Universidad Autónoma de Nuevo León

Facultad de Ciencias Físico-Matemáticas

Posgrado en Ciencias con Orientación en Matemáticas

Nonparametric control chart with guaranteed in-control performance using cautious learning

por

Gerardo Pérez Arriaga

como requisito parcial para obtener el grado de

MAESTRÍA EN CIENCIAS CON ORIENTACIÓN EN MATEMÁTICAS

13 de junio de 2024

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Los miembros del Comité de Tesis recomendamos que la Tesis «Nonparametric control chart with guaranteed in-control performance using cautious learning», realizada por el alumno Gerardo Pérez Arriaga, con número de matrícula 1722269, sea aceptada para su defensa como requisito parcial para obtener el grado de Maestría en Ciencias con Orientación en Matemáticas.

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Durante la elaboración de este trabajo conté con el soporte de muchas personas especiales para mí. Entre ellas mi novia Keila, quien me dio consejos muy valiosos no solo en aspectos técnicos sino de caracter personal que me ayudaron a mantenerme constante y firme con mi objetivo; Mi familia, quienes conforme progresaba me confirmaban su apoyo y me daban ánimos para continuar; Mis compañeros/conocidos/amigos, que hicieron más amena mi estadía en posgrado y con quienes compartí experiencias muy agradables.

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¡Gracias!

Summary

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Candidato para obtener el grado de Maestría en Ciencias con Orientación en Matemáticas.

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Título del estudio: Nonparametric control chart with guaranteed in-control performance using cautious learning.

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Objectives and methods: To develop a distribution-free control chart with guaranteed in-control performance and a high power in change detection using the Cautious Learning approach seen in Capizzi and Masarotto (2020).

Contributions and conclusions: This work gives a new non-parametric control chart with guaranteed-in-control performance by employing cautious learning.

Monitoring observations with a non-normal distribution will be possible, guaranteeing a in-control performance and offering a high power in change detection. By implementing the learning mechanism of Capizzi and Masarotto (2020) on t-student and gamma observations under SNS, CSNSK, CSNSU, and SRT, the results obtained allow us guaranteeing a minimum number of false alarms and quite effective detection of changes in the monitoring of any process.

Firma del asesor: ________________________________
Dr. Alvaro Eduardo Cordero Franco

Chapter 1

Introduction

1.1 Context

A production process converts raw materials into products, which are expected to have a desired level of quality reflected in the adjustment of each product to the established design requirements, which are the characteristics that the product must have to satisfy the consumer, (Qiu, 2014)

The quality of a product is reflected in essential metrics, which, in turn, represents the quality of the process itself. These metrics can be modelled as random variables two possible sources of variability: common and assignable. Common causes of variation include events that cannot be controlled; therefore, statistics tools are not used in these cases. Instead, other strategies such as insurance and emergency funds are employed. In this study we focus on assignable causes, which encompass situations within the producer's control, so there exists a need to fix it. Checking and fixing the assignable causes of a process can be formally conducted as part of the Statistical Process Control (SPC).

The monitoring process is commonly represented with a graph called *control chart*, which illustrates the evolution of process quality over time. This chart features two horizontal lines, a lower and an upper one, symbolizing the control limits. These limits represent values that, if exceeded by the metrics, is detected an assignable cause of variation. At that point, the process become *out-of-control (OC)*. Conversely, if the metrics remain within these limits, the process is considered *in control (IC)*. The term 'control' refers to variability in the process only because of common causes.

Control charts are applied in a two-phase monitoring scheme. In phase I, a retrospective analysis is conducted with a sample of observations. Subsequently, control limits are adjusted to these sample to identify assignable causes of variation. Observations with an assignable cause of variation (OC-observations) are removed from the sample. This process is repeated until the sample no longer contains OC observations. This set of IC data then serves as a reference sample to monitor the quality of the process in Phase II of monitoring, (Montgomery, 2019)

In phase II, using the reference sample obtained in phase I, new quality observations from the now IC process are monitored. Each sample is checked concerning the control limits previously established to determine if the process remains IC or becomes OC. In the latter scenario, an alarm is issued to report the change, recording the number of monitored observations before the change occurred. In this work, a phase II control chart is created.

The most relevant monitoring schemes that exist are the Shewhart chart (1931) by Shewhart (1926), mainly used to detect isolated and medium to significant changes in the process, the CUSUM chart (1954) by Page (1954), and the EWMA chart (1959) by Robets (1959), these last two employs cumulative information from the process to detect principally gradual and minor changes.

Different control charts exist for different monitor targets, such as the location, dispersion, coefficient of variation, among others, which can be monitored alone or in conjunction with others. Figure 1.1 presents example of a control chart to monitor the mean of a normal process, as example.

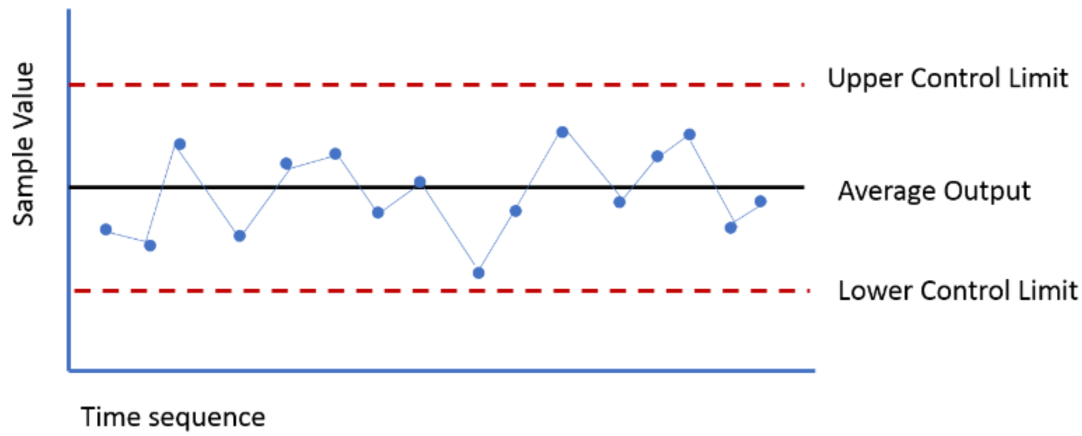

FIGURA 1.1: Control chart

Each point on the graph in 1.1 represents a summary measure of process quality at a time t; the process is considered in control (IC) when a point falls within limits indicated by the dotted lines. Otherwise, the process deemed out of control (OC), and a change signal is issued. The time at which the process ceased to be in-control is denoted by the symbol τ, and it can be known either a priori or a posteriori. The size of the change is denoted by δ, representing the size of the change expressed in standard deviations.

After implementing a control chart to measure its performance in monitoring, consideration is given to the chart's power to detect changes and its capacity to differentiate real changes from those that are not, that is, its sensitivity and specificity. Typically, to evaluate the performance of a control chart, we consider the number of monitored observations after the first alarm, known as run length (RL). Metrics based on the average RL, denoted as ARL, are used to assess IC and OC performance. To study false alarms rates, it is common to obtain the average of IC run-length ARL_0. Traditional control charts are designed to achieve a desired ARL_0. To evaluate the power of the control chart, it is usual to assess the ARL_1 when a real change occurs. These metrics can be obtained through probability theory, when feasible, or through simulation.

The metric ARL_0 is typically set to achieve a good IC performance, observing the resultant OC performance. Practitioners typically consider 370, 500, or 1000 as standards for ARL_0. However, the choice depends on the distribution of the process, along with its parameters, as it determines the probability that the quality will surpass the control limits. In this context, performance metrics can be obtained if the distribution function is a standard normal and the parameters are known. However, if not, either parameter estimation or the distribution of the data could affect the performance of the control chart.

While many existing control charts assume a standard normal probability distribution for the process, in reality, processes are not always normal. Numerous non-parametric methods, most based on ranks, exist to address this issue.

Solving the non-normal distribution problem can involve applying a non-parametric transformation to carry the observations from any distribution to the standard normal without affecting the nature of the data. However, the variation in the chart's performance due to a lack of knowledge about the distribution parameters could be more complex. This problem was initially mentioned by Albers and Kallenberg (2004), and eventually became known as practitioner-to-practitioner variation (PTP). Other authors, including Keefe et al. (2015a) and Jardim et al. (2019a), have proposed strategies to reduce this effect, but with some disadvantages. More recently, Capizzi and Masarotto (2020) introduced a new method called cautious learning, which combines learning capabilities and has a great power in change detection with a low number of false alarms.

Albers and Kallenberg (2004) observed that, although ARL_0 was reached on average, a significant proportion of IC performances during the implementation of a control chart was below the ARL_0, as explained in Figure 1.3. Jardim et al. (2019a) proposed setting the control limits to guarantee an in-control performance 95 % of the time. However, it had the limitation that the detection of changes was significantly affected, as it could not control either the mean nor the variance of the IC performances.

The cautious learning approach by Capizzi and Masarotto (2020), detailed in Chapter 2, guarantees a desired ARL_0 with a high power to detect changes. This approach uses a mix of the approach seen in Jardim et al. (2019a) and a learning propose by Hawkins (1987b). The method involves updating the reference sample until the cumulative likelihood of the phase II sample is sufficiently high, adding all the previous IC monitored observations, and re-estimating the phase I parameters and the control limits, which are previously fixed to obtain a minimum proportion of ARLs below the ARL_0, generally around 5 %. All of this is under the assumption that the IC distribution function is standard normal.

Very few processes follows a normal distribution, so the cautious learning approach by Capizzi and Masarotto (2020) may not be suitable for these processes. However, non-parametric transformations allow the transition from data sets of any distribution to normal observations using a function that preserves the main characteristics of the original data. Here, we choose the sequential normal scores (SNS) proposed by Conover et al. (2017), which employs ranks and the inverse transformation theorem. Additionally, the SNS can be either Conditional Sequential Normal Scores with Known quantile (CSNSK) or Conditional Sequential Normal Scores with unknown quantile (CSNSU) when the distribution is conditioned on the knowledge or the estimation of a sample's quantile. In this work, the SNS and its conditional versions CSNSK and CSNSU, with the median as a condition, are used to monitor observations from any distribution, employing the same reference sample and control limits re-configuration as in Capizzi and Masarotto (2020).

In summary, the present work proposes to link SNS with cautious learning to enhance the control chart through guaranteed in-control performance, learning strategies, and non-parametric transformations.

1.2 MOTIVATION

The reference sample affects the performance of the control chart. In order to illustrate the practitioner-to-practitioner effect, a simulation experiment was conducted. It was assumed that there are 35 subgroups of size n =5 in control, each following a standard normal distribution, with the goal of achieving an ARL_0 of 500 on a Shewhart control chart. In this experiment, 10 000 run-lengths were obtained for compute each ARL_0, and, 1 000 average run-lengths are computed. Two scenarios are considered:

- Known: In this scenario, the parameters of the in-control distribution were assumed to be known
- Estimated: In this scenario, the parameters had to be estimated using sample statistics

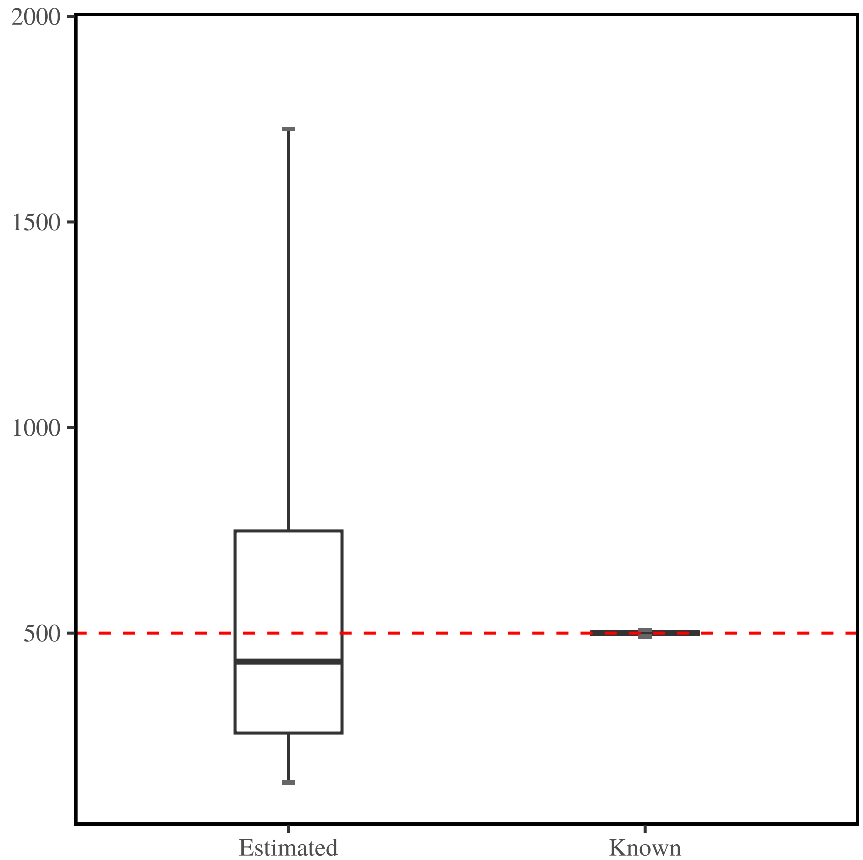

FIGURA 1.2: The effect of parameter estimation on the in-control performance of the Shewhart chart

As depicted in Figure 1.2, the impact of parameter estimation on the IC performance of the Shewhart chart is significant.The variance of the ARL_0 increases, and there there is a notable amount of IC-performances below the desired ARL_0, 21.80 % in this example. This discrepancy leads to a higher rate of false alarms, causing high costs for producers and a reduction in process efficiency.

In most real-life processes, the exact parameters or distribution function are often unknown. Therefore, an improvement in traditional statistical control methods is needed.

Figure 1.3 displays the outcomes of two experiments, one in-control with a target performance ARL_0 = 500 and another out-of-control experiment aimed at detecting a δ =2 size change occurring at the beginning of the process (τ =1). The simulation details are identical to those of the previous experiment, except that this time the cautious learning approach proposed by Capizzi and Masarotto (2020) is employed.

