the previous model can be modified, resulting in a more compact model keeping the same objective function.

The Compact model obtained is:

$$max \sum_{i \in C} d_i y_i$$

$$\sum_{j \in \delta^+(0)} x_{0jt} = g_t \qquad \forall t \in T \tag{3.14}$$

$$\sum_{i \in \delta^-(n+1)} x_{i,n+1,t} = g_t \qquad \forall t \in T \tag{3.15}$$

$$\sum_{j \in \delta^-(i)} x_{jit} = \sum_{j \in \delta^+(i)} x_{ijt} \qquad \forall i \in D, t \in T \tag{3.16}$$

$$\sum_{t \in T} \sum_{j \in \delta^+(i)} x_{ijt} = y_i \qquad \forall i \in C \tag{3.17}$$

$$\sum_{t \in T} \sum_{j+1 \in \delta^+(i)} x_{i,j+1,t} \leq \sum_{t \in T} \sum_{j \in \delta^+(i)} x_{ijt} \qquad \forall i \in C, j \in \{1, ..., n(i) - 1\} \tag{3.18}$$

$$\sum_{t \in T} \sum_{j \in \delta^+(i)} q_t x_{ijt} \geq d_i y_i \qquad \forall i \in C \tag{3.19}$$

$$C^i - M(1 - x_{ijt}) \leq C^j - p_t - c_{ij} \qquad \forall (i, j) \in A, t \in T \tag{3.20}$$

$$C^i - \sum_{t \in T} \sum_{j \in \delta^+(i)} p_t x_{ijt} \geq a_i \qquad \forall i \in D \tag{3.21}$$

$$C^{j+1} - \sum_{t \in T} \sum_{j \in \delta^+(j+1)} p_t x_{j+1,l,t} - C^j \leq \gamma \qquad \forall i \in C, j \in \{1, ..., n(i) - 1\} \tag{3.22}$$

$$C^{j+1} \geq C^j + \sum_{t \in T} \sum_{l \in \delta^+(j)} p_t x_{jlt} \qquad \forall i \in C, j \in \{1, ..., n(i) - 1\} \tag{3.23}$$

$$a_i \leq C^i \leq b_i \qquad \forall i \in V \tag{3.24}$$

$$x_{ijt} \in \{0, 1\} \qquad \forall (i, j) \in A, t \in T \tag{3.25}$$

$$y_i \in \{0, 1\} \qquad \forall i \in C \tag{3.26}$$

Constraints (3.14) and Constraints (3.15) specify the starting location of a tour and the ending location for vehicles type t respectively. Constraints (3.16) stands for flow preservation. Constraints (3.17) state that nodes most be visited once. Constraints (3.18) assure visit order. Constraints (3.19) indicate customers demand must be covered. Constraints (3.20) maintain time consistency for every travel. Constraints (3.21) and (3.24) assure the time window satisfaction. Constraints (3.22) denote the maximum time lag for consecutive deliveries to the same customer and Constraints (3.23) assure that succesive deliveries to the same customer does not overlap in time. Constraints (3.25) and (3.26) represent the variable domains.

3.3 The Compact Model Modified

The Compact model is sensitive to a few more changes that may directly impact its performance because the solver can take advantage of modifying the restriction of flow preservation, dividing it into two clique type constraints. This is considered the second proposed model in this work.

This strategy consists of modeling this routing problem introducing the new variable w_{ti} to the compact model, which is a binary variable indicating if the vehicle of type t travels to customer node i.

The model obtained is:

$$max \sum_{i \in C} d_i y_i$$

subject to Constraints 3.14 - 3.15 and:

$$\sum_{j \in \delta^-(i)} x_{jit} = w_{ti} \qquad \forall i \in D, t \in T \tag{3.27}$$

$$\sum_{j \in \delta^+(i)} x_{ijt} = w_{tj} \qquad \forall i \in D, t \in T \tag{3.28}$$

$$\sum_{t \in T} \sum_{j \in \delta^+(i)} q_k w_{ti} \geq d_i y_i \qquad \forall i \in C \tag{3.29}$$

$$C^i - M(1 - x_{ijt}) \leq C^j - p_t - c_{ij} \qquad \forall (i,j) \in A, t \in T \tag{3.30}$$

$$C^i - \sum_{t \in T} \sum_{j \in \delta^+(i)} p_t w_{ti} \geq a_i \qquad \forall i \in D \tag{3.31}$$

$$C^i - \sum_{t \in T} \sum_{j \in \delta^+(i)} p_t w_{tj} - C^j \leq \gamma \qquad \forall i \in C, j \in \{1, ..., n(i) - 1\} \tag{3.32}$$

$$w_{t,i+1} \geq w_{ti} \qquad \forall t \in T, i \in \{1, ..., n(i)\} \tag{3.33}$$

$$x_{ijt} \in \{0, 1\} \qquad \forall (i,j) \in A, k \in K \tag{3.34}$$

$$y_i \in \{0, 1\} \qquad \forall i \in C \tag{3.35}$$

$$w_{ti} \in \{0, 1\} \qquad \forall i \in C, t \in T \tag{3.36}$$

Constraints (3.27) and (3.28) stands for flow preservation. Customer's demand must be covered by the sum of capacities of the trucks that will perform the deliveries to a customer Constraints (3.29). The time consistency is expressed in Constraints (3.30). Furthermore, Constraints (3.31) ensures a delivery must be made within the time window. Constraints (3.32) implement the maximum time lag. Constraints (3.33) prevent the overlap in time for deliveries to the same customer. In addition, Constraints (3.34 - 3.36) define the nature of the variables.

3.4 Bounds

Upper bounds are obtained by solving the LP relaxation of the models. Moreover, a number of cuts are added in order to improve the convergence to the optimal solution. Kinable *et al.* [17] propose to use the following cuts:

1. For every pair of customers $i, j \in C, i \neq j$, set $y_i = y_j = 1$ and $y_v = 0$, $\forall$ $v \in C$,$v \notin \{i, j\}$
2. Solve the MIP model and whenever it turns out infeasible the inequality $y_i + y_j \leq 1$ may be added to the model, meaning both customers i and j could not be satisfied with no schedule.

As mentioned by Kinable *et al.* [17], cuts can be generated with greater cardinality instead of pairs, however, for the cardinality of the subsets greater than 3, the generation of these cuts are computationally intractable. IBM's Ilog Cplex Solver version 12.8 is used to compute the LP relaxation once the cuts were added, strengthening bounds of the resulting model. Here, cuts are limited to two and three as suggested by Kinable *et al.* [17].

Chapter 4

Experimental Results

4.1 Instances description

The benchmark data that will be used are available at J. Kinable [15]. There are two data sets. Data Set A contains instances with 10-20 customers and 2-5 vehicles, being the smaller ones. Data Set B has up to 50 customers and 20 vehicles. The authors give their computational results in [17], and provides an upper bound on the optimal solution values, the objective value, the gap between the objective value and the bound for the solution methods they proposed.

In order to evaluate the effectiveness of the proposed Compact Models of this work, a comparison of the results for each instance will be given, with the objective of decrease the existing gap between the upper bound and the objective value.

4.2 Results of the Compact Models

First of all, it is important to highlight that the MIP results shown in Kinable *et al.* [17] are influenced by taking an initial solution of their heuristics and by cuts added. Besides, the best results reported by the authors were achieved by our proposed Compact Models (Compact Model and Compact Model Modified) described in the previous chapter.

The CP Model of Kinable *et al.* [17] was not modeled, consequently, it is not analyzed respect to our proposed Compact Models neither in terms of computation times nor the number of used variables and restrictions.

4.2.1 Results from Dataset A

In the considered aspects in which the Compact Models were compared, experiments were performed by testing the proposed models with the best results presented in Kinable *et al.* [17] which corresponds to their Constraint Programming (CP) Model.

A boxplot of the effect of the number of customers in the objective value is presented in Figure 4.1. The horizontal axis represents the number of customers and the vertical one represents the corresponding best solution found. For every instance, a time of 300 seconds was set as stopping criteria. For 5 customers the optimal value is obtained for every instance in the three models. For the group of 10 customers, and especially in the case of 4 vehicles, there is a slight difference for the models, imperceptible in the figure. This is because of an Instance in which the first of our Compact Models could not reach the optimal value. In the case of customers of size 15 and 20, a better performance is clearly observed in the CP Model of Kinable *et al.* [17].

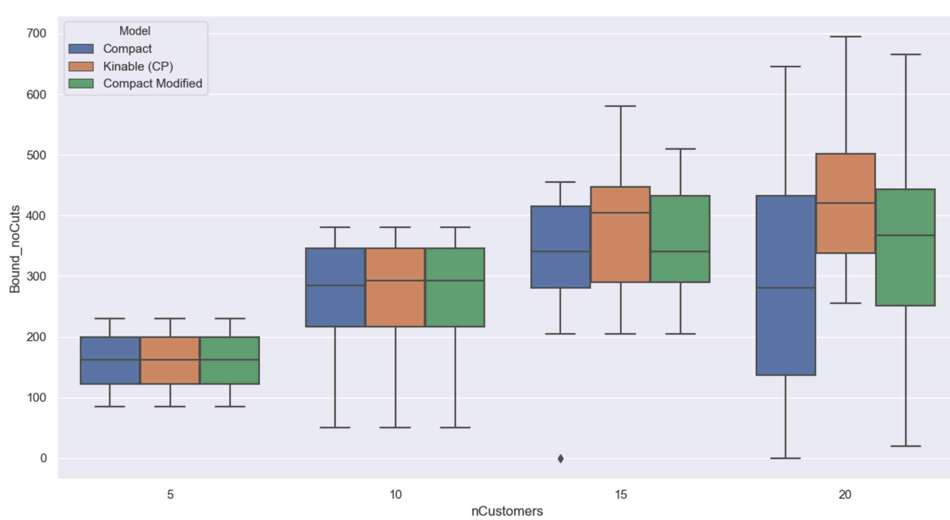


Figure 4.1: Boxplot of objective values respect to the number of customers in all models

Within the instances of 5 customers, a difference in the proposed models is observed in computation times. Although for these small instances, both models achieved optimality, Compact Model Modified converge faster than the Compact Model with an average of 0,021 sec respect to 0,030 sec. In Figure 4.2, one can see a graphical representation of how the Compact Model Modified is able to solve instances in shorter computational time. In this lineplot, the horizontal axis represents the instances of 5 customers and the vertical one the computation time when both models reach optimal values.

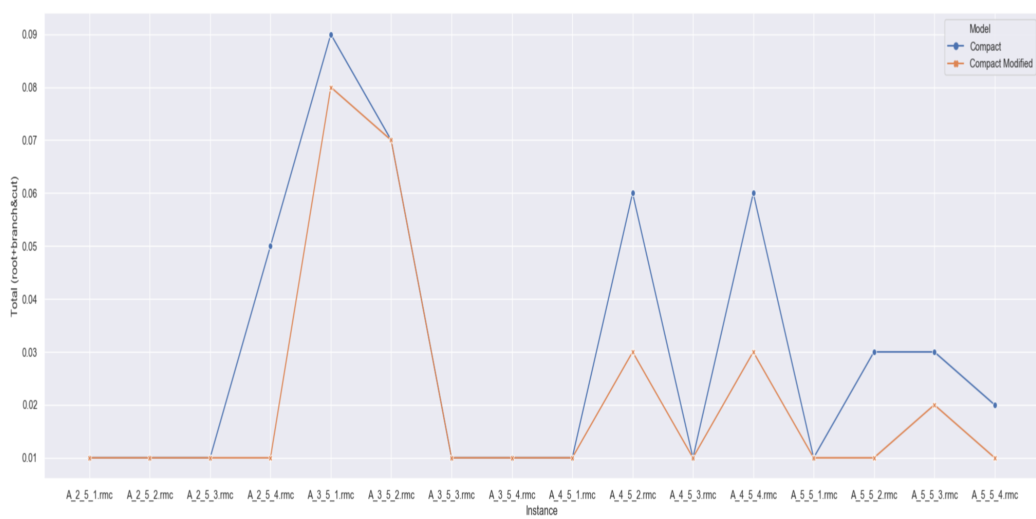


Figure 4.2: Execution times of Instances of 5 customers

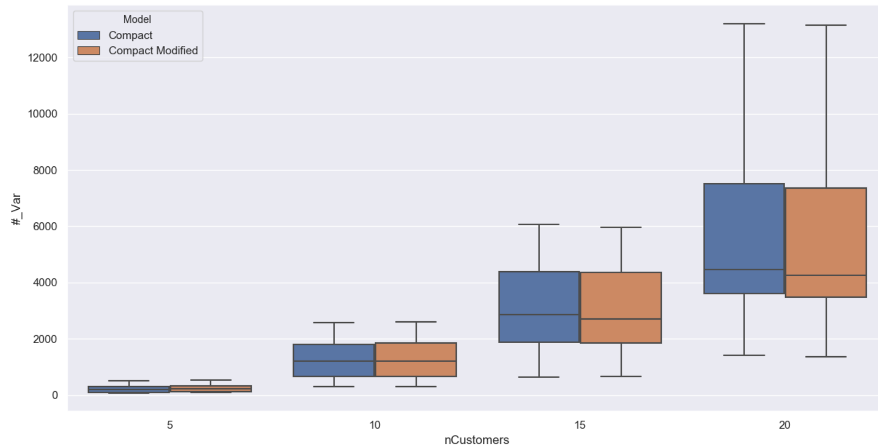
Table 4.1 shows average values of computation time for the other class of instancees within this Dataset.

