The Journal of Engineering

The 14th International Conference on Developments in Power System Protection (DPSP 2018)

Differential protection in power transformers using the statistical second central moment

eISSN 2051-3305
Received on 4th May 2018
Accepted on 23rd May 2018
E-First on 24th August 2018
doi: 10.1049/joe.2018.0234
www.ietdl.org

Hector Esponda Hernandez[1] ✉, Ernesto Vázquez Martínez[1], Manuel Antonio Andrade Soto[1]
[1]*Universidad Autónoma de Nuevo León, México*
✉ *E-mail: espondahdz@gmail.com*

Abstract: This study describes a new differential protection algorithm for power transformers, which is not affected by transient conditions. The proposed algorithm uses the statistical second central moment, statistical variance, to characterise the signal waveform. The goal is to obtain the variance from each differential current after a pre-processing filter stage and to compare it with a threshold. The event will be identified as an internal fault, or otherwise as an inrush current. The algorithm was implemented in MATLAB, and a broad array of cases was carried out using the electromagnetic transient software PSCAD. All results show the algorithm successfully differentiated inrush from the internal fault conditions in over 2000 cases.

1 Introduction

Differential protection is the widest scheme used to protect power transformers against fault currents. Based in KLC, its operation principle sums all currents flowing into and out from the power transformer [1]. Usually, this scheme has a reliable performance identifying faults inside the protection zone. However, the primary challenge this scheme faces is distinguishing between inrush and fault currents [2]. The inrush current is a transient phenomenon that appears when a transformer is energised, and it is characterised by a large magnitude current that is not reflected in the secondary winding of the transformer. These characteristics cause an unbalance in differential currents, resulting in a misoperation of differential protection scheme.

Different methods have been proposed to avoid this problem. Most methods are based on the high specific harmonic content of the inrush to discriminate between inrush and fault currents. It is well-known inrush current has a high content of harmonics, especially the second harmonic, and this characteristic is used to block differential protection at the time the inrush current appears [3]. Other methods proposed to identify between inrush currents from fault currents are based on parameters of the power transformer [4], two-terminal network [5], pattern recognition using principal component analysis (PCA) [6], waveform correlation analysis [7] morphology mathematic (MM) [8], gradient vector angle variation [9], wavelet transform analysis [10], empirical Fourier transform [11], multi-region adaptive [12], neural network evaluation [13], fuzzy logic evaluation [14] and high-order statistic [15]. However, in some cases, these methods operate incorrectly because of several factors as changes in the transformer parameters, current transformer (CT) saturation, modifications in the power system topology, and frequency system variations.

This paper describes a new algorithm for differential protection in power transformers based on the magnitude of the statistical Second Central Moment (SCM) from a differential current. The algorithm applies a delta filter [16] to remove steady-state load conditions, and it calculates the variance from the waveform of the differential current. Based on Bernoulli variance criteria, a threshold was set to discriminate between fault and inrush currents. The threshold is not dependent on the parameters of the transformer.

The paper has been organised as follows. In Section 2, it is described the mathematical foundations. Section 3 shows how the proposed algorithm works. The test system used to evaluate the algorithm performance is shown in Section 4. Section 5 describes and discusses the cases where the algorithm was tested. Finally, conclusions are summarised in Section 6.

2 Mathematical foundations

2.1 SCM of a continuous sinusoidal waveform

The kth central moment m_k of a random variable X is defined as [17]

$$m_k = E\left[(X - E[X])^k\right] \tag{1}$$

where E is the expected value. The SCM for a continuous signal is defined as

$$m_2 = \int_0^T (x - u)^2 \,\mathrm{d}x \tag{2}$$

The SCM, also known as the variance, is a dispersion measure that indicates how far is each point of the variable from its respective mean. This measure can be useful as an indicator to determine when the waveform of a sinusoidal signal has changed, as the mean value will be different from zero. The behaviour of the SCM was analysed in two scenarios: a fully sinusoidal waveform and a half sinusoidal waveform. In the first scenario, a fully sinusoidal waveform was evaluated as shown in Fig. 1*a*. The mean value of the waveform is equal to zero. Therefore, the SCM using (2) will be