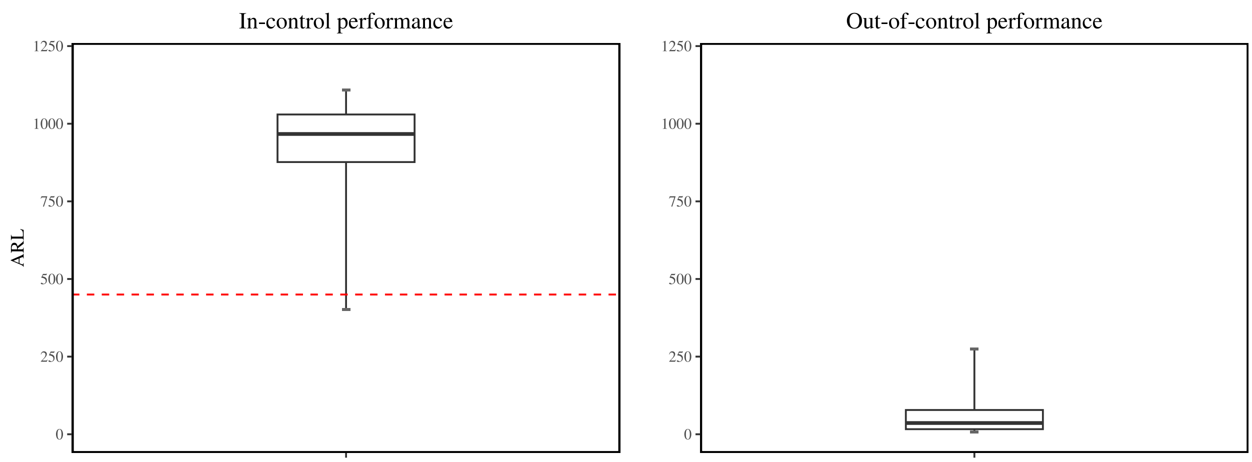

Figura 1.3: In-control and out-of-control performance for a Shewhart chart using cautious learning

When using the cautious learning approach, only 5.90 % of the in-control performances fall below the desired ARL_0. This approach shows high power in changes detection, as reflected in the out-of-control results for a change of δ=2 standard deviations.

Unfortunately, the cautious learning approach applied in the previous experiment was designed specifically for normal observations. Dealing with non-normal distributions introduces challenges in the performance of the control chart, as illustrated in the example presented in Figure 1.4. In this instance, we utilize the cautious learning approach with t-student (4) and gamma(0.5,1) distributions , resulting in all in-control performances falling below the expected levels.

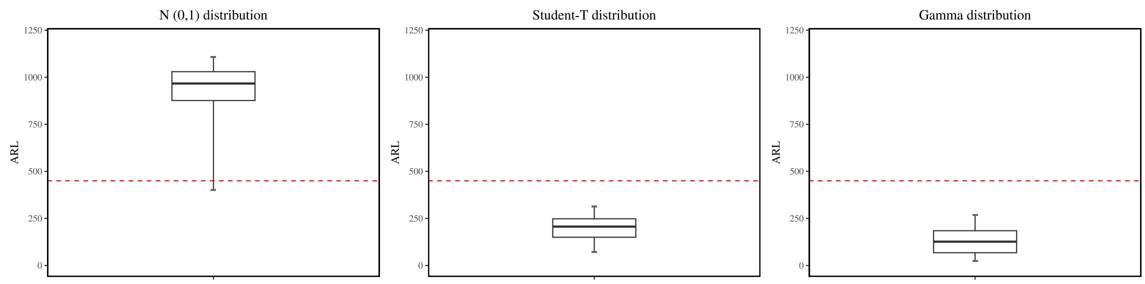

FIGURA 1.4: In-control performance for three different IC distributions

In this research, we propose employing the same control limits designed for cautious learning designed with a normal distribution, following the SNS transformation. The R package 'CautiousLearning' available on CRAN (https://cran.r-project.org/package=CautiousLearning), developed by Capizzi and Masarotto (2020), is utilized for this purpose. Due to the convergence of the SNS to a normal distribution, there is no need to re-calibrate new control limits, this would require a high computational cost.

1.3 SCOPE

It is desired to analyze the effect of applying the SNS transformation for non-parametric monitoring on false alarms and change detection in a control chart utilizing the cautious learning approach. The study utilizes the same control limits established by Capizzi and Masarotto (2020).

1.3.1 RESEARCH QUESTION

Can the cautious learning approach, as proposed by Capizzi and Masarotto (2020), effectively monitor non-normal processes, maintaining consistent results both in-control and out-of-control, by employing the SNS transformation and utilizing the same control limits designed for normal distributions?

1.3.2 HYPOTHESES

Given the convergence of the SNS transformation to a standard normal distribution, as demonstrated by Conover et al. (2017), a non-parametric control chart utilizing this transformation can effectively monitor a process of any distribution. This is achieved by implementing the likelihood-based learning strategy proposed by Capizzi and Masarotto (2020), allowing for consistent results without the necessity to re-calibrate the control limits.

1.4 OBJECTIVES

1.4.1 GENERAL OBJECTIVE

Develop a non-parametric control chart with guaranteed in-control performance that employs cautious learning to accurately monitor process observations from any distribution.

1.4.2 Specific objectives

- Demonstrate that the control limits employed in Capizzi and Masarotto (2020) can be used for monitoring any distribution after applying the SNS transformation, eliminating the need for re-calibration.
- Compare the monitoring performance achieved under different distributions to provide general recommendations for implementing the chart in diverse contexts.

1.5 Contribution

1.5.1 Scientific contribution

To develop the first non-parametric control chart with cautious learning, which is characterized by its guaranteed control performance and high power in detecting real change. This involves implementing the cautious learning approach using the same control limits as proposed by Capizzi and Masarotto (2020), combined with the SNS transformation presented in Conover et al. (2017), leveraging its convergence to the standard normal distribution. Remarkably, the proposed approach eliminates the need for additional calibration, thereby saving significant computational cost.

1.5.2 Contribution for practitioners

This work contributes a new non-parametric control chart that integrates cautious learning and guaranteed-in-control performance. Its application allows for monitoring observations from any distribution, guaranteeing a desired performance and offering higher power in change detection compared to other traditional charts.

Chapter 2

Theoretical framework

Statistical process control (SPC) is the application of statistical tools named control charts in quality process monitoring. Control charts use statistical tests to prove whether the quality value remains within established limits.

The following chapter will delve into the theoretical foundations of SPC, covering its general structure, the design and implementation of the control charts with various monitoring schemes, the effect of estimation on chart performance and different methodologies proposed in the literature to address this issue.This exploration starts from self-methods introduced by Hawkins (1987b) to the cautious learning approach presented by Capizzi and Masarotto (2020). All these aspects are crucial for the development of the current work.

2.1 Phases of SPC

At the moment of start implementing the SPC, the purpose behind its application is determined, whether to establish the initial conditions of the process or to monitor it. Two phases of SPC are considered:

- I. Obtaining the IC distribution of the process
- II. Monitoring of the process

Phase I: Obtaining an IC reference sample

Statistical control begins in this phase, involving the detection of assignable causes of variation, seeking a reference sample with only common causes of variation. This process is iterative, starting with an initial OC sample, and setting control limits as a function of the distribution of this reference sample. These control limits aid in identifying potential observations with assignable cause of variation. If an assignable cause is identified, it is removed. This trial-and-error method is repeated until the sample becomes IC, meaning that all variation in the process is due to common causes. This initial phase of SPC assists practitioners in stabilizing the process before commencing monitoring.

Phase II: Monitoring

As the process generates units over time t, they are stored in random samples of independent observations, each of size n, represented as follows:

$$X_t = X_{t,1}, X_{t,2}, ..., X_{t,n} \quad \forall t = 1, 2, ..., \tag{2.1}$$

each of these samples is referred to as a subgroup.

Subsequently, monitoring is conducted to determine if the process conditions remain stable, as determined in the previous phase. A hypothesis test is performed for this purpose:

$$H_0 : F(X_t) = F_0 \quad vs \quad H_1 : F(X_t) \neq F_0 \quad \forall t = 1, 2, ..., \tag{2.2}$$

here, $F(X)$ represents the distribution function of the process at X, and F_0 is the IC distribution function obtained in the phase I.

The in-control distribution is typically assumed to be normal with parameters μ_0 and σ_0^2. Changes in the location and scale of the process are normally reviewed to evaluate possible deviations in the mean and dispersion of the process. Under the assumption of normality, sequential tests based on the sample mean and variance are used.

Commonly, the monitoring process is conducted using three widely found monitoring schemes in the literature: the Shewhart chart, the EWMA chart, and the CUSUM chart; which are explained in the next Subsection.

2.2 CONTROL CHART

A control chart is an statistical tool used in the monitoring process, offering flexibility in choosing the statistic to use. Here, we will explore the three most commonly used monitoring schemes.

2.2.1 SHEWHART

The Shewhart control chart, proposed by Walter A. Shewhart in 1931 (Shewhart, 1931b) after earlier studies in 1926 Shewhart (1926), was the first monitoring scheme. In the Shewhart control chart, the quality of the process is monitored based on the current sample.Qiu (2014).

However, it is essential to establish the objective for monitoring and decide whether to monitor both the location and the scale of the process. In this work, the focus is on the location parameter, considering that, besides the location parameter, no other parameter changes. This reduces the hypothesis test from (2.2) to the following:

$$H_0 : \mu_i = \mu_0 \quad vs \quad H_1 : \mu_i \neq \mu_0 \quad i > 0, \tag{2.3}$$

μ_0 is the in-control mean of the process, i.e., the location parameter of the F_0 distribution.

As mentioned earlier, a summary measure of the process quality is obtained from each subgroup, denoted by Ti:

$$T_i = \sum_{j=1}^{n} \frac{X_{i,j} - \mu_0}{\sqrt{n}\sigma_0}, \tag{2.4}$$

In this way, the null hypothesis, stating that the mean in control has not changed, is tested.

H_0 is rejected if:

$$T_i < L_i \quad \text{or} \quad T_i > U_i, \tag{2.5}$$

otherwise, the process remains in control.

L_i and U_i represent the lower and upper control limits of the chart at time i, serving as parameters to asses whether the process conditions are maintained.

It is important to note that the statistic T_i possesses the following properties when the process is IC:

$$T_i \sim N(0, 1) \quad i > 0, \tag{2.6}$$

additionally , if the process becomes OC with $\mu_1 = \mu_0 + \delta\sigma$, i.e. a change of size δ standard deviations at time τ:

$$T_i \sim N(0, 1) \quad i < \tau \quad T_i \sim N(\delta, 1) \quad i \geq \tau \tag{2.7}$$

2.2.2 Cumulative sum ($CUSUM$)

While the Shewhart scheme works well for detecting large or isolated changes, it may not be effective in identifying gradual or persistent changes due to its reliance on information from the current period alone. In such cases, monitoring the process with a Shewhart chart becomes less viable.

To address this limitation, alternative monitoring schemes were developed to detect small and persistent changes more efficiently. The CUSUM chart, created by Page (1954) is based on accumulating information from the beginning of phase II to the current period.

Considering individual data $X_1, X_2, ...X_n$, the chart statistic is defined as:

$$C_i = \sum_{j=1}^{i} (X_j - \mu_0) \quad i > 0 \tag{2.8}$$

this is equivalent to:

$$C_0 = 0 \quad C_i = C_{i-1} + (X_i - \mu_0) \quad i > 0$$

The statistic C_i has the following IC and OC distributions, assuming a change of size δ at time τ :

$$C_i \sim N(0, i\sigma_0^2) \quad i < \tau$$

$$C_i \sim N((i - \tau + 1)\delta, i\sigma_0^2) \quad i \geq \tau$$

.

In his work, (Page, 1954) considered monitoring both positive and negative changes with the following two charts.

$$C_i^+ = max\left(0, C_{i-1}^+ + \frac{X_i - \mu_0}{\sigma_0} - k\right) \tag{2.9}$$

$$C_i^- = min\left(0, C_{i-1}^- + \frac{X_i - \mu_0}{\sigma_0} + k\right), \tag{2.10}$$

signaling a change if:

$$C_i^- < -h \quad \text{or} \quad C_i^+ > h \tag{2.11}$$

where $h \geq 0$ represents the control limit used in the charts.

When a change is signaled,equations (2.9) and (2.10) denote the restart mechanism characterizing this chart. In both positive and negative change detection, the value of the statistic returns to 0 after a certain number of monitored observations.

However, one disadvantage of the CUSUM chart is its increasing variation, which becomes more pronounced as the size of the monitored sample increases.

2.2.3 Exponentially weighted moving average $(EWMA)$

In 1959, five years after Page's work with the CUSUM scheme, Robets (1959) proposed the exponentially weighted moving average (EWMA) control chart. That chart also involves accumulating information but assigns weight to each observation in a way that the weight decreases exponentially over time.

The chart statistic is given by:

$$E_0 = \mu_0 \quad E_i = \lambda X_i + (1 - \lambda)E_{i-1} \quad i > 0 \tag{2.12}$$

$\lambda \in (0, 1]$ is the weighting parameter.

The chart statistic E_n has the following IC distribution:

$$E_i \sim N\left(\quad \mu_0, \quad \frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)\sigma^2 \quad \right) \quad i < \tau, \tag{2.13}$$

for large samples the variance of E_i becomes a constant:

$$\lim_{i\to\infty} Var(E_i) = \lim_{i\to\infty} \frac{\lambda}{2-\lambda}\left(1-(1-\lambda)^{2i}\right)\sigma^2 = \frac{\lambda}{2-\lambda}\sigma^2,$$

given its mean, the distribution is now centered on μ_0 instead of 0. Unlike CUSUM, the variability of the statistic remains the same with large sample sizes, making the EWMA chart an excellent option for monitoring processes.

When the process becomes OC, i.e., turns to a $N(\delta, 1)$ distribution at a point τ, the variance of E_n remains the same as shown in (2.13). However, its OC mean is given by:

$$\mu_{E_i,\tau} = \mu_0 + [1-(1-\lambda)^{i-\tau+1}](\mu_1 - \mu_0) \tag{2.14}$$

signalizing a change if:

$$E_i > \mu_0 + \rho\sqrt{\frac{\lambda}{2-\lambda}}\sigma_0 \quad \text{or} \quad E_i < \mu_0 - \rho\sqrt{\frac{\lambda}{2-\lambda}}\sigma_0 \quad i > 0 \tag{2.15}$$

here, the ρ value is chosen as a function of the ARL_0 performance to be achieved.

In this way, three different monitoring schemes are employed, chosen based on the nature of the change to be detected or for comparison. Shewhart charts are used for isolated or medium to big changes; EWMA and CUSUM for gradual or small changes. It has been observed that EWMA is the preferred option for detecting small shifts (Vera do Carmo et al., 2004), especially when they occur early in the process, (Han et al., 2010)

2.3 Antecedents of the reference sample effect over the chart's performance

Regardless of the chosen monitoring scheme, the chart's performance may vary due to differences in the reference samples used by practitioners who implement them. This variation arises during the estimation of distribution parameters and it is referred to as the practitioner-to-practitioner effect. The performance of these charts is further influenced by the prior estimation of in-control parameters, introducing variability to the results.

Parameter estimation from the phase I sample can lead to increased false alarms and impact the efficiency of detecting real changes. (Does et al., 2020). Such effects depend on the errors in estimating the actual parameter values. In the monitoring of localization, overestimating the parameters results in fewer false alarms but slower detection of changes. Conversely, underestimating the parameters leads to frequent false alarm but rapid detection. (Diko et al., 2020).