	Compact Model	Compact Modified
Customers	AVG time (sec)	AVG time (sec)
10	57.303	16.964
15	184.872	140.636
20	236.155	195.329

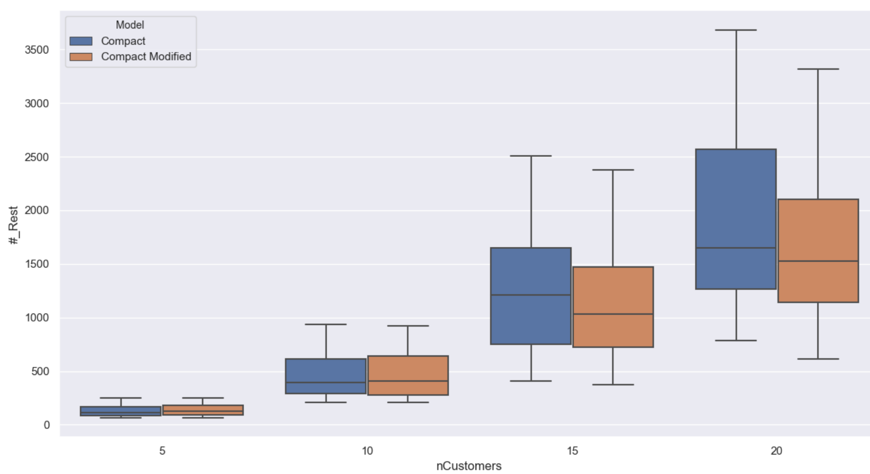
Table 4.1: Average execution times for Instance with 10, 15 and 20 customers

Another important characteristic to analyze is the impact of the number of variables and restrictions. The number of used variables and restrictions of both models are compared and shown in Figure 4.3. The horizontal axis represents each group of customers and the vertical one the number of used variables and restrictions for (a) and (b) respectively.

Compact Models show an overall performance for the number of variables and restrictions as the group of Customers grows, respect to the MIP Model of Kinable *et al.* [17], but differences are clearly seen for the group of 20 customers in both characteristics, and for the group of 15 customers is only appreciable differences in the number of used restrictions.



(a) Boxplot of Number of Variables



(b) Boxplot of Number of Restrictions

Figure 4.3: Boxplots of Number of Variables and Restrictions of the models

Further analysis is shown in Figure 4.4, which allows us to see the relationship between these characteristics. The number of used restrictions in the model are in the horizontal axis whereas the number of used variables are in the vertical one. It can be concluded than in Compact Models although the number of variables grows rapidly with the size of Instances the number of restrictions does not increase as fast as its size, contributing to its overall performance on the largest instances.

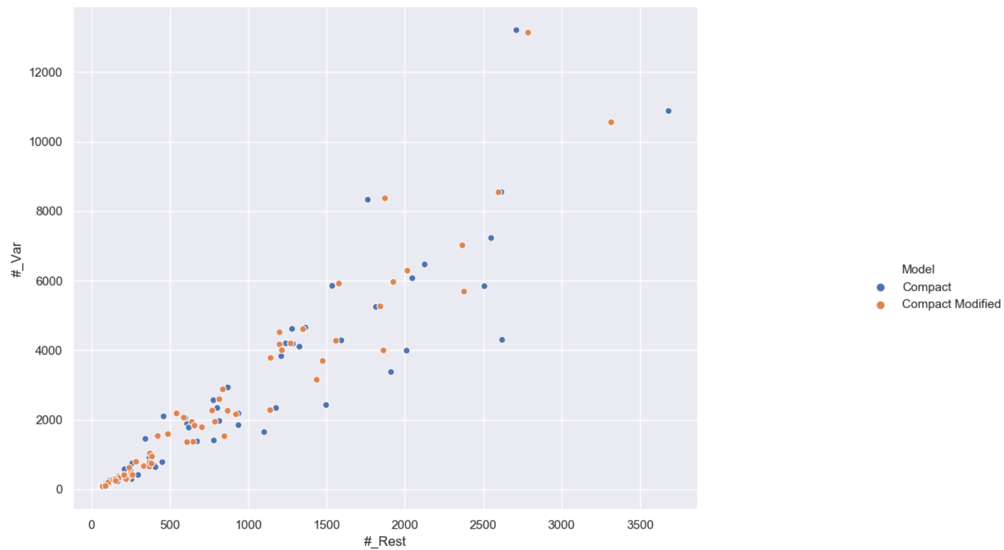


Figure 4.4: Relation of the Number of Variables and Restrictions in all Models with Dataset A

Figure 4.5 represents the number of optimal values reached by each model out of the 64 instances of this dataset. The Compact Model is able to achieve 41 optimal values with an average gap of 17,39%. The CP Model of Kinable *et al.* [17] reached 40 optimal values with an average gap of 4.15% and the Compact Model Modified is capable of reaching 47 optimal values with an average gap of 10.81%. Those contrasting results in gaps are due to differences in the upper bounds of the different models.

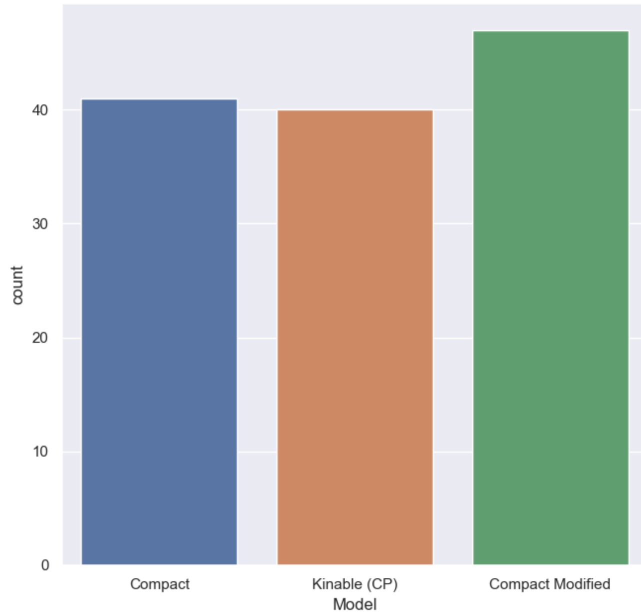


Figure 4.5: Number of reached optimal values by models in Dataset A

Adding cuts

Both proposed models were tested by adding cuts of size two and three and solving the respective MIP Models, in order to strengthen the bounds; then they were compared again with the method which shows best results in Kinable *et al.* [17], the CP Method. This results are shown in Figure 4.6. In this boxplot horizontal axis represents the number of customers and the vertical one represents the obtained objective values of models after the bounds being strengthened.

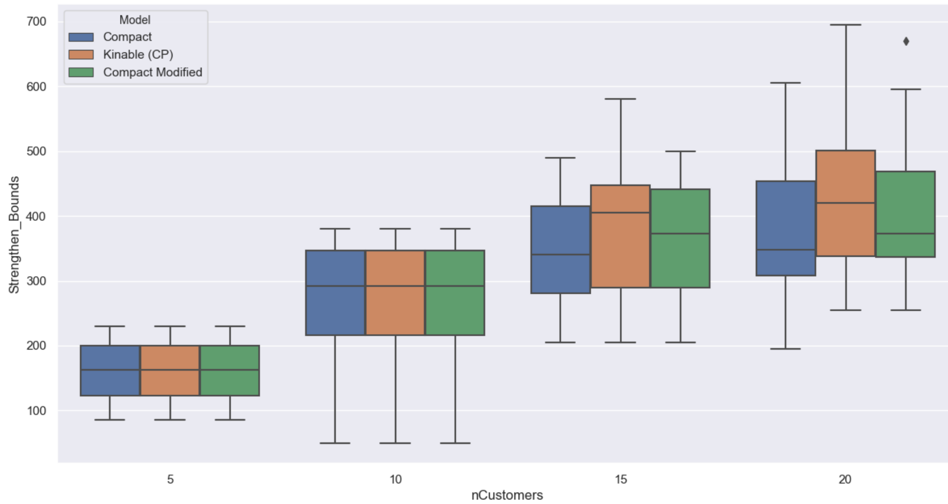


Figure 4.6: Bounds of the strenghten models with Dataset A

The Compact Model is strengthened by adding cuts of size two and three at the same time, and the Compact Model Modified is strengthened by adding cuts of size two because performs slightly better than with cuts of size three (see Appendix A.2).

With cuts being added the Compact Model reaches 48 optimal values. The impact of this strategy represents an increase of 11% in the instances solved to optimality compared to the previous results. The average gap was also improved by decreasing it to 6.11% from 17.39%.

This action also had enhancements in these factors for the Compact Model Modified. A slight increase in instances solved to optimality from 47 to 50 and the average gap had improvements from 10.81% to a 5.53%.

As a conclusion in the group of instances of 15 and 20 customers, the Compact Model Modified show better results than the Compact Model. On the other hand, they still shows slightly worse results than the CP Model of Kinable *et al.* [17]. Compact Model fell 1.96 % as an average of matching at least the CP Model, whereas this difference is of 1.38% as an average for the Compact Model Modified.

4.2.2 Results from Dataset B

In this section as well as in the previous one all considered aspects in which three models were compared, it is taken as a basis the best results among the solution methods in Kinable *et al.* [17]. It is also important to highlight that these authors do not present MIP results for this class in their paper, claiming that it is ineffective in solving this larger instances. Bounds of the proposed models were compared respect to the best bounds presented by the authors, corresponding to their Constraint Programming method.

There are a total of 128 instances for this Dataset. The results of all models are shown in Figure 4.7. The horizontal axis represents the number of customers and the vertical one represents the corresponding best solution found.

Here the time limit of the solver was increased to 600 seconds due to the increasing size of the instance and to match the stopping criteria of Kinable *et al.* [17].

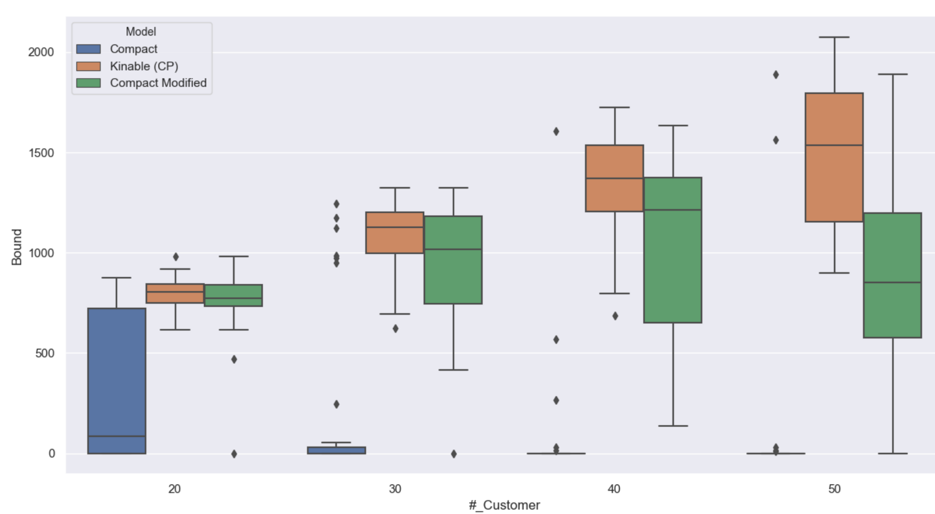
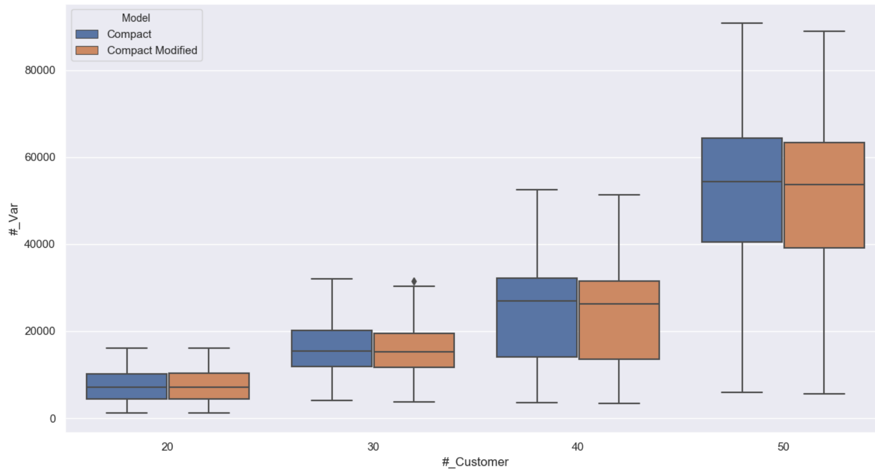


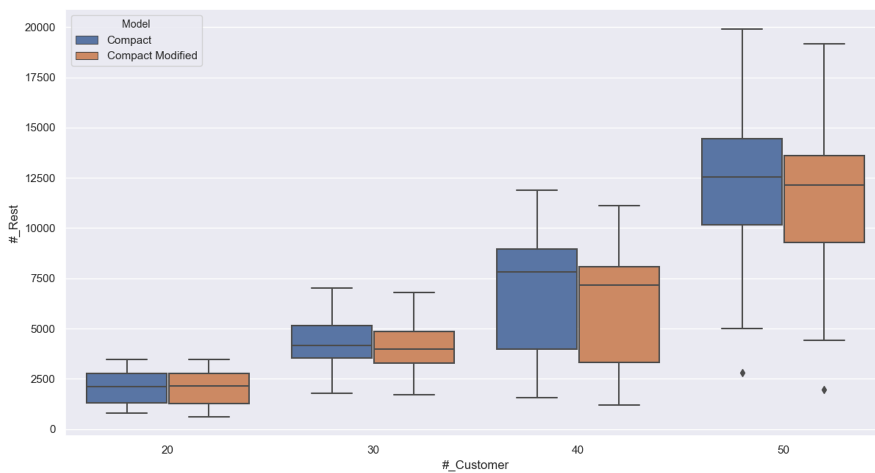
Figure 4.7: Boxplot of objective values respect to the number of customers in the models

Here the Compact model shows low-quality results, especially in instances of 40 and 50 customers, and in 84 cases no customers were visited for this proposed model. Compact Model Modified performs better for this Dataset, being able to achieve greater values of the objective function, meaning more delivered concrete or served customers.