$$y(t) = \frac{1}{T}\int_0^T [A_{\max}\sin(\omega t + \alpha) - u_1]^2 \,\mathrm{d}\omega t \tag{3}$$

$$y(t) = \frac{(A_{\max})^2}{2} \tag{4}$$

While, if the waveform changes as the signal shown in Fig. 1*b*, the mean value will be calculated as the integral evaluation in each semi-period of the sinusoidal as

$$u = \frac{1}{T}\int_0^{T/2} [A_{\max}\sin(\omega t + \alpha)] \,\mathrm{d}\omega t + \int_{T/2}^T [0] \,\mathrm{d}\omega t \tag{5}$$

$$u = \frac{A_{\max}}{\pi} \tag{6}$$

Therefore, the SCM in a half sinusoidal waveform will be

$$y(t) = \frac{(A_{\max})^2}{T}\int_0^T \frac{1}{2}[1 - \cos(2\omega t + 2\alpha)]\,\mathrm{d}\omega t - \frac{1}{T}\int_0^T \left[\frac{A_{\max}}{\pi}\right]\mathrm{d}\omega t \tag{7}$$

$$y(t) = \frac{(A_{\max})^2}{4} - \frac{(A_{\max})^2}{\pi^2} \tag{8}$$

If (4) and (8) are evaluated using sinusoidal signals with an amplitude in the range [−1, +1], the SCM will take the values shown in Table 1. These results establish independently of the magnitude of the sinusoidal waveforms, the value of the SCM in a fully sinusoidal waveform will be 0.5, whereas if a half sinusoidal waveform is evaluated, the SCM value will be 0.1486.

2.2 SCM of a discrete sinusoidal waveform

If the SCM concept is extended to a discrete sinusoidal signal, the result will be the numerical calculation of the statistical variance

$$m_2 = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2 \tag{9}$$

where $\bar{x}$ is the mean value and n is the total number of samples. For a full sinusoidal waveform discretised at 32 samples per cycle, with a sampling frequency of 1920 Hz, and a maximum amplitude between [−1, +1], the numerical value of the variance (considering the mean value $\bar{x} = 0$) is 0.5. Further, for a discrete half sinusoidal waveform, sampled at 32 samples per cycle ($F_s = 1920$ Hz), with an amplitude between [0, +1], the statistical variance (considering the mean value $\bar{x} = 0.3077$), is 0.1478.

2.3 Maximum variance threshold from a Bernoulli distribution

The results obtained from the analysis of SCM of both a continuous and a discretised signal show it can be useful to identify if the analysed signal corresponds to a full sinusoidal waveform or not. If the value of the SCM is lower than 0.5, the signal will be a half sinusoidal waveform. Otherwise, it will be a full sinusoidal waveform. If the SCM concept is extended to the differential protection of the power transformers, it can be applied to identify between inrush and fault currents in a power transformer. The material used to the manufacture the core of the transformer has a non-linear characteristic that causes the current in the second semi-cycle to be zero or almost zero, as the half sinusoidal waveform previously analysed. However, it is necessary to establish a threshold to guarantee the correct identification between an inrush and a fault current because the signals obtained in laboratory or field are not perfect. This limit is based on the Bernoulli distribution and sets the maximum variance that a half sinusoidal waveform can achieve.

If X is a random variable between [a, b] where a and b are the minimum and maximum values, respectively, the maximum variance according to Popoviciu's inequality [18] will be

$$\mathrm{Var}(X) \le \frac{(b-a)^2}{4} \tag{10}$$

However, if it is considered a Bernoulli distribution where the values of a random variable are between [0, 1], the maximum variance using (10) will 0.25 [19]

$$\mathrm{Var}(X) \le \frac{(1-0)^2}{4} = 0.25 \tag{11}$$

Therefore, the maximum variance in a half sinusoidal waveform in a range [0, +1] will be 0.25. Hence, a variance upper bound can be established to differentiate between inrush and fault currents. If the variance is equal or lower than 0.25, the signal will be identified as an inrush current. Otherwise, it will be a fault.