In this study, parameters updating is implemented, particularly for the initial mean and standard deviation, including the mean and variance of the normal scores derived from the Phase I sample. Additionally, the sample median is considered in the case of conditional sequential normal scores, incorporating learning mechanisms shown in Capizzi and Masarotto (2020). These estimates are dynamically updated during monitoring. Psarakis et al. (2014) suggests that updating estimations during monitoring can mitigate the practitioner-to-practitioner effect. However, but Huberts et al. (2019) warns that such updates might be counterproductive if the chart fails to detect the real changes in the process. In this context, our approach is supported by the findings of Capizzi and Masarotto (2020), demonstrating that applying cautious learning on normal observations is effective in change detection. These results remain applicable to the sequential normal scores because its convergence to a normal distribution, as indicated in Conover et al. (2017).

Over the years, various strategies have been proposed to reduce the practitioner-to-practitioner effect. The key literature concerning on this topic is outlined below.

2.3.1 Self-Starting

In Hawkins (1987b), the self-starting approach for CUSUM charts is introduced, involving the expansion of the reference sample size by incorporating monitored observations as they become available. This approach aims to reduce variability in IC performance by diminishing estimation errors, resulting in larger ARL_0 and smaller ARL_1.

Subsequently, Keefe et al. (2015a) extended the self-starting approach to address the practitioner-to-practitioner effect. Notably, this marked the first time in the literature where the effect of the estimation error on the performance of a self-starting chart was systematically studied. The study revealed minimal variance in performances for both small ($n = 2, 5, 10$) and large ($n = 1000, 2000, 5000$) size samples. However, a challenge emerged during implementation, particularly with small initial samples, where the addition of phase II observations to the reference sample had a substantial influence. If a change occurred in the initial observations, it could be hidden due to the early update of the phase I sample with OC data.

While the self-starting procedure aids in reducing the PPT effect, it carries a risk of not detecting premature changes in the process.

2.3.2 Guaranteed in-control performance ($GICP$)

As an alternative to the self-starting approach, Jardim et al. (2019b) proposed a focus on achieving the desired IC performance ARL_0 instead of over the variation. They introduced a GICP control chart under the exceedance criterion in probability:

$$p_\alpha = P(CARL_0 \geq (1-\alpha)ARL_0) = 1-\beta, \tag{2.16}$$

this criterion establishes that a sufficiently high proportion of in-control performances will exceed the desired ARL_0. The α and β parameters are chosen based on to the desired level of guaranteed performance. In Albers and Kallenberg (2004), an equivalent application of the exceedance-probability criterion is discussed.

To increase the ratio p_α, their proposed method involves calculating the control limits for the chart while maintaining equation (2.16). According to their study, a reference sample of at least 250 observations is required to have guaranteed IC performance at a level of 90 %. Furthermore, Jardim et al. (2019a) extend the analysis to the average and variance, AARL and SDARL, of the in-control performances, revealing that the chart may not control these values, potentially impacting change detection. Thus, although it is possible to guarantee IC performance, balancing it with learning strategies is necessary.

Apart from Jardim et al. (2019a), there are few studies on GICP. Notably, Gandy and Kvaløy (2013) proposed bootstrap methods to compute control limits guaranteeing IC performance. Goedhart et al. (2017) addressed the PPT effect in the implementation of the $\bar{X}$ Shewhart chart. Two years before Faraz et al. (2015) worked with the S^2 chart monitoring variance. Recently, Merlo et al. (2022), explored GICP in multivariate SPC, and Diaz Pulido et al. (2023) implemented GICP in process with finite time horizons.

2.3.3 Cautious Learning with GICP

To guarantee IC performance without affecting change detection, Capizzi and Masarotto (2020) introduced a methodology known as cautious learning, which is employed in monitoring normal processes. This approach combines self-starting and guaranteed in-control performance, updating the reference sample until sufficient evidence is obtained that the phase II distribution is equal to the IC distribution. Likelihood of the estimations for μ_0 and σ_0 based on cumulative phase II data serve as reference.

In the experiments conducted under this methodology, a guaranteed IC performance was observed with variances and averages considerably lower than those reported by Jardim et al. (2019a). In terms of change detection, it exhibited superior performance, showing a significant reduction in variability for changes of any size and under any of the Shewhart, EWMA, and CUSUM schemes.

The results presented by Capizzi and Masarotto (2020) represented an advancement, as they achieved good IC performance alongside fast change detection. However, it is important to note the constraint of assuming normality in the observations.

2.4 Non-parametric control charts

Real-world monitoring conditions are often far from ideal, then, the distribution of the reference is frequently non-normal. To address this limitation, non-parametric were introduced by Bhattacharya and Frierson Jr (1981). These charts utilize ranks transformations in the data to obtain normal observations without altering their inherent characteristics. In the present study, a control chart with guaranteed in-control performance, incorporating cautious learning for samples of any distribution, is employed. For this purpose, a non-parametric chart is introduced using the sequential normal scores transformation (SNS) as presented in Conover et al. (2017). Additionally, for comparison purposes, the sequential ranks transformation (SRT) mentioned in Parent (1965b) is considered.

To underscore the significance of employing non-parametric statistics in statistical process control, one can refer to the works of Celano et al. (2016) on a Shewhart chart with the sign statistic. Further insights into the power of these charts compared to the parametric ones can be observed in Chakraborti and van de Wiel (2008). For a comprehensive review of non-parametric control charts, Chakraborti et al. (2001), Chakraborti (2004) and Chakraborti and Graham (2019b) provide insightful analyses.

2.4.1 Key points in the research

- Keefe et al. (2015a) were the firs to consider the practitioner-to-practitioner variation in the application of self-starting charts. This consideration aimed to reduce deviations in in-control performances. However, it introduced a potential problem in change detection when adding observations to the reference sample too early.

- Four years later Jardim et al. (2019a) proposed a method to guarantee an in-control performance of the charts by estimating parameters based on the exceedance probability criterion. Although achieving a minimal proportion of below-target in-control performance, this approach significantly altered the average performance and its deviation, potentially affecting change detection.

- In an effort to balance the goals of guaranteeing in-control performance and maintaining effective change detection, Capizzi and Masarotto (2020) introduced cautious learning for monitoring location in normal processes. This new learning approach differs from self-starting in that the update to the reference sample is not instantaneous but follows a likelihood criterion. It achieves good results in both in-control and out-of-control scenarios but is limited by its reliance on the assumption of normality in the observations.

- Non-normality in observations would not be a problem if non-parametric transformations are employed. The SNS by Conover et al. (2017) and the SRT by Parent (1965b) are well-suited to such situations.

2.4.2 SUMMARY TABLE

The table below lists the most essential consulted papers during this work. In the table contents, *GICP* refers to guaranteed in-control performance, *CL* refers to cautious learning,*S-S* refers to self-starting, and *PTP* refers to practitioner-to-practitioner variation.

Title	**Author(s)**	**GICP**	**Learning**	**Conditional performance**	**Observations**
Are estimated control charts in control?	W. Albers & W.C.M. Kallenberg	✗	✗	✓	Introduce the exceedance probability criterion to deal with the fact that IC performance can be reached on average but with a significant proportion of performances below ARL_0 due to the estimation error
Effect of the Amount of Phase I Data on the Phase II Performance of S-2 and S Control Charts	Eugenio Kahn Epprecht & Subha Chakraborti	✓	✗	✓	A good option for addressing the estimation error is not treating control limits like constants but as variables that change during phase II
Some Recent Developments on the Effects of Parameter Estimation on Control Charts	Stelios Psarakis, Angeliki K. Vyniou and Philippe Castagliola	✓	✗	✓	There is a need to develop new control charts that give more importance to the estimation error
On the design of control charts with guaranteed conditional performance under estimated parameters	Ronald J.M.M. Does, Rob Goedhart & William H. Woodall	✓	✗	✓	The exceedance probability criterion proposed by Albers and Kallenberg (2004) is highly recommended for dealing with the estimation effect
Self-Starting Cusum Charts for Location and Scale	Douglas M. Hawkins	✗	S-S	✗	Self-starting charts do not require a phase I reference sample because it would be formed during the phase II
The Conditional In-Control Performance of Self-Starting Control Charts	Matthew J. Keefe, William H. Woodall & L. Allison Jones-Farmer	✗	S-S	✓	The S-S approach helps to reduce variation in performance; however, there is a risk of adding new observations too early, which may contaminate it.
Two perspectives for designing a phase II control chart with estimated parameters: The case of the Shewhart $\bar{X}$ Chart	Felipe S. Jardim, Subhabrata Chakraborti & Eugenio K. Epprecht	✓	✗	✓	While IC performance is guaranteed, there is no control over the mean and variance of the ARL.
Guaranteed in-control control chart performance with cautious parameter learning	Giovanna Capizzi & Guido Masarotto	✓	CL	✓	It has GICP and an excellent OC performance. However, it is applicable only for normal process monitoring
Adaptive CUSUM chart with cautious parameter learning	Jun Li	✓	CL	✓	It serves as a good alternative to the cautious learning of Capizzi and Masarotto (2020)
Improved control chart performance using cautious parameter learning	Leo C.E. Huberts, Rob Goedhart & Ronald J.M.M. Does	✓	CL	✓	Different selections of learning constants A and B can improve the chart's performance

Chapter 3

Methodology

In this section, we will discuss two non-parametric methods: the Sequential Normal Scores transformation (and its conditional variants) by Conover et al. (2017) and the Sequential Ranks transformation by Parent (1965b). We adapt these non-parametric transformations in a cautious learning approach and compare their performances across different distributions, using the control limits proposed for normal processes in Capizzi and Masarotto (2020).

Additionally, we will describe key features of the proposed control chart approach, such as guaranteed in-control performance (GICP) and the implementation of cautious learning (CL).

3.1 Non-parametric statistics

In this subsection, we will discuss the non-parametric statistics used in the development of this work, which include:

- Normal Scores transformation by (Conover et al., 2017)
- Conditional Normal Scores transformation by (Conover et al., 2017)
- Standardized Ranks transformation

A reference simple of size m ,$Y = \{Y_1, Y_2, ..., Y_m\}$ and monitoring subgroups $X = \{X_1, X_2, ..., X_n\}$ with $X_i = \{X_{i1}, X_{i2}, ..., X_{in}\} \quad i, n \in \mathbb{Z}^+$ will be considered.

The Normal Scores transformation

The simple range R_{ij} of $X_{ij} \quad j = 1, 2, ..., n$ relative to Y is defined as:

$$RT_{ij} = \sum_{Y_j \in Y} I(Y_j < X_{ij}) + 1, \tag{3.1}$$

Normal Scores (NS) are then obtained from an estimator of $F(x)$, denoted as P_i:

$$P_{ij} = \frac{RT_{ij} - 0.5}{m + 1} \tag{3.2}$$

Conover et al. (2017) demonstrated not only that the sequence $\{P_{ij}\}$ is mutually independent but also that its distribution is uniform (0,1) according to the law of large numbers and the Glivenko-Cantelli Theorem by Tucker (1959). With this, they generated independent observations that converge to a standard normal distribution using the inverse transformation theorem:

$$Z_{ij} = \phi^{-1}\left(P_{ij}\right), \tag{3.3}$$

here, ϕ^{-1} represents the inverse of the normal distribution function. Furthermore, as a con-sequence of the central limit theorem:

$$Z_i = \frac{\sum_{i=1}^{n} Zij}{\sqrt{n}} \sim N(0,1) \tag{3.4}$$

Conditional Normal Scores transformation

When information about a parameter (quantile) θ and its distribution function $F(\theta)$ of the sample Y is available, the NS are conditioned on that value:

$$R_{ij|\theta} = \begin{cases} \sum_{Y_j \in Y_L} I(Y_j < X_{ij}) + 1 & \text{if } X_{ij} \leq \theta \\ \sum_{Y_j \in Y_U} I(Y_j < X_{ij}) + 1 & \text{if } X_{ij} > \theta, \end{cases} \tag{3.5}$$

Y_L represents a subset containing the Y_j values lower than θ, and Y_U a subset containing the Y_j values greater than θ. The calculation of P_{ij}, now $P_{ij|\theta}$, is as follows:

$$P_{ij|\theta} = \begin{cases} F_\theta \dfrac{RT_{ij|\theta} - 0.5}{|Y_L| + 1} & \text{if } X_{ij} \leq \theta, \\ F_\theta + (1 - F_\theta)\dfrac{RT_{ij|\theta} - 0.5}{|Y_U| + 1} & \text{if } X_{ij} > \theta, \end{cases} \tag{3.6}$$

here, $|Y_L|$ and $|Y_U|$ are the cardinalities of Y_L and Y_U, respectively. Conditional Normal Scores with a known quantile (CNSK), $Z_{ij|\theta}$, are obtained as:

$$Z_{ij|\theta} = \phi^{-1}(P_{i|\theta}), \tag{3.7}$$

and Z_i is obtained by applying (3.4) to (3.7).

When the value of θ is unknown, $\hat{\theta}$ and $F(\theta)$ are used to obtain the Conditional Normal Scores with an unknown quantile (CNSU).

The Standardized Ranks transformation

It's evident that the ranks RT_{ij} defined in (3.1) are discretely uniform distributed in $\{1, m+1\}$. Therefore, the standardized sequential rank at time t=i of the j-th observation in the subgroup , SRT_{ij}, is computed as follows:

$$SRT_{ij} = \frac{RT_{ij} - \frac{m+2}{2}}{\sqrt{\frac{((m+1)^2 - 1)}{12}}},$$

similarly, as with Z_i:

$$SRT_i = \frac{\sum_{i=1}^{n} SRT_{ij}}{\sqrt{n}} \sim N(0,1), \tag{3.8}$$

The normal scores and standardized sequential ranks will be utilized to monitor observations from heavy-tailed [1] and asymmetric distributions , complemented by the application of learning strategies. Subsequently, their capacity to guarantee IC performance and detect changes will be compared. The complete implementation will be thoroughly discussed in subsection 3.2.1.

To illustrate how rank and normal score statistics work, as well as the difference it makes to use a reference sample, the following example is given.

[1] "Heavy-tailed" within the meaning of definition 2.4 and theorem 2.6 found in Foss et al. (2011)

Suppose you have information about the grades of 20 high school students in a calculus course. You want to obtain a measure that establishes the position of each student in the group. For this you obtain the ranks and normal scores, as shown below.