The number of variables and restrictions involved in the proposed models were also analyzed. Results are shown in Figure 4.8. Horizontal axes represent in both subfigures the groups of customers. In (a) the vertical axis represents the number of variables used in both models and in case of (b) the number of restrictions used. Similar overall behavior in these characteristics is obtained for the Compact Model as well as in Dataset A. For the group of Instances of 30, 40 and 50 customers, the Compact Model Modified uses a little less amount of variables and restrictions than the Compact Model. For 20 customers hardly any difference can be perceived.



(a) Boxplot of Number of Variables



(b) Boxplot of Number of Restrictions

Figure 4.8: Boxplots of Number of Variables and Restrictions of both models

Computational time is another factor of interest. Results are shown in Table 4.2. As conclusion it can be said that Compact Model takes almost the entire stopping criteria of 600 seconds to solve the instances as average, on the other hand, Compact Model Modified reaches its results in lower computational times especially for instances of 20 and 30 customers.

	Compact Model	Compact Model Modified
	AVG time (sec)	AVG time (sec)
20 customers	504.97	181.06
30 customers	557.97	386.38
40 customers	592.06	512.22
50 customers	581.75	581.44

Table 4.2: Average computation time to solve instances for both models

The similar following Table 4.3 shows another resume of computation times, but with the difference of taking into account only the instances with that, the proposed Models found feasible tours.

Notice that there for the Compact Model there are significant variations. The decreasing average times in every group of customers are due to all the missing instances in which the model took all the stopping criteria of time trying to solve those instances. In the case of the Compact Model Modified, differences are only in the group of 20 and 30 customers. This is because of the absence of only one of these instances in these groups respectively.

	Compact Model	Compact Model Modified
	AVG time (sec)	AVG time (sec)
20 customers	467.68	167.55
30 customers	477.72	379.48
40 customers	548.60	512.22
50 customers	480.6	581.44

Table 4.3: Average computation time to solve instances for both models

Adding cuts

A final comparison is made between both proposed models with strengthen bounds by the action of adding cuts and the model proposed in Kinable *et al.* [17] based on CP. In their paper, the CP Model performed better than their heuristic procedure for this dataset, so it will be compared the best obtained bounds for the problem in this section as mentioned before.

Regarding these obtained bounds a boxplot is shown in Figure 4.9. The horizontal axis represents the group of Instances with the respective amount of customers and the vertical one represents the corresponding objective values (bounds) obtained.

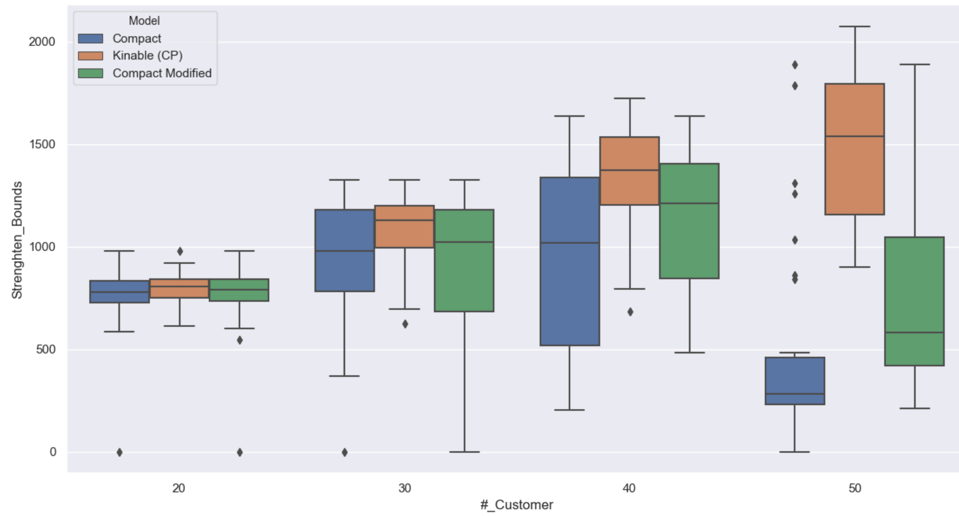


Figure 4.9: Results of the Compact models using cuts of size two and three and the CP Model of Kinable *et al.* [17] with Dataset B

As a result, the Compact Model Modified performs better than the Compact Model especially in the larger Instances of 40 and 50 customers. But Kinable *et al.* [17] CP Model still shows better results in general.

Adding these cuts allow Compact Model to reach 38 optimal values with an average gap of 34.79% in comparison with a figure of 11 optimal values and a higher average gap of 82.7% without the influence of cuts, these because this model could not find any tour in 82 cases out of 128, this evidences why this action causes a high improvement in results of this model.

In the case of the Compact Model Modified small enhancements were also evidenced. A total of 41 optimal values and an average gap of 2.43% were reached in comparison to 36 optimal values and an average gap of 3.40% without this influence.

Further analysis is carried out comparing the obtained Bound with the Upper Bound of the CPLEX Solver, this is considered a measure of the existing GAP for each instance of the Dataset. Figure 4.10 show a boxplot with these values. The horizontal axis represents the group of Instances with the respective amount of customers and the vertical one represents the corresponding value of the GAP in percent.

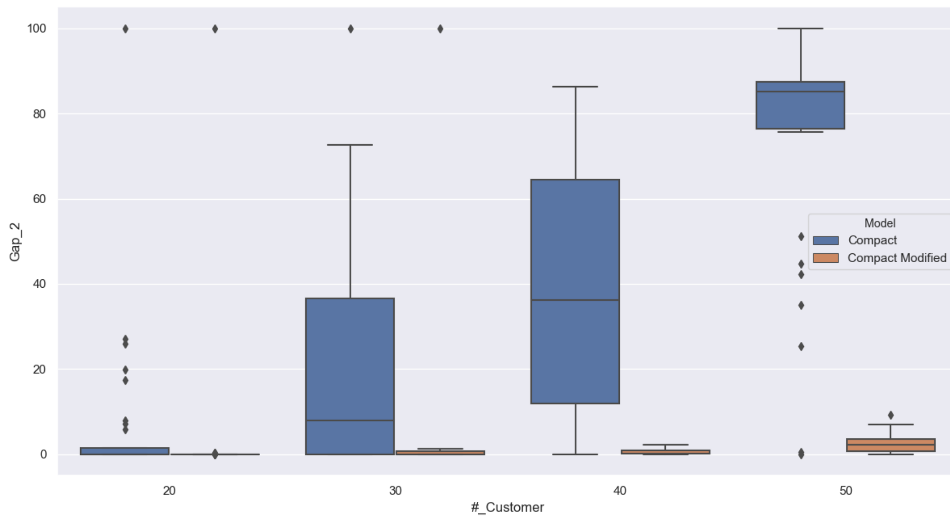


Figure 4.10: Gap of the Compact model using cuts of size two and three with Dataset B

There are remarkable breaches in all groups of customers, with greater differences as the size of instance grows. In this boxplot, the distinction between the two models seems fairly straightforward, reporting the Compact Model Modified better quality results. For the Compact Model Modified which uses slightly fewer variables and restrictions, it seems these are critical in terms of the contribution in finding solutions, helping this model to find better tours than the Compact Model in a total of 85 cases of this dataset.

Chapter 5

Conclusions and Future Work

In this chapter general conclusions of the work are presented as well as the future work to develop in order to continue the study of this routing problem and its areas for improvement.

5.1 Conclusions

The CDP is one of the several variants of rich vehicle routing problems and it is NP-Hard [17]. In the literature, multiple solution approaches have been proposed for the Concrete Delivery Problem, either exact methods, and heuristics. Our objective with this work is the contribution to methodological purposes, although the problem has been studied in real-case scenarios.

In this work, two Mixed Integer Programming formulations were proposed to solve the CDP. A MIP Model presented in Kinable *et al.* [17] was used as a guideline in order to understand the problem. Then, based on a graph representation, more compact formulations were proposed. These formulations reduce the number of variables and restrictions involved in the problem with overall performance as the instance size grows and present more efficient results in finding solutions.

In addition, the Compact Model Modified is the only one capable of finding solutions for the larger instances (40 and 50 customers) without the influence of cuts, although the Compact Model is able to tackle more of these instances when bounds are strengthened.

In general, the main contribution of this thesis is the analysis of the best formulations proposed in other relevant works for the CDP, and based on that, two more compact MIP models are proposed, showing encouraging results.

5.2 Future Work

As future work, we plan to implement and test the MIP model proposed by Kinable *et al.* [17] under the same condition as our compact models. Indeed, in their paper, the authors present results obtained with their model only on instances of dataset A, using the solution of their heuristic as an initial solution. In our study, we are interested in the performance of the models to find a feasible solution without a starting point. Besides, even on dataset B, our compact model can find a feasible solution contrary to what Kinable *et al.* [17] claim about their formulation. Recently Sulaman *et al.* [35] also propose a new set of larger instances for the CDP. Hence, we are considering carrying out further experimentation with these instances in order to have a better evaluation of the performance of our models.

Finally, we consider studying efficient heuristic approaches to tackle efficiently such problems. In particular, we would like to design specific local searches for routing problems with maximum time lag constraints. Such constraints are generally difficult to handle since even a small move in a solution yields most of the time to infeasible solutions. These local searches may be in the future extended to other classes of optimization problems.

Appendix A

Results of the Models from Dataset A

A.1 Tables

Table A.1: Comparison between all models without the influence of cuts in Dataset A

		Kinable's Model			Compact Model			Compact Model Modified		
Instance	Clientes	UB	Bound	Gap	UB	Bound	Gap	UB	Bound	Gap
A_2_5_1.rmc	5	85	85	0.0	85	85	0.0	85	85	0
A_2_5_2.rmc	5	160	160	0.0	160	160	0.0	160	160	0
A_2_5_3.rmc	5	105	105	0.0	105	105	0.0	105	105	0
A_2_5_4.rmc	5	105	105	0.0	105	105	0.0	105	105	0
A_3_5_1.rmc	5	205	205	0.0	205	205	0.0	205	205	0
A_3_5_2.rmc	5	115	115	0.0	115	115	0.0	115	115	0
A_3_5_3.rmc	5	125	125	0.0	125	125	0.0	125	125	0
A_3_5_4.rmc	5	190	190	0.0	190	190	0.0	190	190	0
A_4_5_1.rmc	5	140	140	0.0	140	140	0.0	140	140	0
A_4_5_2.rmc	5	150	150	0.0	150	150	0.0	150	150	0
A_4_5_3.rmc	5	165	165	0.0	165	165	0.0	165	165	0
A_4_5_4.rmc	5	230	230	0.0	230	230	0.0	230	230	0
A_5_5_1.rmc	5	200	200	0.0	200	200	0.0	200	200	0
A_5_5_2.rmc	5	200	200	0.0	200	200	0.0	200	200	0
A_5_5_3.rmc	5	220	220	0.0	220	220	0.0	220	220	0
A_5_5_4.rmc	5	175	175	0.0	175	175	0.0	175	175	0
A_2_10_1.rmc	10	50	50	0.0	50	50	0.0	50	50	0
A_2_10_2.rmc	10	150	150	0.0	150	150	0.0	150	150	0
A_2_10_3.rmc	10	220	220	0.0	220	220	0.0	220	220	0
A_2_10_4.rmc	10	150	150	0.0	150	150	0.0	150	150	0
A_3_10_1.rmc	10	205	205	0.0	205	205	0.0	205	205	0
A_3_10_2.rmc	10	230	230	0.0	230	230	0.0	230	230	0
A_3_10_3.rmc	10	480	305	36.5	305	305	0.0	305	305	0
A_3_10_4.rmc	10	300	300	0.0	300	300	0.0	300	300	0
A_4_10_1.rmc	10	440	170	61.4	360	270	25.0	310	310	0
A_4_10_2.rmc	10	370	370	0.0	370	370	0.0	370	370	0