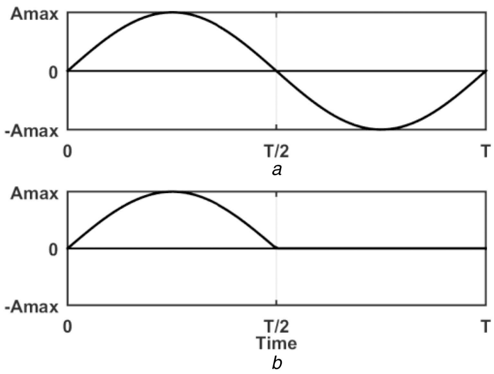

Fig. 1 *Areas considered in the evaluation of the SCM in a sinusoidal waveform*
(a) Full waveform, and ***(b)*** Half waveform

Table 1 SCM behaviour

Type of sinusoidal waveform	Magnitude
half waveform	0.1486
fully waveform	0.5

Differential protection of power transformers based on the SCM concept using the Bernoulli threshold was analysed in over 2000 simulation events. In Fig. 2, there are shown four representative examples from inrush (graph Figs. 2*a* and *c*), and fault currents (Figs. 2*b* and *d*) evaluated in four different transformers. In each transformer, there were changed parameters as the power size, voltage and manufacture material. The transformer characteristics are shown in Table 2.

The SCM from each event is shown in Fig. 3. The Bernoulli threshold was put it as a dashed line that limits the inrush and fault zones. In the case of the inrush currents from T1 (green) and T3 (blue), the SCM did not cross the threshold when the transformers were energised, as it was expected. Nevertheless, the events occurred in T2 (red) and, T4 (black) were identified as faults because the SCM crossed the established limit to inrush currents. All the faults evaluated were inside the differential protection zone.

However, the waveform of differential currents can be distorted (e.g. CT saturation [20]) as shown in Fig. 4. The differential currents with the CTs saturated from an inrush current, and an internal fault are shown in Figs. 4*a* and *c*, respectively. The evaluations indicated the SCM not crossed the threshold in the inrush current as shown in Fig. 4*b*, but it passed in the fault condition (see Fig. 4*d*). All results obtained show the SCM can be used as the basis of an algorithm to identify inrush current in a power transformer during transient conditions.

3 Proposed algorithm

The new proposed algorithm is shown in Fig. 5. The algorithm is divided into fourth stages as follows.

3.1 Data window

The proposed algorithm takes as input signals the secondary CT currents from both sides of the power transformer. The algorithm uses a moving data window where the input signals are scaled, and shift compensated to calculate the differential currents per each phase as

$$I_{\mathrm{DIFFABC}} = [I_{AB} - I_{ab},\quad I_{BC} - I_{bc},\quad I_{CA} - I_{ca}] \tag{12}$$

where I_{AB}, I_{BC}, I_{CA}, I_{ab}, I_{bc}, and I_{ca} are the currents from the high and the low side of the power transformer, respectively.

After that, the SCM is calculated to determine the kind of event. If the event is identified as an inrush current, the window will acquire a new sample, and the process will be restarted. Otherwise, the event will be determined as a fault current, and a signal trip will

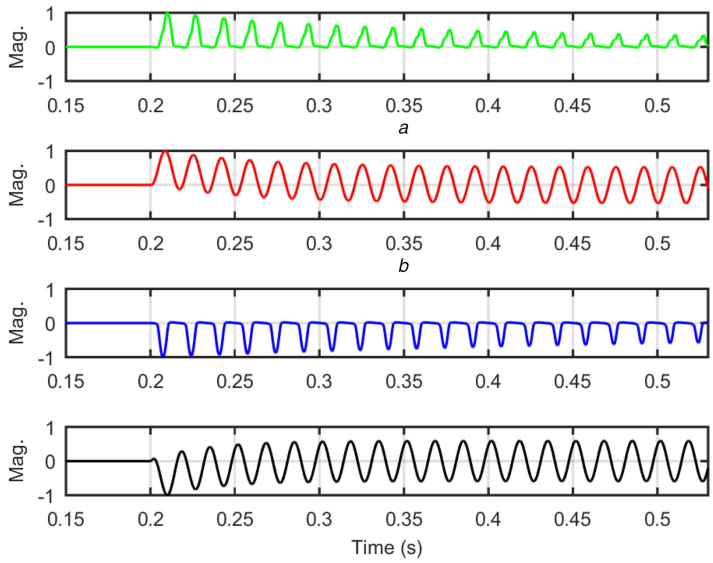

Fig. 2 *Four demonstrative simulation signals*
(a) Inrush current T1, phase A, ***(b)*** Fault current T2, phase A, ***(c)*** Inrush current T3, phase B and ***(d)*** Fault current T4, phase C