Student	Grade	Rank	RT	NS
1	72.30	14	0.61	0.00
2	47.50	6	-0.78	-0.67
3	95.00	18	1.30	0.97
4	14.90	2	-1.47	-1.15
5	17.30	3	-1.30	-0.52
6	93.30	17	1.13	0.67
7	80.40	15	0.78	0.37
8	98.80	20	1.65	1.53
9	98.40	19	1.47	0.97
10	55.60	9	-0.26	-0.39
11	50.20	7	-0.61	-0.47
12	59.30	11	0.09	-0.10
13	68.90	13	0.43	0.00
14	57.00	**10**	**-0.09**	**-0.27**
15	81.30	16	0.95	0.52
16	22.30	4	-1.13	-1.01
17	10.30	1	-1.65	-1.89
18	44.00	5	-0.95	-0.67
19	65.90	12	0.26	0.13
20	52.10	8	-0.43	-0.32

TABLA 3.1: Example: SNS and SRT without a reference sample

In the image above, the most outstanding students are highlighted in green, the students in the middle are highlighted in black, and the students with the lowest scores in the group are highlighted in red. You can see how each statistic reflects this by sending them to the extreme values of the normal distribution.

Now, let's imagine that in addition to the current group's grades we have a record of the previous generation, which consisted of 30 students. Now what is desired is a measure of each student's performance in relation to the grades obtained in the previous semester.

Student	**Grade**
1	34.9
2	56.8
3	37.2
4	5.1
5	13.6
6	66.4
7	49.9
8	51.8
9	72.0
10	46.0
11	78.2
12	73.9
13	91.4
14	30.8
15	92.1
16	2.8
17	67.8
18	37.3
19	85.1
20	20.8

TABLA 3.2: Example: Reference sample of 20 students

Student	**Grade**	**Rank | Ref**	**RT | Ref**	**NS | Ref**
1	72.30	21	0.64	0.51
2	47.50	14	-0.17	-0.08
3	95.00	30	1.68	2.14
4	14.90	4	-1.33	-1.06
5	17.30	4	-1.33	-1.06
6	93.30	29	1.56	1.66
7	80.40	24	0.98	0.81
8	98.80	30	1.68	2.14
9	98.40	30	1.68	2.14
10	55.60	17	0.17	0.16
11	50.20	**15**	**-0.06**	**0.00**
12	59.30	18	0.29	0.25
13	68.90	20	0.52	0.42
14	57.00	18	0.29	0.25
15	81.30	24	0.98	0.81
16	22.30	6	-1.10	-0.81
17	10.30	3	-1.44	-1.21
18	44.00	13	-0.29	-0.16
19	65.90	18	0.06	0.08
20	52.10	16	0.06	0.08

TABLA 3.3: Example: SNS and SRT with a reference sample

These measures have changed, and it is noticeable that now the top 3 students (students 3, 8 and 9) occupy the same position, because they outperform all the grades obtained in the previous year. Whereas without the reference sample they occupied different positions.

3.2 CAUTIOUS LEARNING WITH GICP

A combination of self-starting and guaranteed in-control performance, as discussed in sections 2.3.1 and 2.3.2 is cautious learning by Capizzi and Masarotto (2020). This approach is a parametric monitoring tool that updates the reference sample using a cumulative likelihood criterion and employs the same control limits as seen in Jardim et al. (2019a).

Assuming a Phase I reference sample following a standard normal distribution of size m, $Y_1, Y_2, \ldots, Y_m$, cautious learning initially establishes control limits for monitoring with basis on the fulfillment of condition 2.16. It employs the stochastic approximation algorithm by Polyak and Juditsky (1992) and Ruppert (1988). Subsequently, monitoring begins with the arrival of phase II data $X_1, X_2, \ldots$

Learning and updating in the reference sample occur according to the following parameter:

$$d_{i+1} = \begin{cases} 1 & \text{if } q_i < Ad_i - B, \\ d_i + 1 & \text{otherwise }, \end{cases} \tag{3.9}$$

where:

$$q_i = \sum_{r=i-d_i+1}^{i} \left(\frac{X_r - \bar{Y}_{i-d_i}}{s_{i-d_i}} \right)^2 \tag{3.10}$$

$$\bar{Y}_i = \frac{1}{m+i} \sum_{j=1}^{m+i} Y_j \quad i = 0, 1, \ldots \tag{3.11}$$

$$s_i^2 = \frac{1}{m+i-1}\sum_{j=1}^{m+i}(Y_j - \bar{Y}_i)^2; \quad i = 0, 1, ..., \tag{3.12}$$

A and B are learning constants established based on the size of the change to detect; for the current context, $A = 1.5$ and $B = 50$ are appropriate. The parameter d_i counts how many observations were observed before the cumulative likelihood was sufficiently high. Once its value return to 1, the chart learns from the information provided by those d_i phase II observations. Learning in the chart involves updating the IC estimations as $\hat{\mu}_0 = \bar{Y}_0$ $\widehat{\sigma_0^2} = s_0^2$. The control limits are recalculated as follows:

$$L_{i-d_i} = L_\infty + \Delta_L\sqrt{\frac{m}{m+i-d_i}}, \tag{3.13}$$

where: L_∞: the value that the parameter L_{i-d_i} would have in the case of known parameters

Δ_L: Parameter calculated to guarantee an IC performance.

Monitoring continues until an alarm is issued. Given: $E\left[\sum_{r=i-d_i+1}^{i}\left(\frac{X_r - \bar{Y}_{i-d_i}}{s_{i-d_i}}\right)^2\right] = 1$, the chart learns in average every 100 observations.

L_i is not directly the control limit but serves to obtain it. For the three control charts described in Chapter 2, the control limits are as follows:

UCL: Upper control limit **LCL**: Lower control limit

Recent studies shows other ways to re-estimate parameters under the idea of cautious learning. Li (2022) introduced an adaptive CUSUM chart used to update the parameters. Additionally, Huberts et al. (2022) reviewed various choices from learning parameters A and B in Capizzi and Masarotto (2020).

Scheme	LCL	UCL
Shewhart	$\mu_0 - \sigma_0 L_{i-d_i}$	$\mu_0 + \sigma_0 L_{i-d_i}$
CUSUM	$-L_{i-d_i}$	L_{i-d_i}
EWMA	$\mu_0 - \sigma_0\sqrt{\lambda/(2-\lambda)}L_{i-d_i}$	$\mu_0 + \sigma_0\sqrt{\lambda/(2-\lambda)}L_{i-d_i}$

TABLA 3.4: The control limits for each monitoring scheme

In this work, cautious learning with GICP is extended to monitoring data from non-normal distributions. We assess the performance of the proposed chart with normal, t-sudent, and gamma distributions in Shewhart, EWMA, and CUSUM schemes. We compare the performance of the proposed chart with a sequential implementation of the NS, CNSK and CNSU statistics, which are then referred to in this study as SNS, CSNSK and CSNSU. Additionally, we evaluate the performance of the SRT transformations.

The following algorithm explains the proposed methodology.

3.2.1 ALGORITHM I: A NON-PARAMETRIC CONTROL CHART WITH CAUTIOUS LEARNING

0. Given a reference sample of size m Y ($Y_i \sim N(\mu_0, \sigma_0)$), two learning constants A and B, the learning parameters Δ_L and L_∞, and the GICP values α and β:

1. Do $i = 1$:
2. Receive X_i and apply the selected no-parametric transformation with respect to Y. Store the result in Z_i.
3. Check Z_i with respect to the control limits of the selected monitoring scheme.
   - **3.1**. If the process is OC, set $r = i$ and proceed to step 7
   - **3.2**. If the process remains IC, go to step 4
4. Calculate q_i using equation (3.10).
5. If q_i reaches the threshold from the equation (3.9):
   - **4.1**. Update Y by adding all the monitored observations after the last updating
   - **4.2**. Re-estimate the IC parameters μ_0 and σ_0 based on the updated Y
   - Recalculate the control limits using equation (3.13).
6. Increment i by 1 and return to step 2.
7. Return r and exit

The following diagram graphically presents the described algorithm:

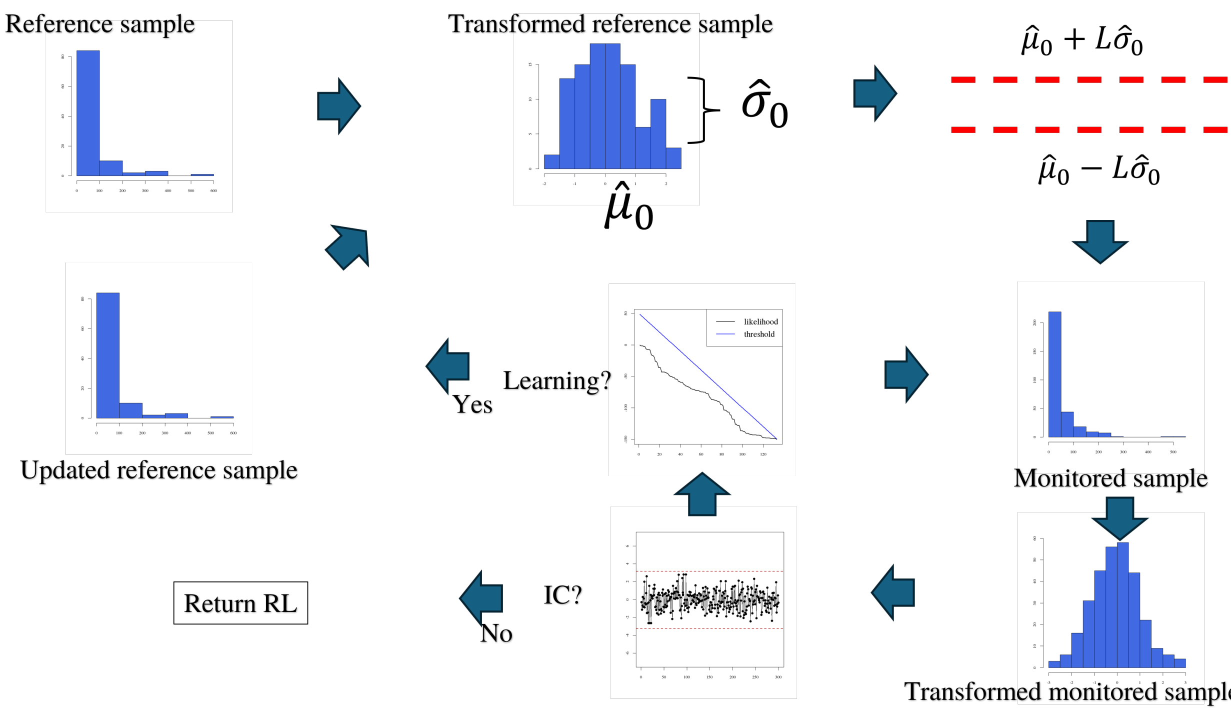

FIGURA 3.1: This algorithm monitors the process until it gets out of control, with the option to learn when there is sufficient evidence that what is currently monitored is indeed in-control.

Chapter 4

Experimental Results

This chapter evaluates the performance of the proposed chart based on the SNS transformation compared to the SRT transformation using the cautious learning approach. Key points of interest include:

- **Compliance with the Exceedance Criterion in Probability:** Tables 4.1, 4.2, and 4.3 review the ratio for all employed statistics , along with $AARL_0$ and $SDARL_0$ metrics. The aim is to maintain the proportion of false alarms at approximately 5 %, ensuring $AARL_0$ is close to 500 with minimal deviation.

- **Performance in control for observations with non-normal distribution:** In the same tables 4.1, 4.2, and 4.3, as well as in figures 4.1, 4.2, and 4.3, a detailed review of these results is conducted. It presents an illustrative comparison between the methods used for various sample sizes and IC distributions.

  Due to the convergence of the employed transformations to the normal distribution, control performance is expected to remain consistent across different distributions.

- **Process shift detection:** Figures 4.4 to 5.14 assess the out-of-control performance of the chart, comparing CL with non-parametric counterparts SNS, CSNSK, CSNSU, and SRT under different distributions and shifts. The proposed chart is expected to enhance results compared to Capizzi and Masarotto (2020) for non-normal distributions and be a viable alternative under the assumption of normality.

4.1 In-control performance

Monte Carlo simulations were conducted to measure the performance of the proposed chart under different parameters.

Parameter	Selection
Monitoring scheme	Shewhart (5), EWMA (ρ =0.2), CUSUM (k=0.5)
Reference sample size	50, 100, 500
In-control distribution	$N(0,1)$, T_4, $G(0.5,1)$
Method	CL, SNS, CSNSK, CSNSU, SRT

Shewhart chart is evaluated with subgroups for optimal performance, while CUSUM and EWMA charts exhibit good performance even with individuals.

Notation used in this chapter includes:

- **CL:** Cautious learning
- **SNS:** Sequential normal scores with cautious learning

- **CSNSK:** Conditional sequential normal scores to a known median with cautious learning

- **CSNSU:** Conditional sequential normal scores to an unknown median with cautious learning

- **SRT:** Standardized sequential ranks transformation with cautious learning

Shewhart															
	Normal					*t*-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	857.08	851.53	**2 951.14**	743.07	758.69	193.51	841.10	**2 946.18**	749.83	754.76	123.43	832.51	**2 937.22**	741.08	749.02
$SDARL_0$	189.90	370.42	452.19	155.98	206.51	167.44	167.44	423.09	189.56	169.61	73.00	175.74	429.65	159.25	196.70
$p_{0.10}$	0.058	0.018	0.000	0.040	0.041	0.996	0.012	0.000	0.060	0.065	1.000	0.024	0.000	0.038	0.054
EWMA															
	Normal					*t*-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	873.57	801.13	**2 154.01**	758.61	755.98	249.29	805.75	**2 138.07**	747.01	768.74	172.71	798.75	**2 163.09**	745.80	765.35
$SDARL_0$	187.50	139.79	310.23	157.50	164.15	85.77	134.18	326.18	173.07	157.38	92.96	128.40	339.43	170.16	151.79
$p_{0.10}$	0.049	0.007	0.000	0.028	0.040	0.981	0.005	0.000	0.049	0.030	0.996	0.006	0.000	0.054	0.018
CUSUM															
	Normal					*t*-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	966.40	846.02	**1 257.82**	797.73	807.57	272.10	839.81	**1 257.73**	802.94	812.96	242.14	849.42	**1 257.31**	794.78	799.97
$SDARL_0$	228.88	151.37	196.83	180.72	181.93	89.70	143.78	205.75	180.17	168.43	114.59	158.25	199.94	181.42	175.75
$p_{0.10}$	0.055	0.011	0.000	0.047	0.043	0.971	0.013	0.000	0.049	0.032	0.965	0.017	0.000	0.046	0.035