Table A.1 continued from previous page

		Kinable's Model			Compact Model			Compact Model Modified		
A_4_10_3.rmc	10	470	340	27.7	410	370	9.8	375	375	0
A_4_10_4.rmc	10	285	285	0.0	285	285	0.0	285	285	0
A_5_10_1.rmc	10	350	350	0.0	350	350	0.0	350	350	0
A_5_10_2.rmc	10	345	345	0.0	345	345	0.0	345	345	0
A_5_10_3.rmc	10	285	285	0.0	285	285	0.0	285	285	0
A_5_10_4.rmc	10	380	380	0.0	380	380	0.0	380	380	0
A_2_15_1.rmc	15	215	215	0.0	215	215	0.0	215	215	0
A_2_15_2.rmc	15	360	220	38.9	447.76	290	35.2	450	290	36
A_2_15_3.rmc	15	415	190	54.2	315	205	34.9	205	205	0
A_2_15_4.rmc	15	255	255	0.0	255	255	0.0	255	255	0
A_3_15_1.rmc	15	605	0	100.0	330	330	0.0	330	330	0
A_3_15_2.rmc	15	395	395	0.0	395	395	0.0	395	395	0
A_3_15_3.rmc	15	465	240	48.4	290	290	0.0	290	290	0
A_3_15_4.rmc	15	740	370	50.0	440	440	0.0	440	440	0
A_4_15_1.rmc	15	655	0	100.0	655	0	100.0	625	295	53
A_4_15_2.rmc	15	650	120	81.5	650	305	53.1	650	330	49
A_4_15_3.rmc	15	580	395	31.9	430	430	0.0	430	430	0
A_4_15_4.rmc	15	725	10	98.6	550	415	24.5	570	455	20
A_5_15_1.rmc	15	630	0	100.0	590	455	22.9	590	430	27
A_5_15_2.rmc	15	695	0	100.0	695	360	48.2	695	510	27
A_5_15_3.rmc	15	465	315	32.3	350	350	0.0	350	350	0
A_5_15_4.rmc	15	600	60	90.0	600	415	30.8	555	480	14
A_2_20_1.rmc	20	920	15	98.4	920	0	100.0	895	20	98
A_2_20_2.rmc	20	735	35	95.2	270	270	0.0	270	270	0
A_2_20_3.rmc	20	850	35	95.9	260	260	0.0	260	260	0
A_2_20_4.rmc	20	770	240	68.8	587.81	335	43.0	355	355	0
A_3_20_1.rmc	20	830	0	100.0	415	290	30.1	340	340	0
A_3_20_2.rmc	20	845	0	100.0	820	0	100.0	730	200	73
A_3_20_3.rmc	20	695	0	100.0	695	110	84.2	645	225	65
A_3_20_4.rmc	20	819	165	79.9	760	440	42.1	637	435	32
A_4_20_1.rmc	20	660	35	94.7	660	255	61.4	635	380	40
A_4_20_2.rmc	20	575	355	38.3	425	425	0.0	425	425	0
A_4_20_3.rmc	20	815	25	96.9	755	95	87.4	690	220	68
A_4_20_4.rmc	20	735	355	51.7	640.83	455	29.0	465	465	0
A_5_20_1.rmc	20	900	395	56.1	875	645	26.3	800	665	17
A_5_20_2.rmc	20	925	0	100.0	925	145	84.3	925	415	55
A_5_20_3.rmc	20	750	375	50.0	595	590	1.0	600	595	1
A_5_20_4.rmc	20	735	205	72.1	710	430	39.4	594	495	17

Table A.2: Comparison between all models with bounds strengthen in Dataset A

		Kinable's Model			Compact Model			Compact Model Modified		
Instance	Clientes	UB	Bound	Gap	UB	Bound	Gap	UB	Bound	Gap
A_2_5_1.rmc	5	85	85	0.0	85	85	0.00	85	85	0
A_2_5_2.rmc	5	160	160	0.0	160	160	0.00	160	160	0
A_2_5_3.rmc	5	105	105	0.0	105	105	0.00	105	105	0
A_2_5_4.rmc	5	105	105	0.0	105	105	0.00	105	105	0
A_3_5_1.rmc	5	205	205	0.0	205	205	0.00	205	205	0
A_3_5_2.rmc	5	115	115	0.0	115	115	0.00	115	115	0
A_3_5_3.rmc	5	125	125	0.0	125	125	0.00	125	125	0
A_3_5_4.rmc	5	190	190	0.0	190	190	0.00	190	190	0
A_4_5_1.rmc	5	140	140	0.0	140	140	0.00	140	140	0
A_4_5_2.rmc	5	150	150	0.0	150	150	0.00	150	150	0
A_4_5_3.rmc	5	165	165	0.0	165	165	0.00	165	165	0
A_4_5_4.rmc	5	230	230	0.0	230	230	0.00	230	230	0
A_5_5_1.rmc	5	200	200	0.0	200	200	0.00	200	200	0
A_5_5_2.rmc	5	200	200	0.0	200	200	0.00	200	200	0
A_5_5_3.rmc	5	220	220	0.0	220	220	0.00	220	220	0
A_5_5_4.rmc	5	175	175	0.0	175	175	0.00	175	175	0
A_2_10_1.rmc	10	50	50	0.0	50	50	0.00	50	50	0
A_2_10_2.rmc	10	150	150	0.0	150	150	0.00	150	150	0
A_2_10_3.rmc	10	220	220	0.0	220	220	0.00	220	220	0
A_2_10_4.rmc	10	150	150	0.0	150	150	0.00	150	150	0
A_3_10_1.rmc	10	205	205	0.0	205	205	0.00	205	205	0
A_3_10_2.rmc	10	230	230	0.0	230	230	0.00	230	230	0
A_3_10_3.rmc	10	305	305	0.0	305	305	0.00	305	305	0
A_3_10_4.rmc	10	300	300	0.0	300	300	0.00	300	300	0
A_4_10_1.rmc	10	310	240	22.6	310	300	3.23	310	310	0
A_4_10_2.rmc	10	370	370	0.0	370	370	0.00	370	370	0
A_4_10_3.rmc	10	445	350	21.3	385	375	2.60	375	375	0
A_4_10_4.rmc	10	285	285	0.0	285	285	0.00	285	285	0
A_5_10_1.rmc	10	350	350	0.0	350	350	0.00	350	350	0
A_5_10_2.rmc	10	345	345	0.0	345	345	0.00	345	345	0
A_5_10_3.rmc	10	285	285	0.0	285	285	0.00	285	285	0
A_5_10_4.rmc	10	380	380	0.0	380	380	0.00	380	380	0
A_2_15_1.rmc	15	215	215	0.0	215	215	0.00	215	215	0
A_2_15_2.rmc	15	320	275	14.1	290	290	0.00	290	290	0
A_2_15_3.rmc	15	205	205	0.0	205	205	0.00	205	205	0
A_2_15_4.rmc	15	255	255	0.0	255	255	0.00	255	255	0
A_3_15_1.rmc	15	330	330	0.0	330	330	0.00	330	330	0
A_3_15_2.rmc	15	425	395	7.1	395	395	0.00	395	395	0
A_3_15_3.rmc	15	330	280	15.2	290	290	0.00	290	290	0
A_3_15_4.rmc	15	475	420	11.6	440	440	0.00	440	440	0
A_4_15_1.rmc	15	545	415	23.9	545	225	58.72	545	345	37
A_4_15_2.rmc	15	610	455	25.4	555	395	28.83	555	445	20
A_4_15_3.rmc	15	450	410	8.9	430	430	0.00	430	430	0
A_4_15_4.rmc	15	515	435	15.5	490	490	0.00	490	490	0
A_5_15_1.rmc	15	590	445	24.6	530	410	22.64	530	420	21
A_5_15_2.rmc	15	695	495	28.8	695	330	52.52	695	535	23
A_5_15_3.rmc	15	395	335	15.2	350	350	0.00	350	350	0
A_5_15_4.rmc	15	520	500	3.8	520	445	14.42	520	500	4
A_2_20_1.rmc	20	255	255	0.0	255	255	0.00	255	255	0
A_2_20_2.rmc	20	270	270	0.0	270	270	0.00	270	270	0
A_2_20_3.rmc	20	260	260	0.0	260	260	0.00	260	260	0
A_2_20_4.rmc	20	380	345	9.2	355	355	0.00	355	355	0
A_3_20_1.rmc	20	345	280	18.8	340	340	0.00	340	340	0

Table A.2 continued from previous page

		Kinable's Model			Compact Model			Compact Model Modified		
A_3_20_2.rmc	20	415	310	25.3	435	370	15.02	415	415	0
A_3_20_3.rmc	20	360	325	9.7	360	320	11.11	360	360	0
A_3_20_4.rmc	20	480	435	9.4	485	470	3.09	480	480	0
A_4_20_1.rmc	20	585	480	17.9	585	335	42.74	585	390	33
A_4_20_2.rmc	20	440	405	8.0	425	425	0.00	425	425	0
A_4_20_3.rmc	20	425	300	29.4	440	325	26.14	440	330	25
A_4_20_4.rmc	20	500	445	11.0	465	465	0.00	465	465	0
A_5_20_1.rmc	20	760	635	16.4	760	605	20.39	760	690	9
A_5_20_2.rmc	20	645	460	28.7	635	195	69.29	635	410	35
A_5_20_3.rmc	20	645	565	12.4	596	590	0.97	595	590	1
A_5_20_4.rmc	20	560	485	13.4	560	450	19.64	555	500	10

Table A.3: Comparison of all models in terms of used variables and restrictions in Dataset A

		Kinable's Model		Compact Model		Compact Model Modified	
Instance	nCustomers	#_Var	#_Rest	#_Var	#_Rest	#_Var	#_Rest
A_2_5_1.rmc	5	357	416	98	82	110	84
A_2_5_2.rmc	5	306	359	89	70	99	71
A_2_5_3.rmc	5	259	306	76	66	85	68
A_2_5_4.rmc	5	1182	1307	323	247	329	233
A_3_5_1.rmc	5	1480	1616	494	223	535	252
A_3_5_2.rmc	5	890	991	371	172	415	212
A_3_5_3.rmc	5	380	439	74	73	86	72
A_3_5_4.rmc	5	790	884	146	109	162	113
A_4_5_1.rmc	5	501	572	141	87	161	100
A_4_5_2.rmc	5	1179	1298	319	174	344	198
A_4_5_3.rmc	5	695	782	175	91	197	108
A_4_5_4.rmc	5	1179	1298	310	152	334	175
A_5_5_1.rmc	5	738	830	87	89	100	91
A_5_5_2.rmc	5	1302	1430	258	118	287	138
A_5_5_3.rmc	5	1302	1430	283	139	298	152
A_5_5_4.rmc	5	2232	2405	225	167	246	156
A_2_10_1.rmc	10	1286	1402	299	257	298	222
A_2_10_2.rmc	10	3752	3976	1890	611	1941	642
A_2_10_3.rmc	10	1496	1624	405	240	412	210
A_2_10_4.rmc	10	3581	3799	1779	622	1836	659
A_3_10_1.rmc	10	2391	2554	416	299	417	263
A_3_10_2.rmc	10	3115	3306	1545	419	1594	489
A_3_10_3.rmc	10	7561	7878	2568	779	2596	817
A_3_10_4.rmc	10	3935	4154	681	407	671	335
A_4_10_1.rmc	10	8156	8484	2186	939	2162	923
A_4_10_2.rmc	10	4139	4363	1030	381	1036	374
A_4_10_3.rmc	10	5231	5487	1384	675	1370	650
A_4_10_4.rmc	10	2536	2704	582	212	627	243
A_5_10_1.rmc	10	7269	7580	2102	461	2188	544
A_5_10_2.rmc	10	3959	4180	752	261	796	286
A_5_10_3.rmc	10	5489	5755	1455	345	1538	424
A_5_10_4.rmc	10	4541	4780	920	371	948	387
A_2_15_1.rmc	15	3097	3282	794	458	772	371
A_2_15_2.rmc	15	8272	8607	4190	1286	4199	1272
A_2_15_3.rmc	15	8531	8872	4287	1595	4275	1561
A_2_15_4.rmc	15	4672	4911	2344	804	2269	771
A_3_15_1.rmc	15	13550	13971	2342	1179	2263	870
A_3_15_2.rmc	15	3726	3923	643	409	656	372

Table A.3 continued from previous page

		Kinable's Model		Compact Model		Compact Model Modified	
A_3_15_3.rmc	15	6136	6403	1968	817	1947	789
A_3_15_4.rmc	15	9146	9483	1650	1103	1531	849
A_4_15_1.rmc	15	20251	20772	5846	2508	5690	2378
A_4_15_2.rmc	15	20251	20772	5249	1816	5266	1844
A_4_15_3.rmc	15	8899	9228	2059	600	2065	590
A_4_15_4.rmc	15	24430	25007	3379	1912	3155	1438
A_5_15_1.rmc	15	22528	23085	4654	1366	4610	1351
A_5_15_2.rmc	15	27470	28090	6075	2048	5967	1927
A_5_15_3.rmc	15	7274	7570	785	453	744	384
A_5_15_4.rmc	15	20560	21090	4200	1241	4171	1201
A_2_20_1.rmc	20	20121	20657	10883	3683	10560	3317
A_2_20_2.rmc	20	14899	15351	3992	2012	3694	1476
A_2_20_3.rmc	20	8799	9131	2428	1498	2281	1141
A_2_20_4.rmc	20	7281	7577	3834	1212	3784	1144
A_3_20_1.rmc	20	23873	24433	4302	2621	3998	1864
A_3_20_2.rmc	20	25505	26086	8555	2616	8548	2598
A_3_20_3.rmc	20	18823	19313	6471	2127	6290	2017
A_3_20_4.rmc	20	11991	12369	4106	1328	4006	1216
A_4_20_1.rmc	20	20829	21343	8332	1764	8373	1874
A_4_20_2.rmc	20	12176	12554	1410	782	1363	611
A_4_20_3.rmc	20	29006	29624	7231	2551	7020	2367
A_4_20_4.rmc	20	12176	12554	2933	871	2878	839
A_5_20_1.rmc	20	23209	23760	4618	1281	4520	1201
A_5_20_2.rmc	20	46197	47000	13210	2712	13138	2787
A_5_20_3.rmc	20	18687	19175	1851	939	1788	706
A_5_20_4.rmc	20	31305	31955	5855	1537	5920	1580