Table 2 Transformers parameters

Event	Transformer	MVA	kV	Material
inrush	T1	200	115/13.8	1
fault	T2	300	230/115	2
inrush	T3	500	230/13.8	1
fault	T4	100	115/13.8	2

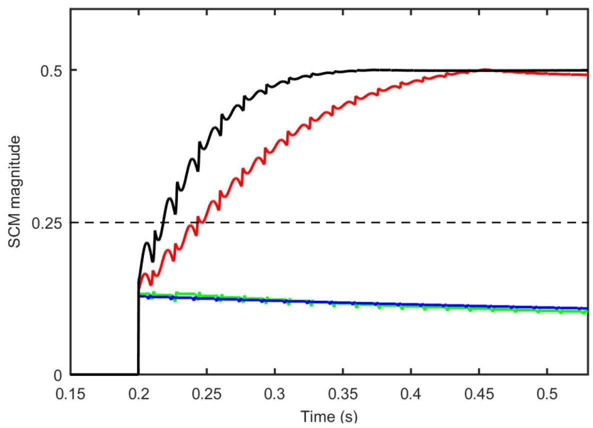

Fig. 3 *SCM evaluation from the four demonstrative signals. Inrush current T1 (green) (b) fault current T2 (red), inrush current T3 (blue), and (d) fault current T4 (black)*

be sent. This information processing gives to the algorithm the advantage that not threshold value is required to start.

3.2 Delta filter

Superimposed quantities are used to heighten any change (delta quantity) that can occur in a signal. A delta filter is used to extract the superimposed quantities eliminating redundant information and highlighting any transient changes in the differential currents. The incremental differential currents will be expressed as

$$\Delta I_{\text{DIFF}}(n) = I_{\text{DIFF}}(n) - I_{\text{DIFF}}(n - nT) \tag{13}$$

where nT represents n periods of the fundamental frequency. Details can be found on [16].

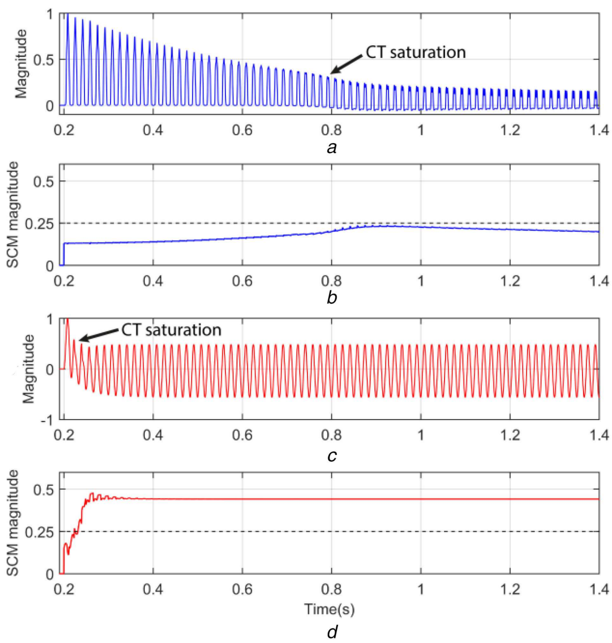

Fig. 4 *SCM evaluation in the presence of CT saturation*
(a) Inrush current, ***(b)*** Its SCM evaluation, ***(c)*** Fault current, ***(d)*** Its SCM evaluation

3.3 Normalisation

The generalisation of the algorithm in any transformer, without depending on its power size, it can be achieved if the input signals are normalised. This normalisation process divides all the data (per window) by the maximum absolute value in each window as is shown in the following equation:

$$I_{DIFF\_N} = \frac{x_i}{|\max(x)|} \tag{14}$$

where I_{DIFF_N} is the differential current normalised and $x = (x_1, x_2, \ldots, x_n)$ are the samples in the data window. This normalisation process makes the input signals to be scaled between $[-1, +1]$.