TABLA 4.1: Cautious Learning IC performance for $m = 50$

Shewhart															
	Normal					t-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	755.90	755.12	**2 679.08**	670.83	668.03	172.18	750.73	**2 678.54**	666.55	671.92	113.97	748.97	**2 670.41**	667.83	675.14
$SDARL_0$	163.34	79.76	206.18	77.28	95.81	72.01	74.10	186.68	75.00	87.58	62.58	76.69	180.54	86.17	89.46
$p_{0.10}$	0.045	0.011	0.000	0.020	0.041	0.996	0.008	0.000	0.024	0.034	0.999	0.009	0.000	0.034	0.032
EWMA															
	Normal					t-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	743.00	715.32	**2 024.62**	658.36	668.43	249.29	713.36	**2 020.53**	662.96	668.73	161.71	710.52	**2 027.25**	665.53	670.30
$SDARL_0$	159.05	67.05	198.36	93.78	79.10	85.77	63.71	201.33	87.01	77.63	79.66	65.25	206.39	84.06	75.97
$p_{0.10}$	0.051	0.006	0.000	0.038	0.023	0.981	0.005	0.000	0.029	0.016	0.994	0.007	0.000	0.027	0.021
CUSUM															
	Normal					t-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	770.90	709.46	**1 053.10**	656.64	664.98	335.70	713.09	**1 053.70**	656.84	663.66	196.14	709.8	**1 048.34**	657.54	666.27
$SDARL_0$	169.70	64.91	86.79	91.38	78.78	120.70	66.39	84.37	87.56	78.85	95.21	56.07	88.64	89.03	76.69
$p_{0.10}$	0.045	0.012	0.000	0.037	0.031	0.864	0.012	0.000	0.036	0.032	0.986	0.004	0.000	0.040	0.028

Tabla 4.2: Cautious Learning IC performance for $m = 100$

Shewhart															
	Normal					t-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	668.51	657.7	**2 370.00**	619.64	619.48	160.54	655.98	**2 370.83**	619.04	618.13	101.95	659.93	**2 366.12**	618.46	622.03
$SDARL_0$	128.88	41.55	47.27	46.30	30.56	51.37	43.28	64.97	43.52	43.46	37.53	42.36	69.51	43.86	41.40
$p_{0.10}$	0.042	0.000	0.000	0.000	0.002	0.997	0.001	0.00	0.001	0.000	1.000	0.000	0.000	0.001	0.001
EWMA															
	Normal					t-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	643.66	624.64	**1 771.74**	598.40	600.23	240.91	626.39	**1 772.72**	597.20	597.94	156.97	624.44	**1 769.41**	595.93	600.23
$SDARL_0$	110.10	68.64	40.99	72.29	66.63	68.18	65.18	40.86	71.70	69.87	51.02	66.81	41.42	71.71	68.29
$p_{0.10}$	0.039	0.009	0.000	0.022	0.017	0.981	0.007	0.000	0.027	0.023	1.000	0.006	0.000	0.024	0.019
CUSUM															
	Normal					t-Student					Gamma				
Metric	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**	**CL**	**SNS**	**SRT**	**CSNSK**	**CSNSU**
$AARL_0$	642.45	628.67	**922.69**	595.74	599.35	303.13	625.49	**923.26**	594.61	599.84	175.45	623.9	**923.74**	590.15	599.72
$SDARL_0$	117.17	63.87	42.37	65.87	68.42	265.63	65.96	42.59	67.35	63.07	56.84	66.78	43.92	68.79	66.27
$p_{0.10}$	0.029	0.005	0.000	0.016	0.017	0.950	0.004	0.000	0.029	0.020	0.997	0.010	0.000	0.026	0.017

Tabla 4.3: Cautious Learning IC performance for $m = 500$

SNS, CSNSU, CSNSK and SRT all work, as they are guaranteed to have a near 5 % IC performance ratio below the ARL_0. On the other hand, CL has practically only false alarms during its monitoring of non-normal processes.

Even though SRT works by having few false alarms, it is of concern that it shows much higher AARLs than the rest, especially in Shewhart and EWMA schemes. This can lead to problems in detection because it would take longer for the chart to detect a real change. The image at the end of this section illustrates the relationship between false alarm probabilities and change detection.

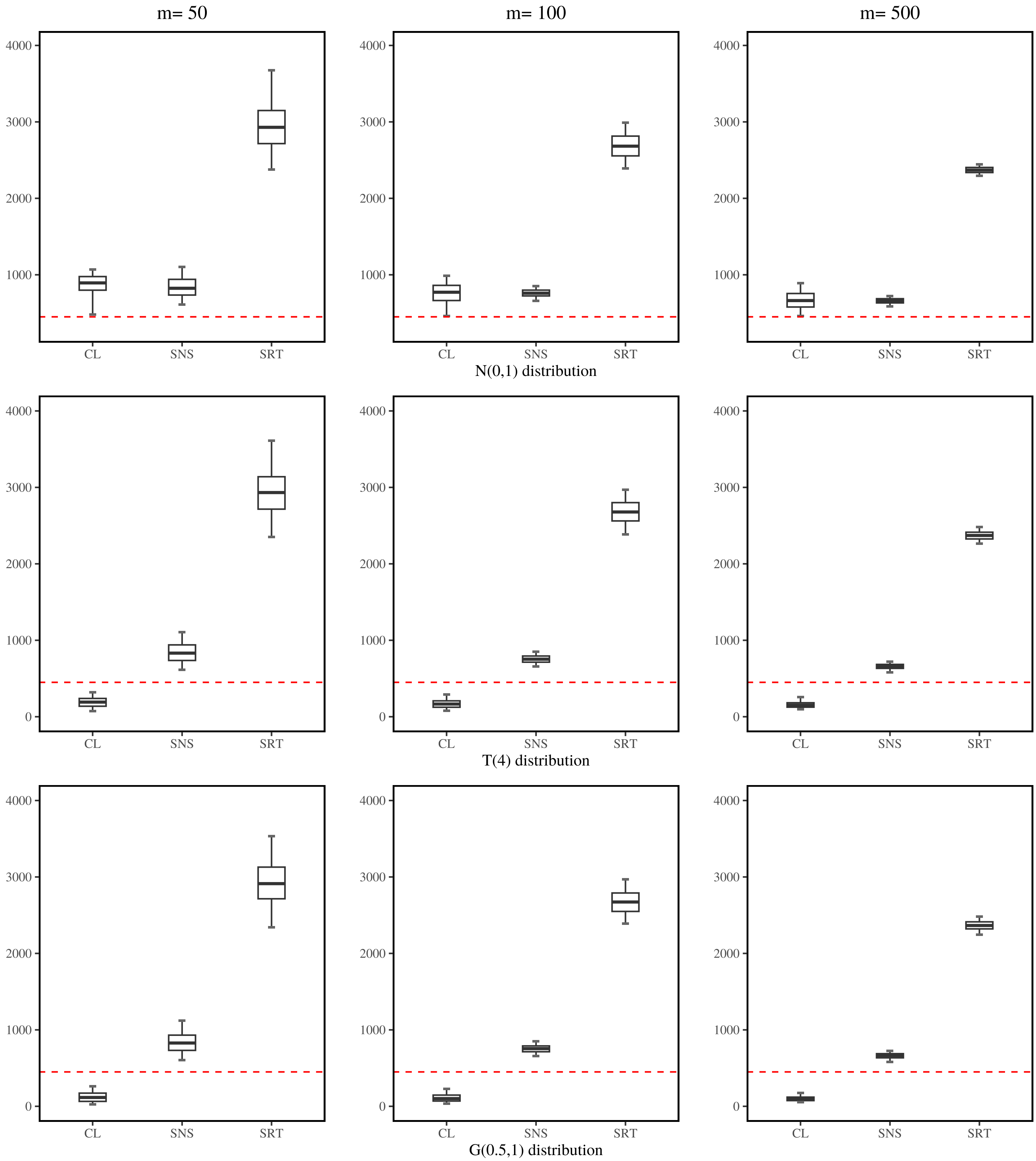

FIGURA 4.1: In-control performances from Shewhart ($n = 5$)

Box-plots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

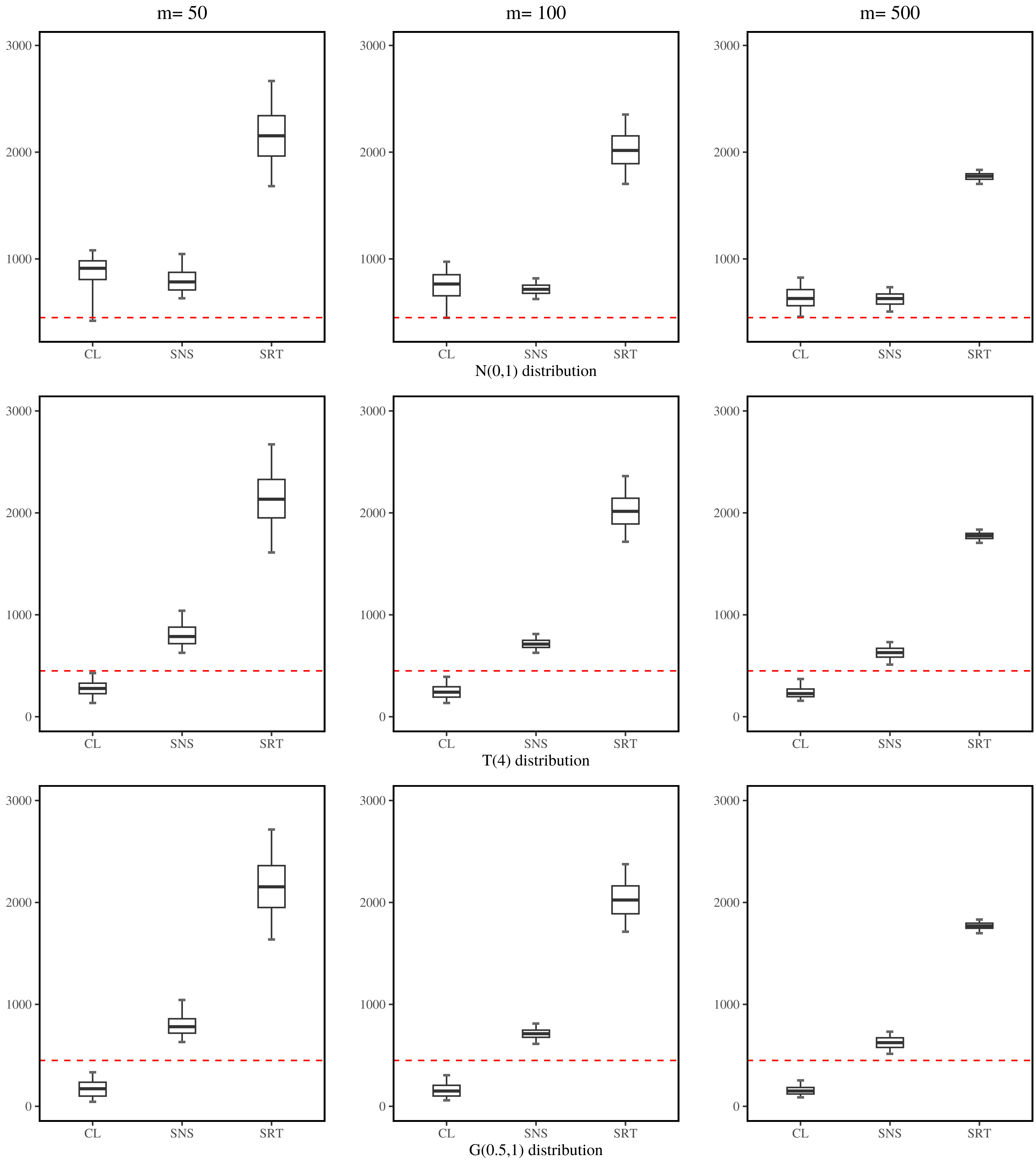

FIGURA 4.2: In-control performances from EWMA (0.2)

Box-plots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

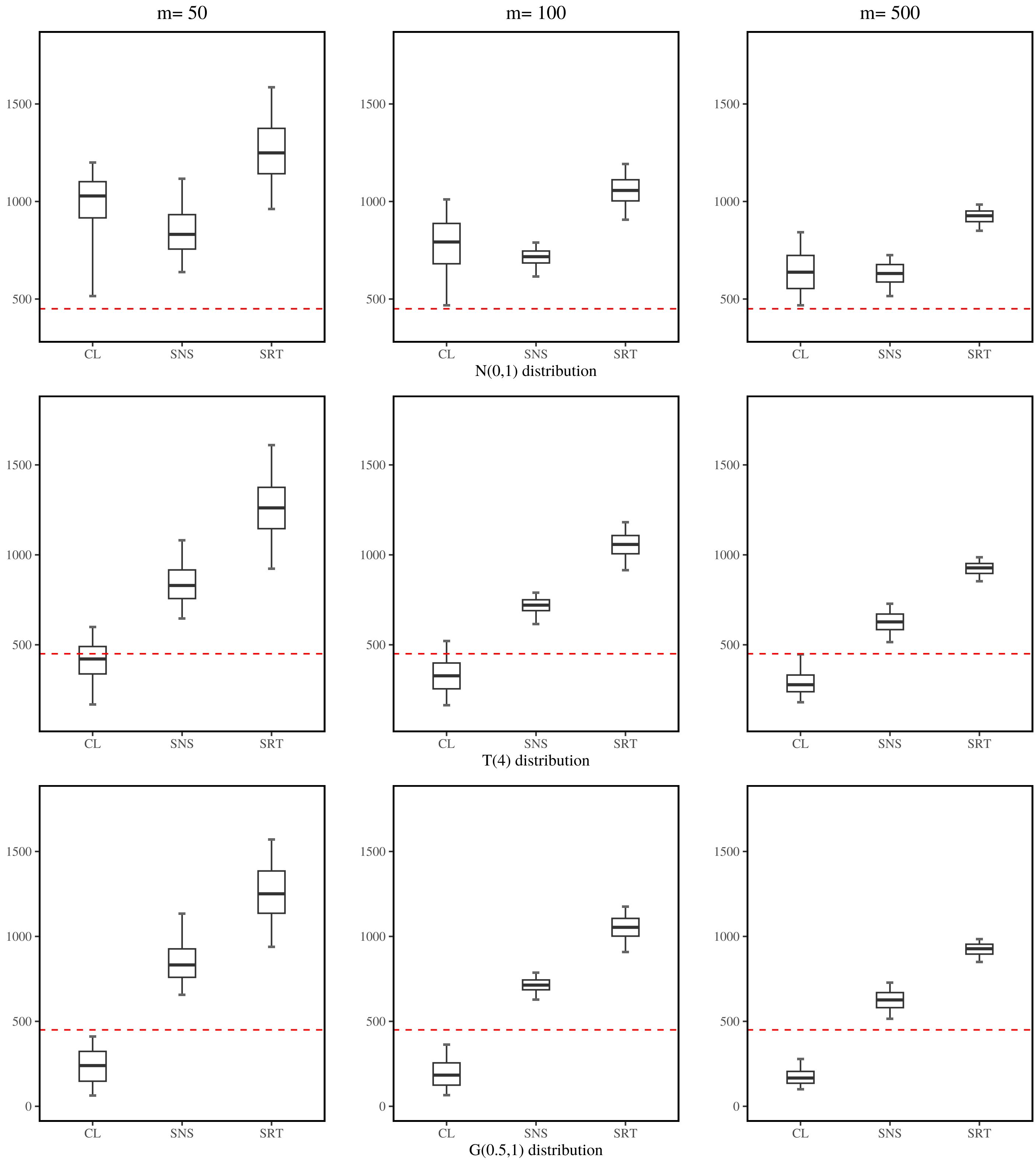

FIGURA 4.3: In-control performances from CUSUM (0.5)

Box-plots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

Under the assumption of normality in the observations, the non-parametric transformations yield excellent in-control results, guaranteeing in-control performance, with a particular emphasis on SNS and its variants. However, SRT proves to be highly conservative, resulting in very large ARLS. The next step is to test the OC performance of the chart, considering the exceptional results observed in the IC scenario.