Table A.4: Comparison of all models in terms of CPU time in Dataset A

Instance	Customers	Kinable's Model	Compact Model	Compact Model Modified
A_2_5_1.rmc	5	0.05	0.01	0.01
A_2_5_2.rmc	5	0.03	0.01	0.01
A_2_5_3.rmc	5	0.01	0.01	0.01
A_2_5_4.rmc	5	0.46	0.05	0.01
A_3_5_1.rmc	5	0.55	0.09	0.08
A_3_5_2.rmc	5	0.33	0.07	0.07
A_3_5_3.rmc	5	0.04	0.01	0.01
A_3_5_4.rmc	5	0.08	0.01	0.01
A_4_5_1.rmc	5	0.03	0.01	0.01
A_4_5_2.rmc	5	0.34	0.06	0.03
A_4_5_3.rmc	5	0.1	0.01	0.01
A_4_5_4.rmc	5	0.15	0.06	0.03
A_5_5_1.rmc	5	0.07	0.01	0.01
A_5_5_2.rmc	5	0.15	0.03	0.01
A_5_5_3.rmc	5	0.17	0.03	0.02
A_5_5_4.rmc	5	0.34	0.02	0.01
A_2_10_1.rmc	10	0.39	0.03	0.01
A_2_10_2.rmc	10	79.01	21.58	1.02
A_2_10_3.rmc	10	1.41	0.06	0.03
A_2_10_4.rmc	10	11.91	2.18	0.38
A_3_10_1.rmc	10	11.09	0.1	0.03
A_3_10_2.rmc	10	217.2	139.03	9.03
A_3_10_3.rmc	10	300	148.43	58.03
A_3_10_4.rmc	10	23.2	0.06	0.04
A_4_10_1.rmc	10	300	300	127.56

Table A.4 continued from previous page

Instance	Customers	Kinable's Model	Compact Model	Compact Model Modified
A_4_10_2.rmc	10	53.19	3.56	0.37
A_4_10_3.rmc	10	300	300	74.15
A_4_10_4.rmc	10	0.92	0.06	0.03
A_5_10_1.rmc	10	12.2	0.55	0.22
A_5_10_2.rmc	10	1.51	0.12	0.08
A_5_10_3.rmc	10	17.07	0.63	0.33
A_5_10_4.rmc	10	16.73	0.46	0.12
A_2_15_1.rmc	15	7.45	0.29	0.09
A_2_15_2.rmc	15	300	300	300
A_2_15_3.rmc	15	300	300	57.71
A_2_15_4.rmc	15	125.2	25.71	1.17
A_3_15_1.rmc	15	300	6.8	0.67
A_3_15_2.rmc	15	274.15	3.11	0.54
A_3_15_3.rmc	15	300	231.52	79.16
A_3_15_4.rmc	15	300	87.28	2
A_4_15_1.rmc	15	300.04	300	300
A_4_15_2.rmc	15	300	300	300
A_4_15_3.rmc	15	300	192.35	8.62
A_4_15_4.rmc	15	300.01	300	300
A_5_15_1.rmc	15	300	300	300
A_5_15_2.rmc	15	300.01	300	300
A_5_15_3.rmc	15	300	10.9	0.21
A_5_15_4.rmc	15	300.01	300	300
A_2_20_1.rmc	20	300.01	300	300
A_2_20_2.rmc	20	300	75.99	1.6
A_2_20_3.rmc	20	300	41.78	1.73
A_2_20_4.rmc	20	300	300	93.19
A_3_20_1.rmc	20	300.01	300	298.65
A_3_20_2.rmc	20	300.01	300	300
A_3_20_3.rmc	20	300.01	300	300
A_3_20_4.rmc	20	300.01	300	300
A_4_20_1.rmc	20	300.01	300	300
A_4_20_2.rmc	20	300	5.02	0.35
A_4_20_3.rmc	20	300.01	300	300
A_4_20_4.rmc	20	300	300	24.41
A_5_20_1.rmc	20	300.01	300	300
A_5_20_2.rmc	20	300.01	300	300
A_5_20_3.rmc	20	300.01	55.69	5.33
A_5_20_4.rmc	20	300.01	300	300

A.2 Figures

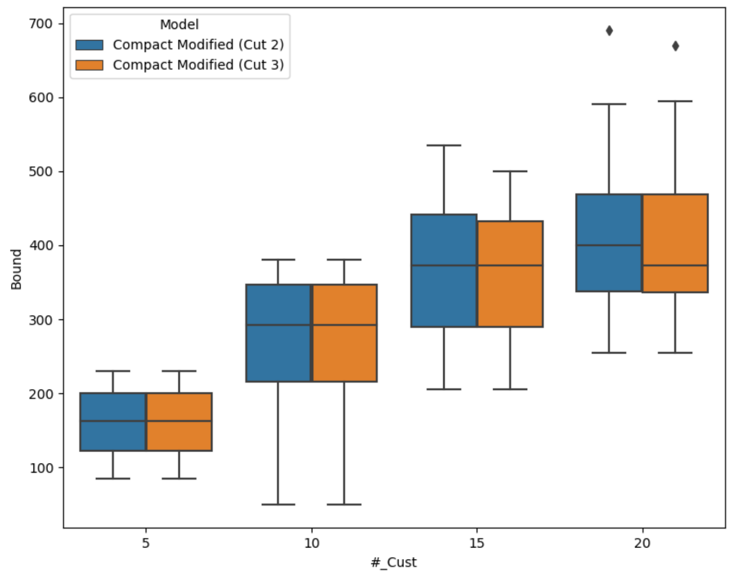


Figure A.1: Effect of applying cuts of size two and three to Compact Model Modified with Dataset A

Appendix B

Results of the Models from Dataset B

B.1 Tables

Table B.1: Comparison between all models without the influence of cuts in Dataset B

		Kinable's Model			Compact Model			Compact Model Modified		
Instance	Customers	UB	Bound	Gap (%)	UB	Bound	Gap (%)	UB	Bound	Gap (%)
B_6_20_1.rmc	20	805	460	42.85714286	760	660	13.16	750	685	8.67
B_6_20_2.rmc	20	855	0	100	835	560	32.93	810	470	41.98
B_6_20_3.rmc	20	760	45	94.07894737	760	485	36.18	730	625	14.38
B_6_20_4.rmc	20	705	485	31.20567376	621	615	0.99	621	615	0.99
B_8_20_1.rmc	20	935	15	98.39572193	935	875	6.42	935	920	1.60
B_8_20_2.rmc	20	865	0	100	865	335	61.27	865	735	15.03
B_8_20_3.rmc	20	655	515	21.3740458	655	655	0.00	655	655	0.00
B_8_20_4.rmc	20	820	20	97.56097561	820	820	0.00	820	820	0.00
B_10_20_1.rmc	20	805	60	92.54658385	805	805	0.00	805	805	0.00
B_10_20_2.rmc	20	825	260	68.48484848	825	825	0.00	825	810	1.82
B_10_20_3.rmc	20	730	0	100	730	730	0.00	730	730	0.00
B_10_20_4.rmc	20	765	10	98.69281046	765	765	0.00	765	765	0.00
B_12_20_1.rmc	20	770	770	0	770	770	0.00	770	770	0.00
B_12_20_2.rmc	20	770	0	100	770	770	0.00	770	770	0.00
B_12_20_3.rmc	20	945	0	100	945	0	100.00	945	775	17.99
B_12_20_4.rmc	20	850	0	100	850	810	4.71	850	850	0.00
B_14_20_1.rmc	20	830	15	98.19277108	830	830	0.00	830	830	0.00
B_14_20_2.rmc	20	695	695	0	695	695	0.00	695	695	0.00
B_14_20_3.rmc	20	840	15	98.21428571	840	670	20.24	840	840	0.00
B_14_20_4.rmc	20	755	20	97.35099338	755	755	0.00	755	755	0.00
B_16_20_1.rmc	20	905	0	100	905	905	0.00	905	905	0.00
B_16_20_2.rmc	20	805	0	100	805	805	0.00	805	805	0.00
B_16_20_3.rmc	20	915	0	100	915	225	75.41	915	915	0.00
B_16_20_4.rmc	20	875	0	100	875	875	0.00	875	875	0.00
B_18_20_1.rmc	20	820	0	100	820	820	0.00	820	820	0.00
B_18_20_2.rmc	20	740	15	97.97297297	740	740	0.00	740	740	0.00
B_18_20_3.rmc	20	775	115	85.16129032	775	775	0.00	775	775	0.00

Table B.1 continued from previous page

		Kinable's Model			Compact Model			Compact Model Modified		
B_18_20_4.rmc	20	840	0	100	840	840	0.00	840	840	0.00
B_20_20_1.rmc	20	875	875	0	875	875	0.00	875	875	0.00
B_20_20_2.rmc	20	770	770	0						
B_20_20_3.rmc	20	980	0	100	980	980	0.00	980	980	0.00
B_20_20_4.rmc	20	765	0	100	765	765	0.00	765	765	0.00
B_6_30_1.rmc	30	1300	55	95.76923077	1300	205	84.23	1292	630	51.22
B_6_30_2.rmc	30	1140	0	100	1140	595	47.81	1092	705	35.44
B_6_30_3.rmc	30	1060	20	98.11320755	1028	275	73.26	904	415	54.09
B_6_30_4.rmc	30	1080	0	100	1045	325	68.90	965	465	51.81
B_8_30_1.rmc	30	1085	0	100	1085	0	100.00	1085	645	40.55
B_8_30_2.rmc	30	1115	0	100	1115	130	88.34	1115	890	20.18
B_8_30_3.rmc	30	1155	0	100	1155	475	58.87	1155	755	34.63
B_8_30_4.rmc	30	1320	0	100	1320	10	99.24	1320	580	56.06
B_10_30_1.rmc	30	1215	0	100	1215	10	99.18	1215	735	39.51
B_10_30_2.rmc	30	1355	0	100	1355	0	100.00	1355	775	42.80
B_10_30_3.rmc	30	1210	0	100	1210	140	88.43	1210	750	38.02
B_10_30_4.rmc	30	1235	0	100	1235	825	33.20	1235	1040	15.79
B_12_30_1.rmc	30	1320	0	100	1320	0	100.00	1320	760	42.42
B_12_30_2.rmc	30	1185	10	99.15611814	1185	1185	0.00	1185	1185	0.00
B_12_30_3.rmc	30	950	0	100	950	865	8.95	950	950	0.00
B_12_30_4.rmc	30	1185	15	98.73417722	1185	1175	0.84	1185	1165	1.69
B_14_30_1.rmc	30	1190	0	100	1190	1180	0.84	1190	1180	0.84
B_14_30_2.rmc	30	1370	0	100	1370	0	100.00	1370	990	27.74
B_14_30_3.rmc	30	1005	950	5.472636816	1005	1005	0.00	1005	1005	0.00
B_14_30_4.rmc	30	1205	0	100	1205	510	57.68	1205	1205	0.00
B_16_30_1.rmc	30	1305	0	100	1305	0	100.00	1305	1295	0.77
B_16_30_2.rmc	30	1175	0	100	1175	145	87.66	1175	1165	0.85
B_16_30_3.rmc	30	1105	0	100	1105	1105	0.00	1105	1095	0.90
B_16_30_4.rmc	30	1090	0	100	1090	990	9.17	1055	1025	2.84
B_18_30_1.rmc	30	1080	0	100	1080	1080	0.00	1080	1080	0.00
B_18_30_2.rmc	30	1205	0	100	1205	1205	0.00	1205	1205	0.00
B_18_30_3.rmc	30	1155	0	100						
B_18_30_4.rmc	30	1125	0	100	1125	1125	0.00	1125	1125	0.00
B_20_30_1.rmc	30	1250	0	100	1250	20	98.40	1250	1240	0.80
B_20_30_2.rmc	30	1325	0	100	1325	0	100.00	1325	1325	0.00
B_20_30_3.rmc	30	1205	0	100	1205	545	54.77	1205	1205	0.00
B_20_30_4.rmc	30	1245	0	100	1245	1245	0.00	1245	1245	0.00
B_6_40_1.rmc	40	1545	0	100	1535	0	100.00	1505	390	74.09
B_6_40_2.rmc	40	1635	0	100	1635	0	100.00	1635	655	59.94
B_6_40_3.rmc	40	1775	0	100	1693	20	98.82	1484	565	61.94
B_6_40_4.rmc	40	1505	0	100	1505	20	98.67	1450	420	71.03
B_8_40_1.rmc	40	1665	0	100	1665	340	79.58	1665	655	60.66
B_8_40_2.rmc	40	1415	10	99.29328622	1290	1155	10.47	1210	1200	0.83
B_8_40_3.rmc	40	1495	0	100	1495	0	100.00	1495	395	73.58
B_8_40_4.rmc	40	1730	0	100	1730	0	100.00	1730	135	92.20
B_10_40_1.rmc	40	1475	0	100	1475	860	41.69	1475	1015	31.19
B_10_40_2.rmc	40	1580	0	100	1580	390	75.32	1580	1335	15.51
B_10_40_3.rmc	40	1605	30	98.13084112	1605	1200	25.23	1550	1365	11.94
B_10_40_4.rmc	40	1455	0	100	1455	405	72.16	1455	1225	15.81
B_12_40_1.rmc	40	1475	0	100	1475	970	34.24	1475	1320	10.51
B_12_40_2.rmc	40	1510	0	100	1510	0	100.00	1510	645	57.28
B_12_40_3.rmc	40	1640	0	100	1640	0	100.00	1640	600	63.41
B_12_40_4.rmc	40	1550	0	100	1550	0	100.00	1550	860	44.52
B_14_40_1.rmc	40	1395	0	100	1395	280	79.93	1395	720	48.39
B_14_40_2.rmc	40	1725	0	100	1725	195	88.70	1725	745	56.81
B_14_40_3.rmc	40	1550	0	100	1550	1360	12.26	1550	1535	0.97
B_14_40_4.rmc	40	1705	0	100	1705	0	100.00	1705	1535	9.97