3.4 Threshold

Once the differential currents were normalised and filtered, the SCM per each differential current was calculated in MATLAB using (9). The algorithm takes a decision comparing the SCM value with the previously established threshold. The discrimination

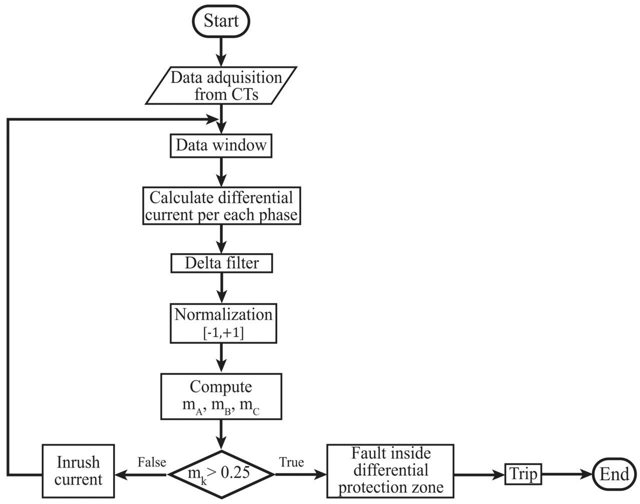

Fig. 5 *Proposed algorithm*

Table 3 Discrimination criteria

Event	Numerical variance value
inrush current	[0 ≤ SCM ≤ 0.25
fault current	[SCM > 0.25]

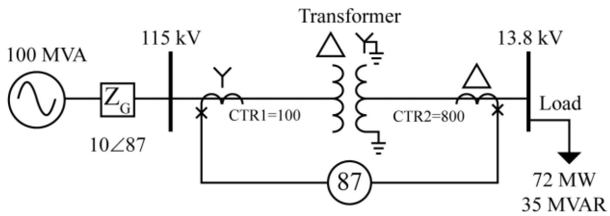

Fig. 6 *Test system*

criteria are shown in Table 3. If the SCM is within the [0, +0.25] range, the event will be identified as an inrush current, and the algorithm will continue evaluating the next window. If any SCM is out of the range, the event will be identified as a fault current inside differential protection zone, and a trip signal will be sent.

4 Test system

The electromagnetic transient software PSCAD was used to simulate the three-phase power system shown in Fig. 6. The system includes a 100 MVA, 115/13.8 kV, 60 Hz transformer in a delta-grounded star (wye) connection. On the high-voltage side, there is a 100 MVA, 115 kV source, with an impedance of 10∠87; on the low side, there is a three-phase 72 MW and 35 MVAR load. The CTs ratios selected using [20] were 500:5 (C200) and 4000:5 (C800) for the high and low sides, respectively.

5 Results

The proposed algorithm was evaluated in distinct scenarios using simulations of the test system performed in PSCAD. The scenarios covered included 128 unloaded and 128 loaded energisations, 704 faults inside and outside of the differential protection zone on both sides of the power transformer, 100 inter-winding faults, 352 CT saturation, 336 parameter transformer variations, 150 non-linear loads, 64 system frequency change, 64 overexcitation, and 176 of combinations of scenarios. It was considered a fault inception time in step increments of 1 ms over a cycle of 60 Hz. In all fault scenarios, it was used a fault resistance of 0.01 Ω. The secondary CT currents signals generated in the PSCAD simulations were exported and read into MATLAB at a sampling frequency of 3.84 kHz. After that, the SCM per phase was calculated in MATLAB using the proposed algorithm. In each scenario, the algorithm response was compared to the traditional differential protection using the harmonic blocking (HB). In Fig. 7, the evaluation of an