4.2 Out-of-control performance

Having established the IC scenarios and confirmed the convergence of statistics to the normal distribution, this section examines the chart's power. The comparison focuses on the SNS, CSNSK, CSNSU, and SRT statistics.

Simulations were conducted to evaluate the out-of-control performance of the chart when the change occurs at the start of monitoring (τ =1) or after 150 observations (the chart's learning is every 100 on average). The goal is to explore the effectiveness of the proposed tool compared to the parametric alternative, recognizing the limitations of the latter when the normality assumption is not met. For any non-normal distribution, the review is limited to the non-parametric proposals, emphasizing the differences based on the monitoring scheme and distribution.

Parameter	**Scenario**
Monitoring scheme	Shewhart, EWMA, CUSUM
Reference sample size	100
In-control distribution	$N(0,1)$, T_4, $G(0.5,1)$
Method	CL, SNS, CSNSK, CSNSU, SRT
δ	1
τ	1,150

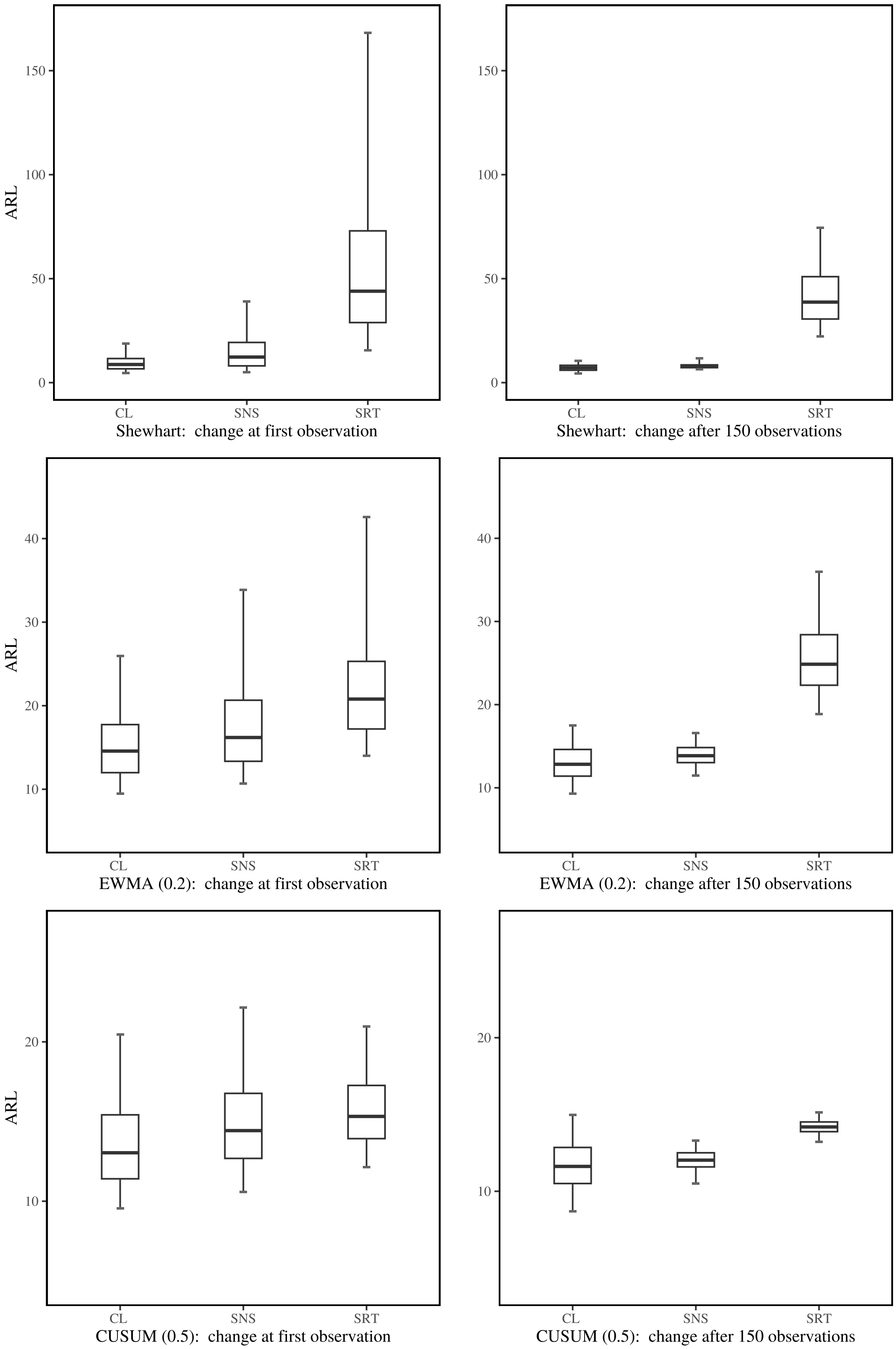

Figura 4.4: Out-of-control performances for N(0,1) distribution

Box-plots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

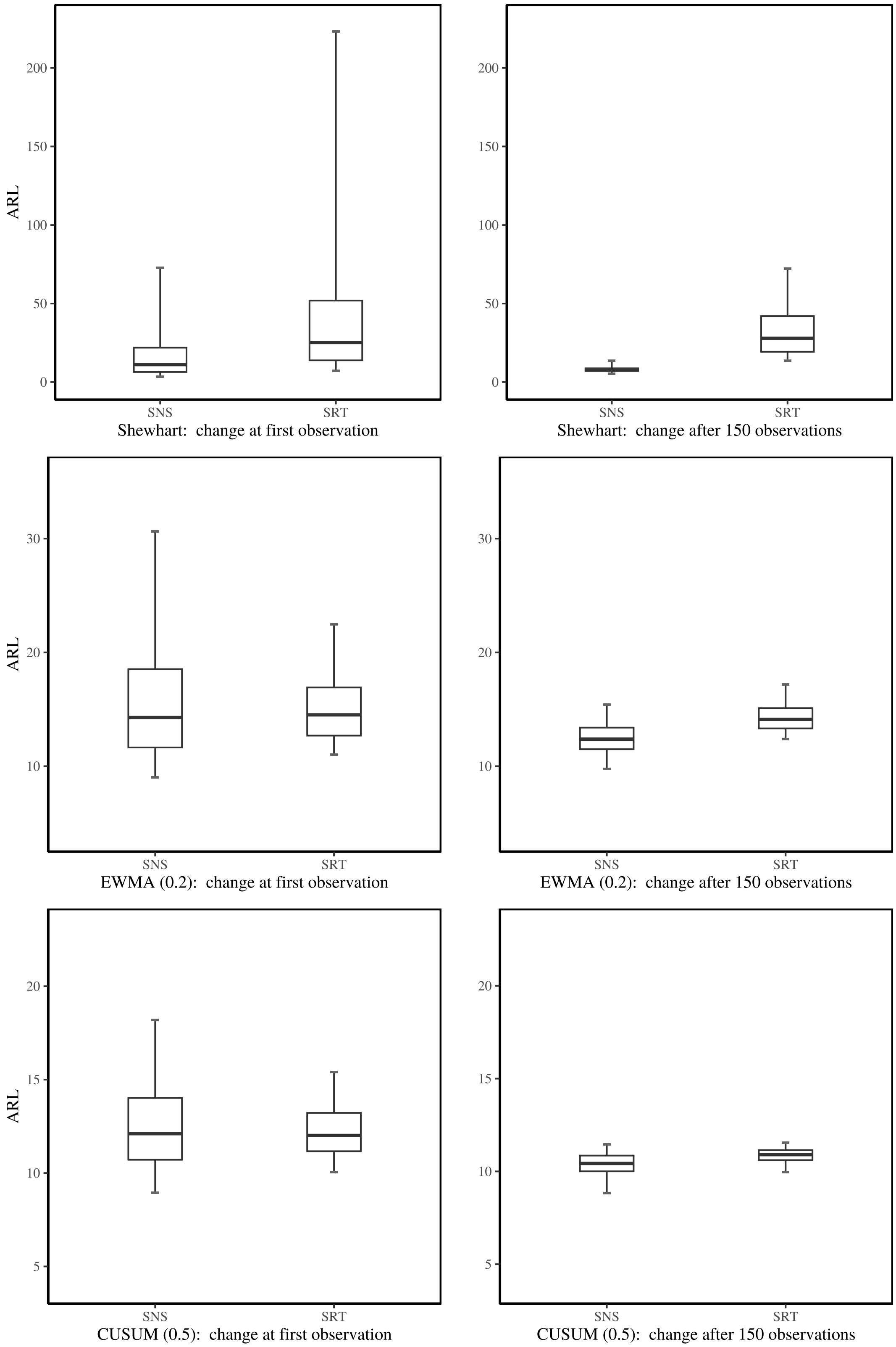

Figura 4.5: Out-of-control performances for T_4 distribution

Box-plots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

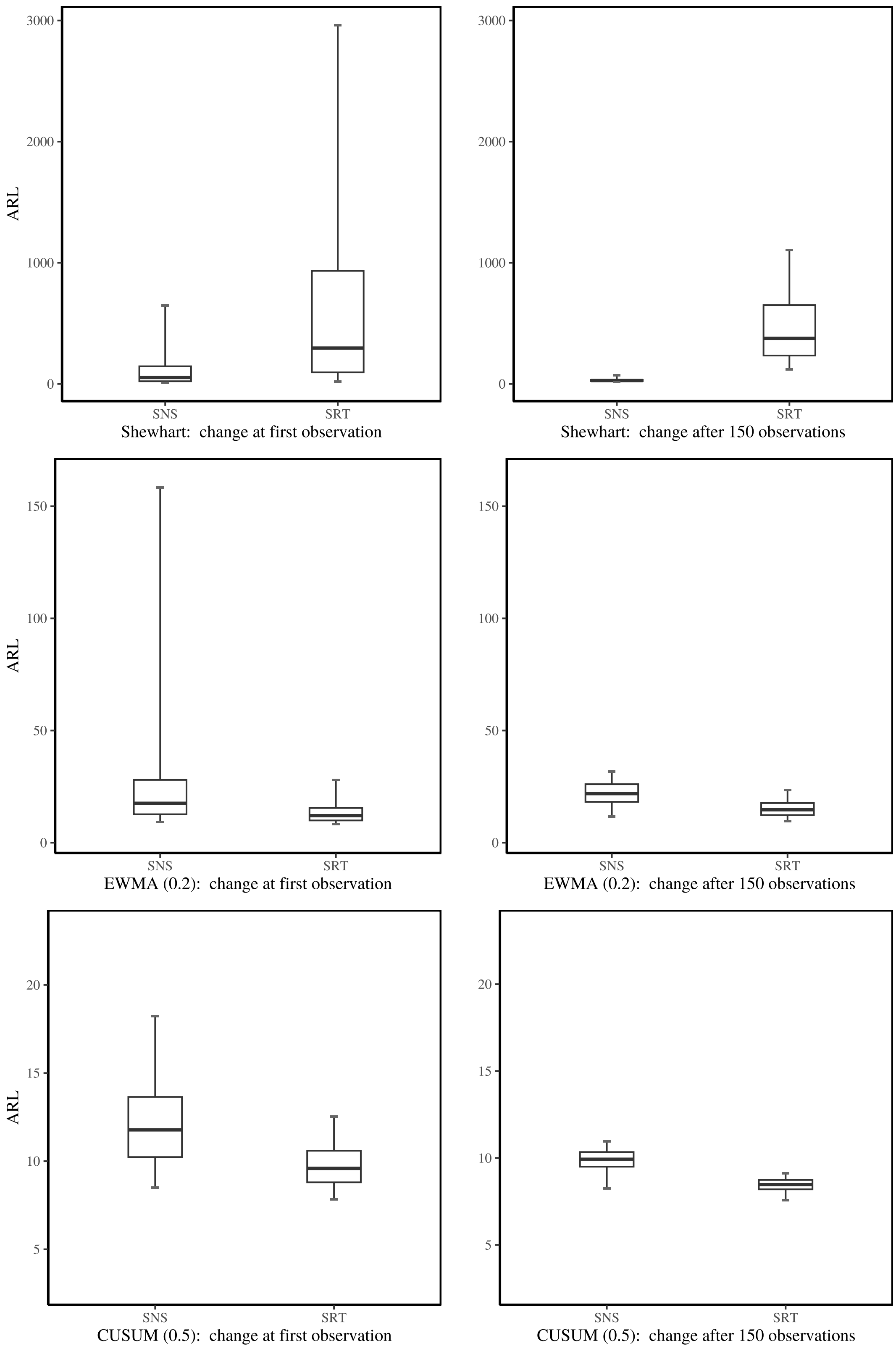

Figura 4.6: Out-of-control performances for G(0.5,1) distribution

Box-plots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

In t-student or Gamma observations, it can be observed that under EWMA and CUSUM schemes, there is no significant difference between SNS and SRT for positive changes. Both transformations prove to be highly effective in change detection. However, under Shewhart, the detection by SRT is dramatically slower. For negative changes in an asymmetric distribution, SNS outperforms SRT, even reaching its best performance in the experiment.

The performance of CSNSK and CSNSU was practically the same as SNS, indicating that the condition of the quantile does not significantly affect the chart's performance.

4.3 Discussion

Guaranteed in-control performance

- Tables 5.3, 5.4 and 5.5 demonstrate that the IC performance is guaranteed in practically all scenarios, with $p_{0.10}$ values equal to or even below 5 %, highlighting the SRT with values close to 0.

- Particularly for normal observations, the convergence of non-parametric statistics to such a distribution helps achieve guaranteed in-control performances, as presented in Capizzi and Masarotto (2020). It's essential to note that SRT is very conservative, having a practically null false alarm rate, and for this reason it obtain very high $AARL_0$ values.

- When the normality assumption is not fulfilled, the recorded $p_{0.10}$ values show that it is no longer possible to guarantee in-control performance under the parametric scheme. It leads to practically only false alarms during monitoring. On the other hand, for SNS and SRT, this is successfully achieved, obtaining results very similar to those obtained following the normality assumption.