Table B.1 continued from previous page

		Kinable's Model			Compact Model			Compact Model Modified		
B_16_40_1.rmc	40	1340	0	100	1340	1340	0.00	1340	1340	0.00
B_16_40_2.rmc	40	1580	0	100	1580	0	100.00	1580	1275	19.30
B_16_40_3.rmc	40	1600	0	100	1600	0	100.00	1600	1200	25.00
B_16_40_4.rmc	40	1615	0	100	1615	1605	0.62	1615	1600	0.93
B_18_40_1.rmc	40	1670	0	100	1670	0	100.00	1670	1360	18.56
B_18_40_2.rmc	40	1635	0	100	1635	1355	17.13	1635	1635	0.00
B_18_40_3.rmc	40	1610	0	100	1610	0	100.00	1610	1410	12.42
B_18_40_4.rmc	40	1655	0	100	1655	0	100.00	1655	1350	18.43
B_20_40_1.rmc	40	1695	0	100	1695	1370	19.17	1695	1625	4.13
B_20_40_2.rmc	40	1725	0	100	1725	0	100.00	1725	615	64.35
B_20_40_3.rmc	40	1540	0	100	1540	15	99.03	1540	1400	9.09
B_20_40_4.rmc	40	1530	0	100	1530	130	91.50	1530	1515	0.98
B_6_50_1.rmc	50	1890	0	100	1890	0	100.00	1890	515	72.75
B_6_50_2.rmc	50	2310	0	100	2310	0	100.00	2310	445	80.74
B_6_50_3.rmc	50	1795	0	100	1795	0	100.00	1765	240	86.40
B_6_50_4.rmc	50	2080	0	100	2080	0	100.00	2080	335	83.89
B_8_50_1.rmc	50	1980	0	100	1941	835	56.98	1718	895	47.89
B_8_50_2.rmc	50	1935	0	100	1935	20	98.97	1935	195	89.92
B_8_50_3.rmc	50	1960	0	100	1960	180	90.82	1960	630	67.86
B_8_50_4.rmc	50	1835	0	100	1835	0	100.00	1835	530	71.12
B_10_50_1.rmc	50	2265	0	100	2265	0	100.00	2265	65	97.13
B_10_50_2.rmc	50	1900	0	100	1866	705	62.22	1514	0	100.00
B_10_50_3.rmc	50	2005	0	100	2005	0	100.00	2005	645	67.83
B_10_50_4.rmc	50	1925	0	100	1925	30	98.44	1925	1005	47.79
B_12_50_1.rmc	50	1755	0	100	1755	25	98.58	1755	1200	31.62
B_12_50_2.rmc	50	2000	0	100	2000	0	100.00	2000	695	65.25
B_12_50_3.rmc	50	1825	0	100	1825	0	100.00	1825	685	62.47
B_12_50_4.rmc	50	1940	0	100	1940	15	99.23	1940	1395	28.09
B_14_50_1.rmc	50	2285	0	100	2285	0	100.00	2285	940	58.86
B_14_50_2.rmc	50	2015	0	100	2015	0	100.00	2015	785	61.04
B_14_50_3.rmc	50	2095	0	100	2095	0	100.00	2095	860	58.95
B_14_50_4.rmc	50	2095	0	100	2095	10	99.52	2095	590	71.84
B_16_50_1.rmc	50	2090	0	100	2090	0	100.00	2090	850	59.33
B_16_50_2.rmc	50	1930	0	100	1930	0	100.00	1930	1195	38.08
B_16_50_3.rmc	50	2010	0	100	2010	0	100.00	2010	850	57.71
B_16_50_4.rmc	50	1980	0	100	1980	25	98.74	1980	950	52.02
B_18_50_1.rmc	50	1795	0	100	1795	35	98.05	1795	1625	9.47
B_18_50_2.rmc	50	1930	0	100	1930	0	100.00	1930	1290	33.16
B_18_50_3.rmc	50	2005	0	100	2005	0	100.00	2005	1145	42.89
B_18_50_4.rmc	50	1795	0	100	1795	1660	7.52	1795	1725	3.90
B_20_50_1.rmc	50	2075	0	100	2075	15	99.28	2075	805	61.20
B_20_50_2.rmc	50	1825	0	100	1825	0	100.00	1825	1410	22.74
B_20_50_3.rmc	50	1825	0	100	1825	15	99.18	1825	1565	14.25
B_20_50_4.rmc	50	1890	0	100	1890	1890	0.00	1890	1890	0.00

Table B.2: Comparison between Compact models with the Bounds Strengthen in Dataset B

		Compact Model(cut 2 y 3)			Compact Model Modified (cut 2 y 3)		
Instance	#_Cust	UB	Bound	Gap (%)	UB	Bound	Gap (%)
B_6_20_1.rmc	20	760	715	5.92	750	715	0.05
B_6_20_2.rmc	20	810	590	27.16	810	600	0.35
B_6_20_3.rmc	20	730	585	19.86	730	545	0.34
B_6_20_4.rmc	20	615	615	0.00	621	615	0.01

Table B.2 continued from previous page

		Compact Model(cut 2 y 3)			Compact Model Modified (cut 2 y 3)		
B_8_20_1.rmc	20	935	860	8.02	935	920	0.02
B_8_20_2.rmc	20	865	640	26.01	865	735	0.18
B_8_20_3.rmc	20	655	655	0.00	655	655	0
B_8_20_4.rmc	20	820	820	0.00	820	820	0
B_10_20_1.rmc	20	805	805	0.00	805	805	0
B_10_20_2.rmc	20	825	825	0.00	825	825	0
B_10_20_3.rmc	20	730	730	0.00	730	730	0
B_10_20_4.rmc	20	765	765	0.00	765	765	0
B_12_20_1.rmc	20	770	770	0.00	770	770	0
B_12_20_2.rmc	20	770	770	0.00	770	770	0
B_12_20_3.rmc	20	945	780	17.46	945	835	0.13
B_12_20_4.rmc	20	850	850	0.00	850	850	0
B_14_20_1.rmc	20	830	830	0.00	830	830	0
B_14_20_2.rmc	20	695	695	0.00	695	695	0
B_14_20_3.rmc	20	840	780	7.14	840	840	0
B_14_20_4.rmc	20	755	755	0.00	755	755	0
B_16_20_1.rmc	20	905	905	0.00	905	905	0
B_16_20_2.rmc	20	805	805	0.00	805	805	0
B_16_20_3.rmc	20	915	915	0.00	915	915	0
B_16_20_4.rmc	20	875	875	0.00	875	875	0
B_18_20_1.rmc	20	820	820	0.00	820	820	0
B_18_20_2.rmc	20	740	740	0.00	740	740	0
B_18_20_3.rmc	20	775	775	0.00	775	775	0
B_18_20_4.rmc	20	840	840	0.00	840	840	0
B_20_20_1.rmc	20	875	875	0.00	875	875	0
B_20_20_2.rmc	20	0	0	0			
B_20_20_3.rmc	20	980	980	0.00	980	980	0
B_20_20_4.rmc	20	765	765	0.00	765	765	0
B_6_30_1.rmc	30	1240	785	36.69	1225	685	0.79
B_6_30_2.rmc	30	1105	695	37.10	1105	680	0.62
B_6_30_3.rmc	30	865	625	27.75	865	390	1.22
B_6_30_4.rmc	30	885	550	37.85	885	470	0.88
B_8_30_1.rmc	30	1085	505	53.46	1085	680	0.6
B_8_30_2.rmc	30	1115	855	23.32	1115	850	0.31
B_8_30_3.rmc	30	1155	880	23.81	1155	780	0.48
B_8_30_4.rmc	30	1320	415	68.56	1320	665	0.98
B_10_30_1.rmc	30	1215	770	36.63	1215	610	0.99
B_10_30_2.rmc	30	1355	370	72.69	1355	600	1.26
B_10_30_3.rmc	30	1210	820	32.23	1210	795	0.52
B_10_30_4.rmc	30	1235	960	22.27	1235	1075	0.15
B_12_30_1.rmc	30	1320	895	32.20	1320	720	0.83
B_12_30_2.rmc	30	1185	1175	0.84	1185	1175	0.01
B_12_30_3.rmc	30	950	940	1.05	950	950	0
B_12_30_4.rmc	30	1185	1175	0.84	1185	1185	0
B_14_30_1.rmc	30	1190	1190	0.00	1190	1180	0.01
B_14_30_2.rmc	30	1370	790	42.34	1370	1020	0.34
B_14_30_3.rmc	30	1005	1005	0.00	1005	1005	0
B_14_30_4.rmc	30	1205	1205	0.00	1205	1195	0.01
B_16_30_1.rmc	30	1305	1205	7.66	1305	1230	0.06
B_16_30_2.rmc	30	1175	1160	1.28	1175	1175	0
B_16_30_3.rmc	30	1105	1105	0.00	1105	1105	0
B_16_30_4.rmc	30	1089	1000	8.17	1055	1025	0.03
B_18_30_1.rmc	30	1080	1080	0.00	1080	1080	0
B_18_30_2.rmc	30	1205	1205	0.00	1205	1205	0
B_18_30_3.rmc	30						
B_18_30_4.rmc	30	1125	1125	0.00	1125	1125	0
B_20_30_1.rmc	30	1250	1250	0.00	1250	1250	0
B_20_30_2.rmc	30	1325	1325	0.00	1325	1325	0

Table B.2 continued from previous page

		Compact Model(cut 2 y 3)			Compact Model Modified (cut 2 y 3)		
B_20_30_3.rmc	30	1205	1205	0.00	1205	1205	0
B_20_30_4.rmc	30	1245	1245	0.00	1245	1245	0
B_6_40_1.rmc	40	1260	520	58.73	1260	620	1.03
B_6_40_2.rmc	40	1635	405	75.23	1635	825	0.98
B_6_40_3.rmc	40	1340	485	63.81	1340	485	1.76
B_6_40_4.rmc	40	1160	460	60.34	1160	530	1.19
B_8_40_1.rmc	40	1665	1010	39.34	1665	855	0.95
B_8_40_2.rmc	40	1266	1185	6.40	1212	1200	0.01
B_8_40_3.rmc	40	1495	205	86.29	1495	510	1.93
B_8_40_4.rmc	40	1730	395	77.17	1730	545	2.17
B_10_40_1.rmc	40	1475	1145	22.37	1475	890	0.66
B_10_40_2.rmc	40	1580	1390	12.03	1580	1380	0.14
B_10_40_3.rmc	40	1605	1365	14.95	1550	1325	0.17
B_10_40_4.rmc	40	1455	1205	17.18	1455	1275	0.14
B_12_40_1.rmc	40	1475	1300	11.86	1475	1225	0.2
B_12_40_2.rmc	40	1510	860	43.05	1510	695	1.17
B_12_40_3.rmc	40	1640	275	83.23	1640	930	0.76
B_12_40_4.rmc	40	1550	520	66.45	1550	915	0.69
B_14_40_1.rmc	40	1395	1130	19.00	1395	1250	0.12
B_14_40_2.rmc	40	1725	245	85.80	1725	1005	0.72
B_14_40_3.rmc	40	1550	1490	3.87	1550	1550	0
B_14_40_4.rmc	40	1705	1425	16.42	1705	1545	0.1
B_16_40_1.rmc	40	1340	1330	0.75	1340	1340	0
B_16_40_2.rmc	40	1580	855	45.89	1580	1290	0.22
B_16_40_3.rmc	40	1600	655	59.06	1600	1040	0.54
B_16_40_4.rmc	40	1615	1615	0.00	1615	1600	0.01
B_18_40_1.rmc	40	1670	1185	29.04	1670	1595	0.05
B_18_40_2.rmc	40	1635	1635	0.00	1635	1635	0
B_18_40_3.rmc	40	1610	635	60.56	1610	1475	0.09
B_18_40_4.rmc	40	1655	530	67.98	1655	1165	0.42
B_20_40_1.rmc	40	1695	1505	11.21	1695	1605	0.06
B_20_40_2.rmc	40	1725	510	70.43	1725	750	1.3
B_20_40_3.rmc	40	1540	1030	33.12	1540	1355	0.14
B_20_40_4.rmc	40	1530	1530	0.00	1530	1530	0
B_6_50_1.rmc	50	1800	245	86.39	1800	405	3.44
B_6_50_2.rmc	50	1800	210	88.33	1800	555	2.24
B_6_50_3.rmc	50	1680	410	75.60	1667	210	6.94
B_6_50_4.rmc	50	1815	0	100.00	1815	455	2.99
B_8_50_1.rmc	50	1719	840	51.13	1680	975	0.72
B_8_50_2.rmc	50	1935	245	87.34	1935	525	2.69
B_8_50_3.rmc	50	1960	230	88.27	1960	525	2.73
B_8_50_4.rmc	50	1765	270	84.70	1765	260	5.79
B_10_50_1.rmc	50	2265	210	90.73	2265	220	9.3
B_10_50_2.rmc	50	1558	860	44.80	1507	430	2.51
B_10_50_3.rmc	50	2005	205	89.78	2005	345	4.81
B_10_50_4.rmc	50	1925	450	76.62	1925	890	1.16
B_12_50_1.rmc	50	1755	1310	25.36	1755	1330	0.32
B_12_50_2.rmc	50	2000	285	85.75	2000	545	2.67
B_12_50_3.rmc	50	1825	225	87.67	1825	565	2.23
B_12_50_4.rmc	50	1940	1260	35.05	1940	1320	0.47
B_14_50_1.rmc	50	2285	200	91.25	2285	815	1.8
B_14_50_2.rmc	50	2015	410	79.65	2015	340	4.93
B_14_50_3.rmc	50	2095	485	76.85	2095	875	1.39
B_14_50_4.rmc	50	2095	270	87.11	2095	390	4.37
B_16_50_1.rmc	50	2090	280	86.60	2090	425	3.92
B_16_50_2.rmc	50	1930	385	80.05	1930	810	1.38
B_16_50_3.rmc	50	2010	450	77.61	2010	930	1.16