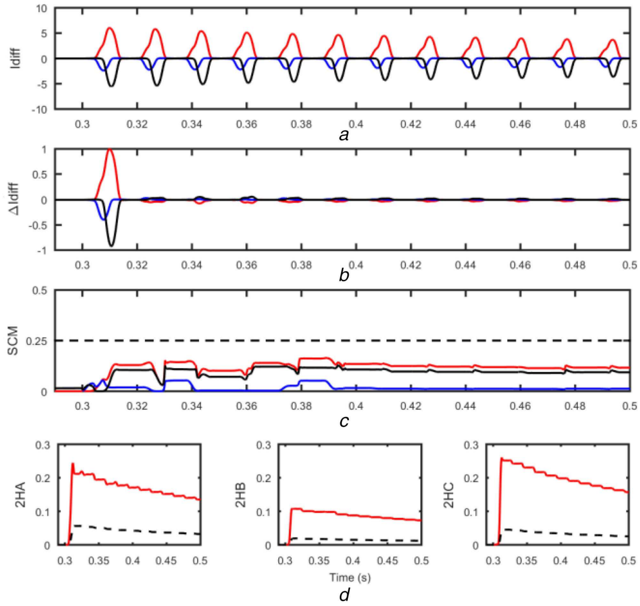

Fig. 7 *Loaded transformer energisation*
(a) Differential currents, ***(b)*** Incremental differential currents, ***(c)*** SCM magnitude behaviour and ***(d)*** Ratio of second harmonic content respect the fundamental

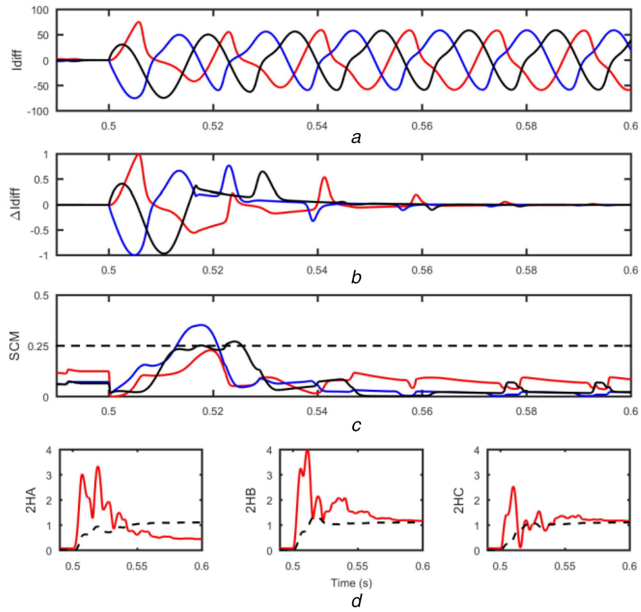

Fig. 8 *Three-phase internal fault*
(a) Differential currents, ***(b)*** Incremental differential currents, ***(c)*** SCM magnitude behaviour and ***(d)*** Ratio of second harmonic content respect the fundamental

inrush current generated by the loaded transformer energisation is shown. The normalisation and filtered of the differential currents are shown in Fig. 7*b*. The transformer was energised at 0.25 s. In this case, all the SCM signals not exceeded the limit of 0.25 as shown in Fig. 7*c*, and consequently, the algorithm identified the event as an inrush current. However, the HB blocked the misoperation of differential elements because the ratio of second harmonic content (solid line) is higher than the 12% of fundamental established (dashed line) as shown in Fig. 7*d*.

A three-phase fault inside the differential protection zone, at the low voltage side, after a transformer energisation, was evaluated (see Fig. 8). The fault was incepted in 0.5 s. In this scenario, two of the SCM signals crossed the threshold, and the algorithm identified the event as a fault as shown in Fig. 8*c*. Further, the ratio of the second harmonic induced a delay in the operation of differential elements as shown in Fig. 8*d* because the CT saturation introduces harmonic content to differential signals.

External faults can lead to a misoperation of differential elements because the saturation of CTs distorts the signals