- Figures 5.1, 5.2 and 5.3 provide a graphical summary, showcasing the difference between SRT results and the rest of the tested statistics while still remaining above the desired performance. The variation between the $ARLs$ is practically minimal, decreasing as the sample size increases.

OUT-OF-CONTROL PERFORMANCE

- Figure 4.4 shows very similar results among almost all the statistics, with a notable exception under the Shewhart scheme. In this case, SRT, which had been very conservative in the IC scenario, presents the slowest detection among the evaluated statistics . On the side of EWMA and CUSUM schemes, the differences are practically imperceptible, with SRT being slightly faster than SNS and its variants for change detection, especially for Gamma observations.

- Figures 4.5 and 4.6 present the same observations as 4.4, with the difference that we are evaluating a scenario where normality is not fulfilled in the observations, yielding mixed results among the different statistics. In this case, there would not be a favorite option, but any of these could serve as a good alternative in monitoring. Particularly, SRT performs better under CUSUM and EWMA schemes, while SNS and its variant conditions outperform SRT by far when a Shewhart chart is used.

- Figures 4.7-4.14 extend the analysis by detailing what happens to the detection under different magnitudes of change. This allows us to observe that the symmetry of the distribution has a strong influence, as under an asymmetric Gamma distribution, the detection is practically instantaneous when the change is negative. Conversely, in a Normal distribution, given its symmetry, there is no noticeable difference between positive and negative changes. Also, in these figures, simulations of different positive magnitude changes for CL and SNS were presented.

4.4 PRACTICAL EXERCISE

As a practical application of the tool developed in this study, records on health in Mexico were considered, obtained from Secretaría de Salud (2022) . Data of people from the 32 states of the republic were taken.

The database includes 4363 records, each with 36 variables, including hypertension risk, blood pressure, body mass index, among other bio-metric measures. This database was analyzed by the control chart developed, after applying a logistic regression classification model to it. [1]

First, data cleaning was performed, eliminating duplicate or missing records, leaving us with 1540 data from 8 different variables. Subsequently, a logistic regression model was adjusted, where the objective was to classify each patient as at-risk or not at-risk for hypertension, considering the risk variable as dependent on the rest.

Age	Waist (cm)	Blood pressure	Sleep time (hours)	BMI	Exercise (h)	**Hypertension risk**
57	115.4	130	2	33.38	260	1
32	79.1	83	4	22.61	290	0
...	...	...	...	...	...	...

TABLA 4.4: Data set from Kaggle. Encuesta Nacional de Salud y Nutrición México 2022

The model obtained was as follows:

[1]A brief explanation of the use of this can be seen in Appendix II at the end of the document

$$\hat{y} = -15.7003 - 0.9653x_1 - 0.0005x_2 + 0.0697x_3 + 0.3206x_4 \tag{4.1}$$

- y: Hypertension risk measure
- x_1: Hours of sleep
- x_2: Minutes of exercise per week
- x_3: Blood preassure
- x_4: Body Mass Index

80 % of the total base was considered to build it, leaving the remaining 20 % (308 observations) to measure its performance and for subsequent risk monitoring.

The performance obtained in the regression model was as follows:

Prediction / Observed	No risk	Risk
No risk	95	18
Risk	24	171

TABLA 4.5: 88 % of people at risk of hypertension were correctly classified as at-risk group.

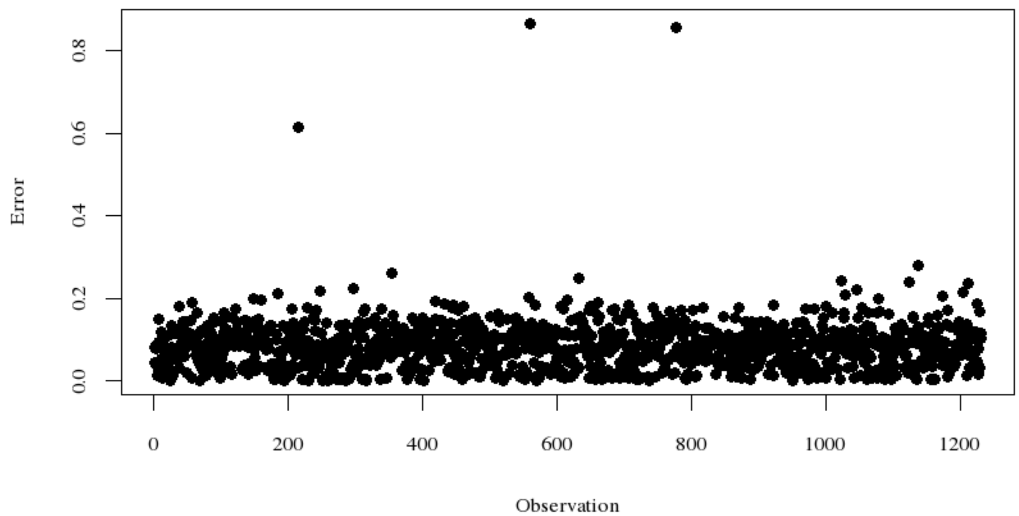

FIGURA 4.7: Variation in model-adjusted risk probability

Once the classification was obtained, the risk status of each individual was monitored, using a reference sample of 50 persons with an average risk-probability of 0.29 and standard deviation of 0.22. This would seek to detect a change in hypertension risk within the population, with this change being at least the error shown by the model. Below are the results obtained by the monitoring, using the EWMA (0.2), and CUSUM (0.5) schemes and considering a positive change [2] of 1.5 standard deviations for Shewhart and 1 standard deviation for EWMA and CUSUM. The change point was considered at τ=1 for shewhart, at τ=1,150 for EWMA and at τ=1,150,300 for CUSUM.

[2]Considering the deviations of the model, which behave mostly in the range of 0 to 20 % (Figura 4.7), detecting a change of δ=1.5,2, which is equivalent to 22 % and 33 %, is reliable.

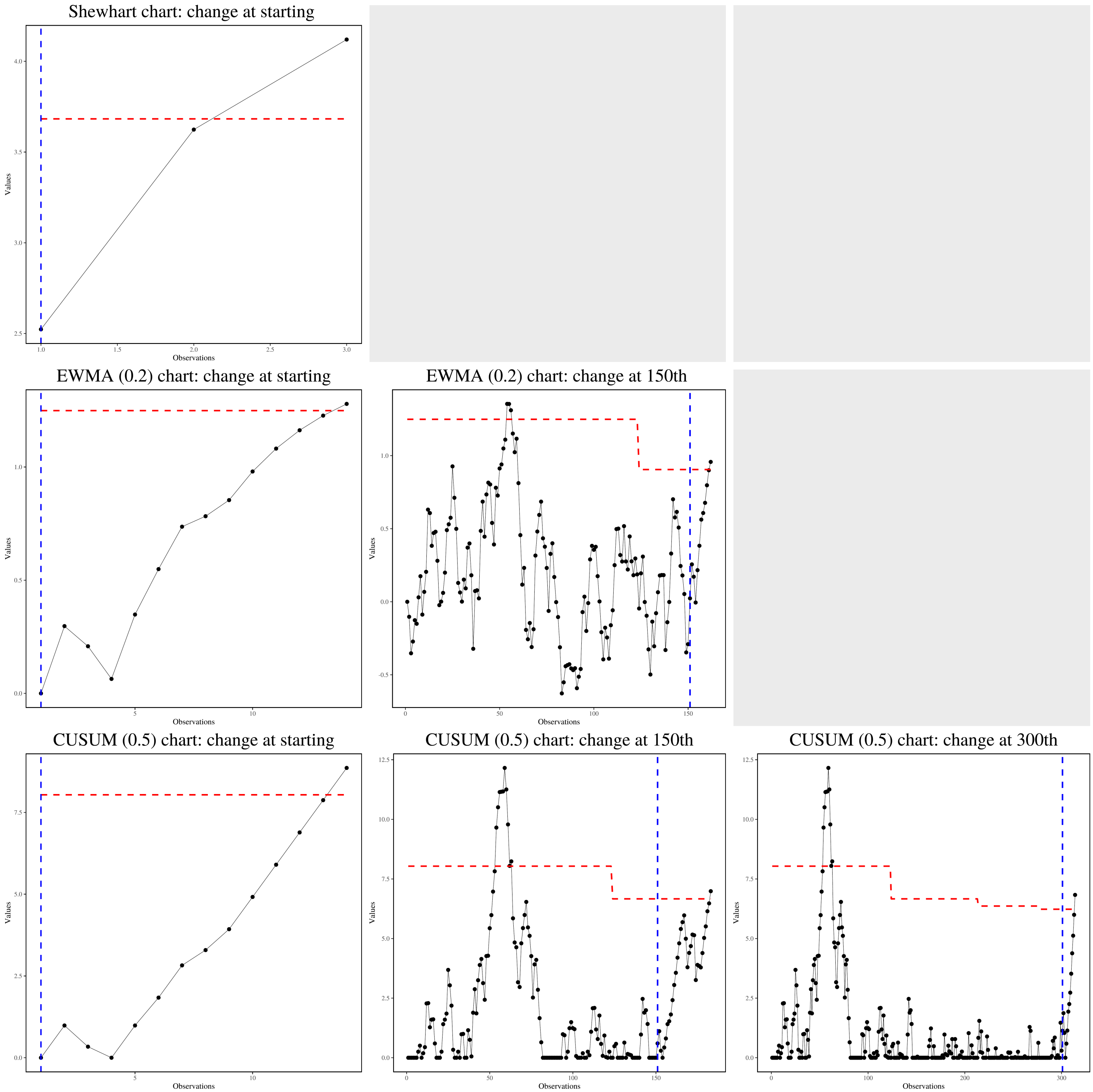

FIGURA 4.8: Prevention of hypertension in patients

In general, the chart performs quite well under all 3 schemes, both in a small and in a large change. The detection is fast under the Shewhart scheme, with only 3 observations, while for EWMA and CUSUM, the 1 standard deviation change was detected more quickly as the control chart learned, this is observed once in EWMA and up to 3 times in CUSUM.

The response time to a health emergency is represented by the speed of detection of this tool, which is very quick.

Chapter 5

Conclusions and future research

The proposed non-parametric control tool works in the monitoring of processes coming from normal and non-normal distributions, such as t-student and Gamma. It also gives an added value the use of cautious learning, where it extends the results obtained by Capizzi and Masarotto (2020), for the first time in the literature.

The application of this tool is simple by virtue of the convergence of the statistics used, since it was demonstrated that it works using the same control limits as in the parametric case, thus saving a considerable amount of computational time.

Within the in-control monitoring, each statistic guarantees a minimum performance, highlighting that the SRT presents too conservative results. As for change detection, all statistics perform well, with SNS and its variants under the Shewhart scheme being the best choice, while in the EWMA and CUSUM charts any statistic could well be chosen with consistently good results.

The developed tool can be extended to the monitoring of other population parameters such as variance or coefficient of variation, so its application would be extended if developed for multivariate processes.

Autobiographic summary

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Tesis:
NONPARAMETRIC CONTROL CHART WITH GUARANTEED IN-CONTROL PERFORMANCE USING CAUTIOUS LEARNING

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Appendix

As an appendix to this thesis there are 3 main documents, the first one, which is shown below, is a research work derived from the present project. The following two complement the explanation given in this paper, either by way of examples or theoretical material.

5.1 The Sequential Normal Scores Approximation as a Bridge Between Parametric and Nonparametric Statistical Process Monitoring

Alvaro E. Cordero-Franco, Gerardo Pérez-Arriaga, Víctor G. Tercero-Gómez, Luis A. Benavides-Vázquez, William J. Conover Cordero-Franco et al.

Emphasis on power has led to the creation of increasingly complex, and even doubtful, approaches in nonparametric process monitoring research. A proper method is as useful as it is likely to be used. Computational complexity, design limitations, and reduced interpretability are common barriers that practitioners face when choosing a chart. Sequential normal scores (SNS) transformation, a nonparametric sequential linear rank transformation where only the most recent observations are ranked and transformed into independent normal scores, offers a solution that alleviates some of these problems by extending the use of methods designed for independent and normal observations. Alternatives using normal scores show power levels comparable to the best tradition of rank transformation methods when in-control behavior is matched. The normal approximation reduces the need to rely on special lookup tables and case-by-case numerical calibrations. By avoiding re-ranking, computational complexity is reduced. Finally, as a bonus, interpretation is facilitated by using a Gaussian reference. With the natural updating capabilities of SNS, novel approaches such as cautious learning with guaranteed performance are readily available for practitioners with no special set-up required but a pre-processing transformation. Location monitoring results show robustness to distribution with power equivalent to parametric counterparts when assumptions hold.

5.1.1 INTRODUCTION

The increasing emphasis on data-driven approaches in the era of Industry 4.0 has driven the need for robust *statistical process monitoring* (SPM) techniques in manufacturing and service processes. Control charts have been widely adopted as a primary tool for online process monitoring since their introduction by Shewhart (1931a), with traditional methods predominantly relying on parametric assumptions. However, these assumptions may not hold in practice due to various reasons, such as noncompliance with desired distributional assumptions or limited availability of reliable data. As a result, nonparametric approaches have emerged as an alternative solution to address these challenges.

Nonparametric SPM methods offer flexibility by not assuming specific probability distributions for the underlying process. Instead, they rely on appropriate transformations that allow for distributional assessment, even when the original data distribution remains unknown. Two commonly used transformations in nonparametric SPM are dichotomization and rank transformation, the latter being favored for its higher power (Conover and Iman, 1981).

Rank transformations, based on the application of traditional parametric procedures to the ranks of the data, have demonstrated strong statistical properties (Conover and Iman, 1981; Zimmerman, 1992). These transformations provide an opportunity to leverage well-established parametric methods that assume normality, enabling practitioners already familiar with classical parametric approaches to adopt nonparametric SPM techniques (Conover, 2012).

Although the nonparametric SPM literature has proposed various rank-based methods to monitor process behavior (Li et al., 2010; McDonald, 1990), these approaches often rely on numerical routines, lookup tables, and distributional approximations to ensure desired performance in specific application scenarios. Moreover, some existing nonparametric methods suffer from limitations such as low detection capability, excessive variation in run length, and dependence on numerical analysis, tables or poor approximations for their implementation (McDonald, 1990).

A method is as good as it is likely to be used. Power to detect a change in a process is important, however, it is only a factor to consider when choosing a method to use. Easy-of-use and interpretation are key elements for practitioners. Sadly, most research mostly focus on power, leaving aside the latter elements. Approaches with these elements, including high power, can be constructed with the *sequential normal scores* (SNS) transformation, a technique introduced by Conover et al. (2017). Such a transformation extends the application of existing methods that assume normality to the monitoring of non-normal measurements. The SNS transformation is a nonparametric sequential linear rank transformation that addresses several limitations of existing nonparametric approaches. It offers the advantages of reduced computational complexity, improved interpretability, and extended applicability of methods designed for independent and normally distributed observations. By avoiding the need for reranking and utilizing a Gaussian reference, the SNS approach reduces reliance on special lookup tables and case-by-case numerical calibrations, thus simplifying implementation. When scores are not obtained sequentially they are called *normal scores* (NS) for simplicity.