Table B.2 continued from previous page

		Compact Model(cut 2 y 3)			Compact Model Modified (cut 2 y 3)		
B_16_50_4.rmc	50	1980	250	87.37	1980	250	6.92
B_18_50_1.rmc	50	1795	1035	42.34	1795	1570	0.14
B_18_50_2.rmc	50	1930	265	86.27	1930	600	2.22
B_18_50_3.rmc	50	2005	210	89.53	2005	1260	0.59
B_18_50_4.rmc	50	1795	1785	0.56	1795	1705	0.05
B_20_50_1.rmc	50	2075	345	83.37	2075	1400	0.48
B_20_50_2.rmc	50	1825	230	87.40	1825	755	1.42
B_20_50_3.rmc	50	1825	365	80.00	1825	1440	0.27
B_20_50_4.rmc	50	1890	1890	0.00	1890	1890	0

Table B.3: Comparison of all models in terms of used variables and restrictions in Dataset B

		Kinable's Model		Compact Model		Compact Model Modified	
Instance	#_Cust	#_Var	#_Rest	#_Var	#_Rest	#_Var	#_Rest
B_6_20_1.rmc	20	23147	23707	3700	1212	3637	1128
B_6_20_2.rmc	20	49798	50648	8782	2636	8650	2471
B_6_20_3.rmc	20	41438	42208	10064	1984	10161	2085
B_6_20_4.rmc	20	13322	13732	1172	790	1122	614
B_8_20_1.rmc	20	39291	40073	4981	1572	4893	1463
B_8_20_2.rmc	20	70803	71873	13508	3023	13471	3051
B_8_20_3.rmc	20	38178	38948	4415	1181	4484	1238
B_8_20_4.rmc	20	32853	33563	4071	1184	4004	1117
B_10_20_1.rmc	20	36081	36865	3528	1155	3478	1081
B_10_20_2.rmc	20	37292	38090	3626	1184	3568	1142
B_10_20_3.rmc	20	60939	61975	9176	2040	9188	2112
B_10_20_4.rmc	20	68994	70100	7038	2142	7065	2146
B_12_20_1.rmc	20	40447	41321	3367	1116	3323	1065
B_12_20_2.rmc	20	84777	86067	7085	2413	7084	2406
B_12_20_3.rmc	20	122534	124096	14920	3456	14797	3474
B_12_20_4.rmc	20	97311	98697	8548	2720	8344	2623
B_14_20_1.rmc	20	113511	115079	12865	2681	12841	2733
B_14_20_2.rmc	20	78846	80144	5400	1423	5487	1481
B_14_20_3.rmc	20	111004	112554	7884	3055	7638	2831
B_14_20_4.rmc	20	94239	95663	6988	2091	6939	2035
B_16_20_1.rmc	20	150662	152552	14558	3035	14481	3136
B_16_20_2.rmc	20	121212	122902	4112	2049	3891	1462
B_16_20_3.rmc	20	153783	155693	15322	3238	15157	3280
B_16_20_4.rmc	20	135537	137327	9010	3154	8661	2883
B_18_20_1.rmc	20	133235	135079	8012	2486	7841	2337
B_18_20_2.rmc	20	118200	119934	6518	1737	6517	1746
B_18_20_3.rmc	20	121135	122891	6809	2195	6821	2175
B_18_20_4.rmc	20	149170	151124	12879	2978	12878	3088
B_20_20_1.rmc	20	89868	91430	4621	1350	4506	1268
B_20_20_2.rmc	20	69700	71070	1781	969	1693	738
B_20_20_3.rmc	20	212304	214730	16101	3267	16086	3333
B_20_20_4.rmc	20	141225	143195	10661	2298	10707	2383
B_6_30_1.rmc	30	54276	55136	9186	2918	8906	2648
B_6_30_2.rmc	30	86551	87661	13955	3559	13964	3541
B_6_30_3.rmc	30	74068	75088	12129	3431	12059	3385
B_6_30_4.rmc	30	79496	80556	12886	3821	12878	3784
B_8_30_1.rmc	30	111541	112869	20385	3734	20325	3788
B_8_30_2.rmc	30	113438	114778	14431	3884	14332	3844
B_8_30_3.rmc	30	121186	122574	14852	4203	14516	3913

Table B.3 continued from previous page

		Kinable's Model		Compact Model		Compact Model Modified	
B_8_30_4.rmc	30	156971	158563	30303	6139	29945	6033
B_10_30_1.rmc	30	166570	168290	16883	5157	16324	4634
B_10_30_2.rmc	30	207535	209465	32077	6887	31564	6798
B_10_30_3.rmc	30	166570	168290	25759	5271	25436	5243
B_10_30_4.rmc	30	86614	87830	8498	2537	8092	2237
B_12_30_1.rmc	30	228697	230821	28248	5456	27972	5360
B_12_30_2.rmc	30	97321	98677	8326	2388	8046	2114
B_12_30_3.rmc	30	127442	129006	10964	3116	10740	2955
B_12_30_4.rmc	30	93047	94371	4040	2387	3806	1852
B_14_30_1.rmc	30	113521	115059	4173	2497	3954	1908
B_14_30_2.rmc	30	290479	292989	30822	6552	30273	6404
B_14_30_3.rmc	30	83114	84418	5578	1794	5477	1701
B_14_30_4.rmc	30	222421	224607	15730	4790	15615	4624
B_16_30_1.rmc	30	296103	298743	20055	7021	19733	6782
B_16_30_2.rmc	30	250156	252576	16145	4889	15981	4775
B_16_30_3.rmc	30	222933	225213	15236	4594	15127	4517
B_16_30_4.rmc	30	215443	217683	7002	4033	6320	2711
B_18_30_1.rmc	30	246550	249046	14149	3820	13817	3514
B_18_30_2.rmc	30	290480	293196	15964	4074	15806	3971
B_18_30_3.rmc	30	272476	275104	15548	4657	15292	4528
B_18_30_4.rmc	30	272476	275104	15100	4087	15015	4000
B_20_30_1.rmc	30	348643	351735	26993	5176	26738	5167
B_20_30_2.rmc	30	381049	384285	20163	5688	19493	5148
B_20_30_3.rmc	30	338161	341205	17634	4789	17212	4507
B_20_30_4.rmc	30	343382	346450	17778	4847	17326	4390
B_6_40_1.rmc	40	163556	165086	28748	8400	28480	8190
B_6_40_2.rmc	40	179788	181398	30983	7850	30586	7501
B_6_40_3.rmc	40	203361	205081	18177	9793	16921	7094
B_6_40_4.rmc	40	151886	153356	26434	8043	25480	7175
B_8_40_1.rmc	40	123173	124543	15529	4359	14685	3718
B_8_40_2.rmc	40	56573	57463	3515	1577	3353	1198
B_8_40_3.rmc	40	202448	204238	37976	7629	37370	7493
B_8_40_4.rmc	40	274026	276128	52221	10862	51388	10493
B_10_40_1.rmc	40	130115	131595	12622	3400	12307	3082
B_10_40_2.rmc	40	146572	148150	14252	3479	13931	3148
B_10_40_3.rmc	40	146572	148150	7670	3799	7141	2558
B_10_40_4.rmc	40	125593	127045	12242	3116	11804	2811
B_12_40_1.rmc	40	150681	152359	12454	3270	11810	2785
B_12_40_2.rmc	40	311254	313716	39766	8101	39425	8039
B_12_40_3.rmc	40	367716	370402	31916	9197	31002	8362
B_12_40_4.rmc	40	326906	329432	27502	7790	26762	7131
B_14_40_1.rmc	40	298611	301127	33207	6580	33042	6597
B_14_40_2.rmc	40	484571	487807	36709	11877	35046	10269
B_14_40_3.rmc	40	185306	187264	13348	3792	12734	3359
B_14_40_4.rmc	40	236771	238999	17491	5266	16703	4586
B_16_40_1.rmc	40	169888	171838	10477	2643	10217	2449
B_16_40_2.rmc	40	457186	460456	29632	8342	29012	7925
B_16_40_3.rmc	40	462611	465901	28816	8862	28426	8497
B_16_40_4.rmc	40	226736	229006	7433	4047	6930	2939
B_18_40_1.rmc	40	570531	574339	33701	9918	33284	9571
B_18_40_2.rmc	40	285935	288599	16295	4687	15644	4055
B_18_40_3.rmc	40	514308	517918	45079	9638	44506	9373
B_18_40_4.rmc	40	545183	548903	30532	8451	29511	7658
B_20_40_1.rmc	40	655442	659680	18403	10456	16849	7342
B_20_40_2.rmc	40	670004	674290	52532	11585	51384	11122
B_20_40_3.rmc	40	538125	541955	27477	7922	26480	7166
B_20_40_4.rmc	40	531584	535390	26118	7443	25907	7240

Table B.3 continued from previous page

		Kinable's Model		Compact Model		Compact Model Modified	
B_6_50_1.rmc	50	242658	244518	60801	10713	60129	10439
B_6_50_2.rmc	50	348778	351038	61115	18518	58653	16243
B_6_50_3.rmc	50	216841	218591	52731	9014	52316	8926
B_6_50_4.rmc	50	288036	290076	51849	15865	49823	13933
B_8_50_1.rmc	50	346371	348719	23324	11446	21605	7805
B_8_50_2.rmc	50	336456	338768	63984	12576	63063	12259
B_8_50_3.rmc	50	353061	355433	64243	12520	63244	12073
B_8_50_4.rmc	50	295155	297311	55880	11543	54938	11248
B_10_50_1.rmc	50	561978	565150	90742	19915	88854	19160
B_10_50_2.rmc	50	400251	402905	19942	10117	18105	6702
B_10_50_3.rmc	50	445472	448280	66277	13674	65290	13328
B_10_50_4.rmc	50	424617	427355	43480	12703	42744	12139
B_12_50_1.rmc	50	212452	214436	17793	5181	17144	4583
B_12_50_2.rmc	50	529461	532677	46166	13370	45558	12831
B_12_50_3.rmc	50	465956	468964	58370	10256	57718	10045
B_12_50_4.rmc	50	259506	261714	21058	5865	20310	5176
B_14_50_1.rmc	50	820189	824403	61988	18741	60785	17759
B_14_50_2.rmc	50	641409	645119	49252	14235	48527	13587
B_14_50_3.rmc	50	677871	681689	50621	15028	49793	14314
B_14_50_4.rmc	50	690249	694103	52692	15230	50852	13435
B_16_50_1.rmc	50	817493	821873	81897	17198	80590	16654
B_16_50_2.rmc	50	679233	683213	64456	12693	63452	12249
B_16_50_3.rmc	50	719367	723467	70562	13598	69563	13099
B_16_50_4.rmc	50	692483	696503	68364	14040	67242	13663
B_18_50_1.rmc	50	677693	681823	38080	10159	37579	9730
B_18_50_2.rmc	50	742016	746344	41233	10681	39639	9390
B_18_50_3.rmc	50	824593	829163	70259	12522	69209	12115
B_18_50_4.rmc	50	347968	350888	18497	5000	17806	4426
B_20_50_1.rmc	50	977092	982260	76346	15018	75236	14539
B_20_50_2.rmc	50	784329	788945	37248	9550	36212	8579
B_20_50_3.rmc	50	745224	749720	56593	9696	55903	9509
B_20_50_4.rmc	50	246582	249110	5955	2804	5607	1971

Table B.4: Comparison of all models in terms of CPU time in Dataset B

Instance	Customers	Kinable's Model	Compact Model	Compact Model Modified
B_6_20_1.rmc	20	600	600	600
B_6_20_2.rmc	20	600	600	600
B_6_20_3.rmc	20	600	600	600
B_6_20_4.rmc	20	600	600	9
B_8_20_1.rmc	20	600	600	600
B_8_20_2.rmc	20	600	600	600
B_8_20_3.rmc	20	600	600	36
B_8_20_4.rmc	20	600	600	132
B_10_20_1.rmc	20	600	600	13
B_10_20_2.rmc	20	600	600	600
B_10_20_3.rmc	20	600	600	68
B_10_20_4.rmc	20	600	62	14
B_12_20_1.rmc	20	586	588	0.45
B_12_20_2.rmc	20	600	600	49
B_12_20_3.rmc	20	600	600	600
B_12_20_4.rmc	20	600	600	316
B_14_20_1.rmc	20	600	600	4
B_14_20_2.rmc	20	424	421	1
B_14_20_3.rmc	20	600	600	93
B_14_20_4.rmc	20	600	45	2