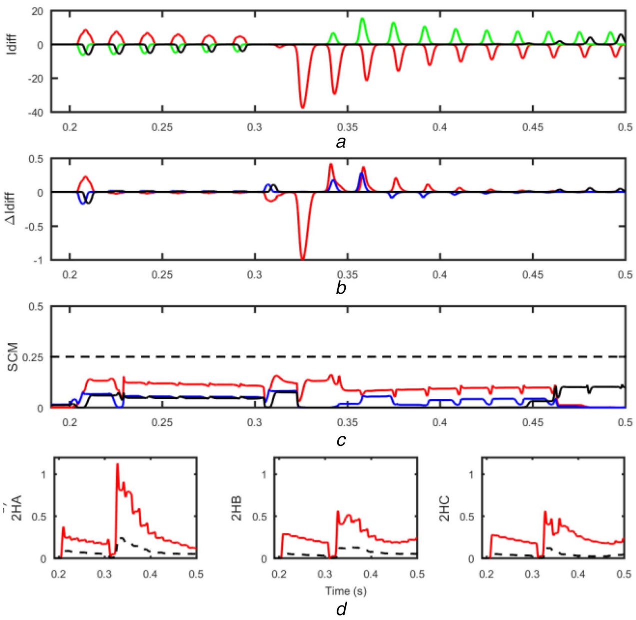

Fig. 9 *Three-phase external fault*

(a) Differential currents, ***(b)*** Incremental differential currents, ***(c)*** SCM magnitude behaviour and ***(d)*** Ratio of second harmonic content respect the fundamental

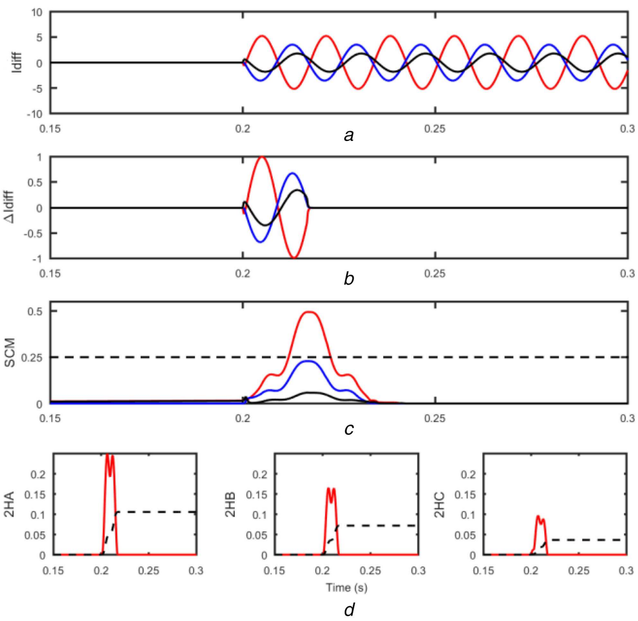

Fig. 10 *Inter-winding fault*

(a) Differential currents, ***(b)*** Incremental differential currents, ***(c)*** SCM magnitude behaviour and ***(d)*** Ratio of second harmonic content respect the fundamental

reproduced. By this reason, a three-phase external fault at the low side after a transformer energisation was evaluated (see Fig. 9). The transformer was energised at 0.2 s, and the external fault occurred at 0.3 s. As it was expected, all SCM signals not passed the threshold as shown in Fig. 9*c*. In this scenario, the HB blocked the misoperation of all differential elements because the second harmonic content was greater than the percentage established, as shown in Fig. 9*d*.

An inter-winding fault scenario is evaluated in Fig. 10. The internal fault (winding to ground) was incepted at the same time the transformer was energised. The fault was incepted at 0.2 s at 10% of the winding in the low-voltage side. In this case, the SCM signal of phase *A* exceeded the threshold, and the algorithm identified the event as a fault, as shown in Fig. 10*c*. HB correctly operated as shown in Fig. 10*d*.

6 Conclusions

This paper proposes a new algorithm based on the SCM to discriminate between inrush and fault currents in the differential protection of a power transformer. The statistical SCM is used to highlight the characteristic patterns in a waveform. The proposed algorithm uses the differential current signals as input signals to calculate the SCM signal per each phase. All the SCM signals are compared to an established threshold to identify the event. If any SCM is larger than the threshold, the event is detected as a fault inside the differential protection zone, otherwise, it will be an inrush current. The algorithm showed an effective discrimination between faults inside the differential protection zone from inrush currents. These results show the proposed algorithm can be used as the basis for a new transformer differential protection.

7 Acknowledgments

The authors gratefully acknowledge to CONACYT for the financial sponsorship for this research (project 257068).

8 References

[1] Ziegler, G.: '*Numerical differential protection*' (Publicist Pub, Erlangen, Germany, 2012, 2nd edn.), pp. 10–16

[2] Xiangning, L., Jing, M., Qing, T., *et al.*: '*Electromagnetic transient analysis and novell protective relaying techniques for power transformers*' (Wiley-IEEE Press, Singapore, 2015), pp. 1–8

[3] Hamilton, R.: 'Analysis of transformer inrush current and comparison of harmonic restraint methods in power transformer', *IEEE Trans. Ind.*, 2013, **49**, (4), pp. 1890–1899

[4] Mostafaei, M., Haghjoo, F.: 'Flux-based turn-to-turn fault protection for power transformers', *IET Gener. Transm. Distrib.*, 2016, **10**, (5), pp. 1154–1163

[5] Ma, J., Wang, Z., Yang, Q., *et al.*: 'A two terminal network-based method for discrimination between internal faults and inrush currents', *IEEE Trans. Power Deliv.*, 2010, **25**, (3), pp. 1599–1605

[6] Vázquez, E., Mijares, I., Chacon, O., *et al.*: 'Transformer differential protection using principal component analysis', *IEEE Trans. Power Deliv.*, 2012, **23**, (3), pp. 1394–1404

[7] Tian, X., Li, Y., Li, X.: 'Hybrid algorithm for traction transformer differential protection based on intrinsic mode function energy entropy and correlation dimension', *IET Gener. Transm. Distrib.*, 2014, **8**, (7), pp. 1274–1283

[8] Wu, W., Ji, T., Li, M., *et al.*: 'Using mathematical morphology to discriminate between internal and inrush current of transformers', *IET Gener. Transm. Distrib.*, 2016, **10**, (1), pp. 73–80

[9] Alencar, R.J., Bezerra, U.H., Ferreira, M.D.: 'A method to identify inrush currents in power transformers protection based on the differential current gradient', *Electr. Power Syst. Res.*, 2014, **111**, pp. 78–84

[10] Medeiros, R.P., Costa, F.B., Silva, K.: 'Power transformer differential protection using the boundary discrete wavelet transform', *IEEE Trans. Power Deliv.*, 2016, **31**, (5), pp. 2083–2095

[11] Murugan, S.K., Simon, S.P., Sundareswaran, K., *et al.*: 'An empirical Fourier transform- based power transformer differential protection', *IEEE Trans. Power Deliv.*, 2017, **32**, (1), pp. 209–218

[12] Dashti, H., Sanaye-Pasand, M.: 'Power transformer using a multiregion adaptive differential relay', *IEEE Trans. Power Deliv.*, 2014, **29**, (2), pp. 777–785

[13] Tripathy, M., Maheshwari, R.P., Verma, H.K.: 'Power transformer differential protection based on optimal probabilistic neural network', *IEEE Trans. Power Deliv.*, 2010, **25**, (1), pp. 102–112

[14] Bejmert, D., Rebizant, W., Schiel, L.: 'Transformer differential protection with fuzzy logic based inrush stabilization', *Electr. Power Syst. Res.*, 2014, **63**, pp. 51–63

[15] Zhang, L., Wu, Q.H., Ji, T.Y., *et al.*: 'Identification of inrush currents in power transformers based on high-order statistics', *Electr. Power Syst. Res.*, 2017, **146**, pp. 161–163

[16] Benmouyal, G., Roberts, J.: 'Superimposed quantities: their true nature and application in relays'. Proc. of 26th Annual Western Protective Relay Conf., Spokane, WA, 1999, pp. 1–18

[17] Sharma, R., Sharma, A., Saini, R., *et al.*: 'Means moments and Newton's inequalities', Available on arXiv:1702.04665 [math.ST]

[18] Sharma, R., Gupta, M., Kappor, G.: 'Some better bounds on the variance with applications', *J. Math. Inequalities*, 2016, **4**, pp. 355–363

[19] Léka, Z.: 'Some inequalities for central moments of matrices', *Linear Algebr. Appl.*, 2016, **496**, pp. 246–261

[20] Stanbury, M., Djekic, Z.: 'The impact of current-transformer saturation on transformer differential protection', *IEEE Trans. Power Deliv.*, 2015, **30**, (3), pp. 1278–1287