Table B.4 continued from previous page

Instance	Customers	Kinable's Model	Compact Model	Compact Model Modified
B_16_20_1.rmc	20	600	600	101
B_16_20_2.rmc	20	600	600	0.14
B_16_20_3.rmc	20	600	600	109
B_16_20_4.rmc	20	600	600	19
B_18_20_1.rmc	20	600	600	7
B_18_20_2.rmc	20	600	600	1
B_18_20_3.rmc	20	600	600	1
B_18_20_4.rmc	20	600	238	5
B_20_20_1.rmc	20	304	295	0.44
B_20_20_2.rmc	20	90	91	0.04
B_20_20_3.rmc	20	600	600	12
B_20_20_4.rmc	20	600	19	2
B_6_30_1.rmc	30	600	600	600
B_6_30_2.rmc	30	600	600	600
B_6_30_3.rmc	30	600	600	600
B_6_30_4.rmc	30	600	600	600
B_8_30_1.rmc	30	600	600	600
B_8_30_2.rmc	30	600	600	600
B_8_30_3.rmc	30	600	600	600
B_8_30_4.rmc	30	600	600	600
B_10_30_1.rmc	30	600	600	600
B_10_30_2.rmc	30	600	600	600
B_10_30_3.rmc	30	600	600	600
B_10_30_4.rmc	30	600	600	600
B_12_30_1.rmc	30	600	600	600
B_12_30_2.rmc	30	600	600	23
B_12_30_3.rmc	30	600	600	591
B_12_30_4.rmc	30	600	116	600
B_14_30_1.rmc	30	600	600	8
B_14_30_2.rmc	30	600	600	600
B_14_30_3.rmc	30	600	600	4
B_14_30_4.rmc	30	600	600	565
B_16_30_1.rmc	30	600	600	564
B_16_30_2.rmc	30	600	600	113
B_16_30_3.rmc	30	600	600	7
B_16_30_4.rmc	30	600	600	600
B_18_30_1.rmc	30	600	600	3
B_18_30_2.rmc	30	600	600	6
B_18_30_3.rmc	30	600	600	7.21
B_18_30_4.rmc	30	600	260	11
B_20_30_1.rmc	30	600	600	20
B_20_30_2.rmc	30	600	600	193
B_20_30_3.rmc	30	600	600	51
B_20_30_4.rmc	30	600	79	5
B_6_40_1.rmc	40	600	600	600
B_6_40_2.rmc	40	600	600	600
B_6_40_3.rmc	40	600	600	600
B_6_40_4.rmc	40	600	600	600
B_8_40_1.rmc	40	600	600	600
B_8_40_2.rmc	40	600	600	149
B_8_40_3.rmc	40	600	600	600
B_8_40_4.rmc	40	600	600	600
B_10_40_1.rmc	40	600	600	600
B_10_40_2.rmc	40	600	600	600
B_10_40_3.rmc	40	600	600	600
B_10_40_4.rmc	40	600	600	600
B_12_40_1.rmc	40	600	600	600

Table B.4 continued from previous page

Instance	Customers	Kinable's Model	Compact Model	Compact Model Modified
B_12_40_2.rmc	40	600	600	600
B_12_40_3.rmc	40	600	600	600
B_12_40_4.rmc	40	600	600	600
B_14_40_1.rmc	40	600	600	600
B_14_40_2.rmc	40	601	601	600
B_14_40_3.rmc	40	600	600	55
B_14_40_4.rmc	40	600	600	600
B_16_40_1.rmc	40	600	600	18
B_16_40_2.rmc	40	600	600	600
B_16_40_3.rmc	40	600	600	600
B_16_40_4.rmc	40	600	343	24
B_18_40_1.rmc	40	601	601	600
B_18_40_2.rmc	40	600	600	7
B_18_40_3.rmc	40	600	600	600
B_18_40_4.rmc	40	600	600	600
B_20_40_1.rmc	40	601	600	600
B_20_40_2.rmc	40	601	601	600
B_20_40_3.rmc	40	601	600	600
B_20_40_4.rmc	40	600	600	538
B_6_50_1.rmc	50	600	600	600
B_6_50_2.rmc	50	600	600	600
B_6_50_3.rmc	50	600	600	600
B_6_50_4.rmc	50	600	600	600
B_8_50_1.rmc	50	600	600	600
B_8_50_2.rmc	50	600	600	600
B_8_50_3.rmc	50	600	600	600
B_8_50_4.rmc	50	600	600	600
B_10_50_1.rmc	50	601	600	600
B_10_50_2.rmc	50	600	600	600
B_10_50_3.rmc	50	600	600	600
B_10_50_4.rmc	50	600	600	600
B_12_50_1.rmc	50	600	600	600
B_12_50_2.rmc	50	601	601	600
B_12_50_3.rmc	50	600	600	600
B_12_50_4.rmc	50	600	600	600
B_14_50_1.rmc	50	601	601	600
B_14_50_2.rmc	50	601	601	600
B_14_50_3.rmc	50	601	601	600
B_14_50_4.rmc	50	601	600	600
B_16_50_1.rmc	50	602	601	601
B_16_50_2.rmc	50	601	601	601
B_16_50_3.rmc	50	601	601	601
B_16_50_4.rmc	50	601	600	600
B_18_50_1.rmc	50	601	600	601
B_18_50_2.rmc	50	601	601	600
B_18_50_3.rmc	50	601	601	601
B_18_50_4.rmc	50	600	600	600
B_20_50_1.rmc	50	601	601	600
B_20_50_2.rmc	50	601	602	600
B_20_50_3.rmc	50	601	601	601
B_20_50_4.rmc	50	600	3	0.62

Bibliography

[1] Applegate, D. y W. Cook (1991), ≪A computational study of the job-shop scheduling problem≫, *ORSA Journal on computing*, **3**(2), págs. 149–156.

[2] Archetti, C. y M. G. Speranza (2012), ≪Vehicle routing problems with split deliveries≫, *International transactions in operational research*, **19**(1-2), págs. 3–22.

[3] Asbach, L., U. Dorndorf y E. Pesch (2009), ≪Analysis, modeling and solution of the concrete delivery problem≫, *European journal of operational research*, **193**(3), págs. 820–835.

[4] Bertsimas, D. J. (1992), ≪A vehicle routing problem with stochastic demand≫, *Operations Research*, **40**(3), págs. 574–585.

[5] Clark, N., J. Dropkin, L. Kaplan y J. Selikoff (2001), ≪Ready mixed concrete truck drivers: Work-related hazards and recommendations for controls≫, *Construction Hygiene and Ergonomics Program, USA, 27p.*

[6] Cordeau, J.-F., M. Gendreau, A. Hertz, G. Laporte y J.-S. Sormany (2005), ≪New heuristics for the vehicle routing problem≫, en *Logistics systems: design and optimization*, Springer, págs. 279–297.

[7] Dantzig, G. B. y J. H. Ramser (1959), ≪The truck dispatching problem≫, *Management science*, **6**(1), págs. 80–91.

[8] Desrochers, M., J. Desrosiers y M. Solomon (1992), ≪A new optimization algorithm for the vehicle routing problem with time windows≫, *Operations research*, **40**(2), págs. 342–354.

[9] Dror, M., G. Laporte y P. Trudeau (1994), ≪Vehicle routing with split deliveries≫, *Discrete Applied Mathematics*, **50**(3), págs. 239–254.

[10] Durbin, M. y K. Hoffman (2008), ≪OR PRACTICE. The dance of the thirty-ton trucks: Dispatching and scheduling in a dynamic environment≫, *Operations research*, **56**(1), págs. 3–19.

[11] Gendreau, M., J.-Y. Potvin *et al.* (2010), *Handbook of metaheuristics*, tomo 2, Springer.

[12] Goetschalckx, M. y C. Jacobs-Blecha (1989), ≪The vehicle routing problem with backhauls≫, *European Journal of Operational Research*, **42**(1), págs. 39–51.

[13] Golden, B. L., E. A. Wasil, J. P. Kelly y I.-M. Chao (1998), ≪The impact of metaheuristics on solving the vehicle routing problem: algorithms, problem sets, and computational results≫, en *Fleet management and logistics*, Springer, págs. 33–56.

[14] Hertz, A., M. Uldry y M. Widmer (2012), ≪Integer linear programming models for a cement delivery problem≫, *European Journal of Operational Research*, **222**(3), págs. 623–631.

[15] J. Kinable, T. (2014), ≪CDPLib≫, `https://sites.google.com/site/cdplib/`, Last accessed on 2019-03-10.

[16] Kesen, S. E. y T. Bektaş (2019), ≪Integrated Production Scheduling and Distribution Planning with Time Windows≫, en *Lean and Green Supply Chain Management*, Springer, págs. 231–252.

[17] Kinable, J., T. Wauters y G. V. Berghe (2014), ≪The concrete delivery problem≫, *Computers & Operations Research*, **48**, págs. 53–68.

[18] Laporte, G. (1992), ≪The vehicle routing problem: An overview of exact and approximate algorithms≫, *European journal of operational research*, **59**(3), págs. 345–358.

[19] Laporte, G., Y. Nobert y S. Taillefer (1988), ≪Solving a family of multi-depot vehicle routing and location-routing problems≫, *Transportation science*, **22**(3), págs. 161–172.

[20] Lei, H., G. Laporte y B. Guo (2011), ≪The capacitated vehicle routing problem with stochastic demands and time windows≫, *Computers & Operations Research*, **38**(12), págs. 1775–1783.

[21] Letchford, A. N. y J.-J. Salazar-González (2019), ≪The capacitated vehicle routing problem: stronger bounds in pseudo-polynomial time≫, *European Journal of Operational Research*, **272**(1), págs. 24–31.

[22] Lin, P.-C., J. Wang, S.-H. Huang y Y.-T. Wang (2010), ≪Dispatching ready mixed concrete trucks under demand postponement and weight limit regulation≫, *Automation in construction*, **19**(6), págs. 798–807.

[23] Lin, S.-W., Z.-J. Lee, K.-C. Ying y C.-Y. Lee (2009), ≪Applying hybrid meta-heuristics for capacitated vehicle routing problem≫, *Expert Systems with Applications*, **36**(2), págs. 1505–1512.

[24] Liu, Z., Y. Zhang y M. Li (2014), ≪Integrated scheduling of ready-mixed concrete production and delivery≫, *Automation in Construction*, **48**, págs. 31–43.

[25] Liu, Z., Y. Zhang, M. Yu y X. Zhou (2017), ≪Heuristic algorithm for ready-mixed concrete plant scheduling with multiple mixers≫, *Automation in Construction*, **84**, págs. 1–13.

[26] Maghrebi, M., T. S. Waller y C. Sammut (2013), ≪Scheduling concrete delivery problems by a robust meta heuristic method≫, en *2013 European Modelling Symposium*, IEEE, págs. 375–380.

[27] Matsatsinis, N. F. (2004), ≪Towards a decision support system for the ready concrete distribution system: A case of a Greek company≫, *European Journal of Operational Research*, **152**(2), págs. 487–499.

[28] Mayteekrieangkrai, N. y W. Wongthatsanekorn (2015), ≪Optimized ready mixed concrete truck scheduling for uncertain factors using bee algorithm≫, *Songklanakarin J. Sci. Technol*, **37**(2), págs. 221–230.

[29] Montoya-Torres, J. R., J. L. Franco, S. N. Isaza, H. F. Jiménez y N. Herazo-Padilla (2015), ≪A literature review on the vehicle routing problem with multiple depots≫, *Computers & Industrial Engineering*, **79**, págs. 115–129.

[30] Naso, D., M. Surico, B. Turchiano y U. Kaymak (2007), ≪Genetic algorithms for supply-chain scheduling: A case study in the distribution of ready-mixed concrete≫, *European Journal of Operational Research*, **177**(3), págs. 2069–2099.

[31] Payr, F. y V. Schmid (2009), ≪Optimizing deliveries of ready-mixed concrete≫, en *2009 2nd International Symposium on Logistics and Industrial Informatics*, IEEE, págs. 1–6.

[32] Ralphs, T. K., L. Kopman, W. R. Pulleyblank y L. E. Trotter (2003), ≪On the capacitated vehicle routing problem≫, *Mathematical programming*, **94**(2-3), págs. 343–359.

[33] Schmid, V., K. F. Doerner, R. F. Hartl y J.-J. Salazar-González (2010), ≪Hybridization of very large neighborhood search for ready-mixed concrete delivery problems≫, *Computers & operations research*, **37**(3), págs. 559–574.

[34] Solomon, M. M. (1987), ≪Algorithms for the vehicle routing and scheduling problems with time window constraints≫, *Operations research*, **35**(2), págs. 254–265.

[35] Sulaman, M., X. Cai, M. Misir y Z. Fan (2017), ≪Simulated Annealing with a Time-Slot Heuristic for Ready-Mix Concrete Delivery≫, en *Asia-Pacific Conference on Simulated Evolution and Learning*, Springer, págs. 39–50.

[36] Tommelein, I. D. y A. Li (1999), ≪Just-in-time concrete delivery: mapping alternatives for vertical supply chain integration≫, en *Proceedings IGLC*, tomo 7, pág. 97.

[37] Wade, A. y S. Salhi (2002), ≪An investigation into a new class of vehicle routing problem with backhauls≫, *Omega*, **30**(6), págs. 479–487.

[38] Zhang, G. y J. Zeng (2014), ≪Optimizing of ready-mixed concrete vehicle scheduling problem by hybrid heuristic algorithm≫, *COMPUTER MODELLING & NEW TECHNOLOGIES*, **18**, págs. 562–569.

Autobiography

Oscar Alejandro Hernández López

Candidato para el grado de Máster en Ciencias
de la Ingeniería con Orientación en Sistemas

Universidad Autónoma de Nuevo León

Facultad de Ingeniería Mecánica y Eléctrica

Tesis:

Study of Mixed Integer Programming Models for the Concrete Delivery Problem

I was born on July 19th, 1993 in Havana City, Cuba. The only child of Oscar Hernández Pérez and Caridad Evarista López Guerra. In 2017, I obtained the degree of Industrial Engineer at Universidad Tecnológica de La Habana "José Antonio Echeverría" (CUJAE), Cuba. I collaborated in teaching and research in subjects of Method Engineering and Time and Motion Study. In 2018, I started working as a Logistic Specialist at the enterprise TRD Caribe, developing activities of Warehouse Management and control of Logistic Key Performance Indicators in the company. One year after, I move to México, to start my master's studies in the Graduate Program in Systems Engineering at Universidad Autónoma de Nuevo León, working under the supervision of Dr. Vincent André Lionel Boyer.