Rank-based methods and sequential approaches have been extensively analyzed in the literature when dealing with process monitoring (Parent, 1965a; Reynolds and Marion, 1975; McDonald, 1990; Hackl and Ledolter, 1992; Gordon and Pollak, 1994; Yakir, 1998; Andreou and Ghysels, 2003; Zou and Tsung, 2010; Li, 2011; Tapang and Pongpullponsak, 2012; Liu et al., 2013; Mao et al., 2013; Liu et al., 2014; Nazir et al., 2013; Zhang, 2014;

Zhang and Chen, 2014; Qiu and Zhang, 2015; Zheng and Chakraborti, 2016). See Chakraborti and Graham (2019a) and Chakraborti and Graham (2019b) for a recent review and overview of different nonparametric monitoring approaches. However, these approaches often rely on numerical analysis or large sample approximations for their implementation. NS and SNS offer an alternative where the normal approximation provides a good fit even with small samples.

Following the SPM literature, Capizzi and Masarotto (2020) disrupted the way we approach monitoring by considering how the estimation of parameters can be addressed and improved with a cautious learning scheme that provides guaranteed in-control performance. They provided a parametric solution to deal with normal observations. Following the approach of Conover (2012), we can extend these results to deal with non-normal observations by using this procedure, with the corresponding decision criteria, on SNS and SRT scores. With the use of these nonparametric transformations, we are proposing, probably for the first time, a nonparametric cautious learning scheme with guaranteed performance. The distribution of the *average run length* (ARL) conditioned on the phase I sample, conditional performance for short, is used for the analysis.

In this paper, we focus our attention on cautious learning approaches with guaranteed performance. However, together with the *supplementary material*, we evaluate the use of NS and SNS as transformations to use with existing parametric approaches that assume normality. This is, NS and SNS are evaluated with a Shewhart $\bar{X}$ chart, an EWMA scheme, and a CUSUM procedure. We compare the run length distribution of these transformations with the parametric counterparts, under in-control and out-of-control scenarios. We also add to the comparison the use of the *rank transformation* (RT) following the sense of Conover (2012), where a RT and a *sequential rank transformation* (SRT) are used with the same monitoring schemes. Control limits used with NS, SNS, RT and SRT are the same as the ones used in a Shewhart, EWMA and CUSUM chart that assume that observations are normal. Hence, we are evaluating the normal approximation of these nonparametric transformations.

To facilitate the presentation of findings, this paper presents results on cautious learning, the modern approach at the time this research was done. However, additional results following similar conclusions about the goodness of NS and SNS can be found as supplementary material where we explore approaches with "no learning", and "incautious learning" procedures and their run-length distribution is evaluated. "No learning" with Shewhart, CUSUM and EWMA schemes were compared with the parametric scheme where parameters are estimated. The unconditional distribution of the run length is used as a way to compare schemes. Incautious learning happens when every new observation is used to improve parameter estimation, or increase the reference sample for nonparametric calculations. Incautious learning methods using Shewhart, CUSUM and EWMA schemes follow the sense of self-starting procedures by Hawkins (1987a) and Quesenberry (1991). *Conditional delay* (CD) distribution after a change-point τ is used to evaluate each procedure. Supplementary material is and optional read, it adds confidence on the use of NS and SNS, nevertheless, conclusions on the effect of these nonparametric transformation remains unchanged.

The purpose of this paper is to assess the normal approximation of NS and SNS, how it compares with RT and SRT, and which approach is a better match when compared with parametric alternatives under normality. The closer the gap between what is expected under normal circumstances, the more reliable and practical the approach is for practitioners.

The rest of the paper is organized as follows. In Section 5.1.2 we described the nonparametric transformations that lead to the SNS transformation. This includes the development of the RT and the SRT. Subsections in Section 5.1.5 also include how these approaches are adapted to different monitoring schemes. These schemes are Shewhart $\bar{X}$, CUSUM, and EWMA to monitor univariate changes in location. Implementation of these monitoring schemes include guaranteed performance approaches and learning procedures to improve performance as trusted data is included to improve estimation of the in-control distribution. Section 5.1.5 shows the performance of the proposed control charts.

5.1.2 Nonparametric transformations and monitoring schemes

The SNS transformation is a nonparametric procedure that converts sequential ranks into asymptotically normally distributed statistics. This transformation can be used in control charts that assume normality for their statistical properties to hold, as presented in Conover et al. (2017). When this sequential statistic is employed, the computational complexity is reduced, since only the ranks of the monitored sample need to be evaluated. Additionally, the normal asymptotic behavior of the transformed scores allows for the use of control limits based on the normal distribution, accommodating different control schemes as discussed in Section 5.1.5.

Subsection 5.1.2.1 provides a description of the rank-based transformations employed in SPM, namely the traditional rank, sequential rank, and SNS transformations. These transformations serve the purpose of converting data into scores, enabling the creation of distribution-free tests for process monitoring. In Subsection 5.1.3, we delve into the concept of learning from the *in-control* (IC) sample during the monitoring process. Emphasizing the significance of practitioner-to-practitioner variation, we highlight its effects on IC performance and explore strategies to mitigate it through the updating of IC sample parameter estimations. Furthermore, Subsection 5.1.4 introduces location charts based on SNS transformations under three distinct learning schemes that can be effectively employed with SNS-transformed-data, ensuring reliable and effective process control.

5.1.2.1 Transformations

Let $Y = \{Y_j\}$, $j = 1, \ldots, m$ an IC sample. The rank transformation of Y_j over the sample Y is defined as

$$R_j = R(Y_j) = \sum_{Y_i \in Y} I(Y_i \leq Y_j), \tag{5.1}$$

where the $I(\cdot)$ indicator function is 1 if it is evaluated as true and 0 otherwise. TRanks are correlated, but their in-control distribution does not depend on the distribution of Y, only depends on the counting permutations. Li et al. (2010) introduced nonparametric control charts for location based on this ranking process, combining the monitored sample $X_i = \{X_{i1}, X_{i2}, \ldots, X_{in}\}$, with the IC sample Y in the set $Y = Y \cup X_i$, to use the Wilcoxon test as monitoring statistic:

$$W_i = \sum_{X_{ij} \in X_i} R(X_{ij}), \tag{5.2}$$

This statistic (5.2) uses the rank of X_{ij} over the combined sample $Y = Y \cup X_i$ in equation (5.1). Li et al. (2010) proposed a CUSUM and an EWMA control chart using (5.2), obtaining their control limits by numerical methods based on the run-length distribution of these charts. Control charts based on this statistic need to compute the ranks of the combined sample $Y \cup X_i, \forall i = 1, 2, \ldots$ to obtain (5.2), which can be computationally intensive for large streams, or to evaluate the performance of control charts for different monitoring schemes.

Another ranking alternative is found on the sequential rank transformation, SRT, where the calculation of the a rank is restricted to the observation being ranked and previous subgroups. A major advantage for using this approach is that sequential ranks are statistically mutually independent, which simplifies its analysis and application. As an example of the difference in traditional and sequential ranking transformation, see the next table of 7 independent observations X. Ranks are correlated, but their in-control distribution depends only on the counting of permutations, not the actual distribution of X. The in-control distribution of the sequential ranks also depends only on the counting of permutations, however they are statistically independent of each other.

TABLA 5.1: Example of rank and sequential rank transformations.

i	**1**	**2**	**3**	**4**	**5**	**6**	**7**
X	3	4.5	8.6	2.3	2.8	1.7	6.6
RT	4	5	7	2	3	1	6
SRT	1	2	3	1	2	1	6

McDonald (1990) proposed a CUSUM chart to monitor the location of a process with sequential rank transformations using a Markov chain model to approximate limiting control limits. The SRT for X_{ij} of the monitored sample X_i, SRT_{ij}, over the reference sample Y is:

$$\mathrm{SRT}_{ij} = \mathrm{SRT}(X_{ij}) = \sum_{Y_k \in Y} I(Y_k \leq X_{ij}) + \sum_{u=1}^{i-1} \sum_{v=1}^{n} I(X_{uv} \leq X_{ij}) + 1, \tag{5.3}$$

whose standardized version is:

$$\text{SRT}^*_{ij} = \frac{\text{SRT}_{ij} - (m + n(i1) + 2)/2}{\sqrt{((m + n(i1) + 1)^2 - 1)/12}}, \tag{5.4}$$

Here, the ranking transformation is derived only for the monitored sample, there is no need to obtain the ranks of the reference sample, which reduces the computational complexity with respect to traditional ranks. When dealing with subgroups size $n > 1$, ranking between observations from the same group adds unneeded dependence. Observations in a subgroups are treated as individuals ranked only with previous subgroup elements.

The sequential normal scores transformation are obtained by transforming sequential ranks with $z = \Phi^{-1}[(SRT - 0.5)/i]$, rankits, where Φ^{-1} is the inverse of the standard normal distribution and i is the sample size at that point in time. Using the example of Table **??**, Table **??** shows the calculations of this transformation.

TABLA 5.2: Example of sequential normal score transformations.

i	**1**	**2**	**3**	**4**	**5**	**6**	**7**
X	3	4.5	8.6	2.3	2.8	1.7	6.6
RT	4	5	7	2	3	1	6
SRT	1	2	3	1	2	1	6
SNS	0	0.674	0.967	-1.150	-0.524	-1.383	0.792

The use of SNS in contro chart is explained in Conover et al. (2017). For the IC sample $Y = \{Y_j\}$, $j = 1, \ldots, m$ and the monitoring observations $X_{ij} \in X_i$, the transformation Z_{ij}

is calculated by

$$Z_{ij} = \Phi^{-1}(P_{ij}), \tag{5.5}$$

where Φ^{-1} is the inverse of the standard normal cumulative distribution function and P_{ij} is defined by:

$$P_{ij} = \frac{SRT_{ij} - 0.5}{m + 1 + n(i - 1)}, \tag{5.6}$$

whereas, the SNS for Y_j in the IC-sample is defined by $Z_j = \Phi^{-1}(P_j)$ where $P_j = \frac{R_j - 0.5}{m}$.

According to Conover et al. (2017), the sequence of Z_{ij} are independent asymptotically standard normal random variables, then, we can use tests based on SNS to create control charts using control limits obtained by assuming a normally distributed process. The asymptotic behaviour of SNS and SRT is evaluated using control charts based on the assumption of normality, under different learning schemes about the IC sample in Section 5.1.5.

5.1.3 Learning procedures

Numerous authors such as Jensen et al. (2006), Saleh et al. (2015), Jardim et al. (2020), have emphasized that the performance of a control chart is closely tied to the IC sample. When using the ARL as a performance metric, it has been observed that a significant number of practitioners experience values of ARL_0 lower than expected, leading to an increase in the false alarm rate. Given the dependence of the performance of the control chart on the IC sample, it becomes crucial to consider the *conditional average run length* (CARL) as a performance metric. The CARL represents the ARL calculated for a specific IC sample $Y = y$, denoted as CARL($Y = y$). To mitigate the impact of the IC sample, Albers and Kallenberg (2004) proposed the exceedance probability criterion as a measure to evaluate the performance of estimated control charts. In terms of CARL, this criterion is formulated as follows:

$$P(\mathrm{CARL}_0 \leq (1-\alpha)(\mathrm{ARL}_0)) = \beta, \tag{5.7}$$

A control chart created under this criterion ensures a small proportion of CARLs under the desired performance.

Furthermore, Keefe et al. (2015b) conducted a study to examine the effect of self-starting control charts, see Hawkins (1987a), on practitioner-to-practitioner variation. They concluded that this incautious learning approach may have an impact on the *out-of-control* (OC) performance since OC observations can be used to estimate the IC parameters.

To address the impact of the exceedance probability criterion on OC performance and reduce the effect of including OC observations in the integrative approach, Capizzi and Masarotto (2020) proposed a cautious learning scheme. Capizzi and Masarotto (2020) updated the estimators of the IC sample at time i, with the estimators $\bar{X}_{i-d_i}$ and $S^2_{i-d_i}$, where $\bar{X}_0$ and S^2_0 are the mean and variance of the reference sample, respectively, and d_i is a delay parameter defined as:

$$d_{i+1} = \begin{cases} 1 & \text{if } \displaystyle\sum_{r=i-d_i+1}^{i} \left(\frac{x_r - \bar{x}_{i-d_i}}{s_{i-d_i}}\right)^2 < Ad_i - B, \\ d_i + 1 & \text{otherwise}, \end{cases} \tag{5.8}$$

where $d_1 = 1$. This rule enables the updating of the parameters only when likelihood of $X_{i-d_i+1}, \ldots, X_i$ is sufficiently high. A and B are learning constants used to achieve the guaranteed IC performance condition, updating the control limits on average for each 100 samples. Also, with each update, the critical values L_i are adjusted with the same rate of convergence than the estimation errors, according to the next formula:

$$L_{i-d_i} = L_\infty + \Delta_L \sqrt{\frac{m}{m+i-d_i}}, \tag{5.9}$$

where L_∞ is the control limit for an IC ARL equal to a nominal ARL_0 with known parameters and Δ_L is obtained to achieve the guaranteed exceedance criterion. This cautious learning approach is applied in Shewhart, CUSUM and EWMA control schemes showing better results in the power of detecting real changes in the process, compared with a no-learning control chart with IC guaranteed performance; whereas its IC performance presents lower variation in the conditional ARL.

In nonparametric schemes, control charts based on SNS and standardized SRT can be seen as incautious learning approaches, because the rank procedure is defined over a reference sample which is updated with each new IC sample. Also, given that they have a limiting standard normal distribution, we can use them with the control limits based on this distribution.

We can also use SNS and SRT to create location distribution-free control charts in a cautious learning approach. For monitoring the sample X_i, lets define Z_i and SRT_i:

$$Z_i = \frac{\sum_{j=1}^{n} Z_{ij}}{\sqrt{n}}, \tag{5.10}$$

$$\text{SRT}_i = \frac{\sum_{j=1}^{n} \text{SRT}_{ij}^*}{\sqrt{n}}, \tag{5.11}$$

Both statistics have a limiting standard normal distribution as $n \to \infty$, therefore, they can be used in the cautious learning control chart of Capizzi and Masarotto (2020). As initial estimators of the normal control chart, we can use the mean and standard deviation of the transformation. The delay mechanism in equation (5.8) used for the estimation of the control chart parameters also updates the reference sample used in the rank procedure.

5.1.4 MONITORING LOCATION SCHEMES

In this research, we present the Shewhart, EWMA and CUSUM control chart for location using the SNS transformation to monitor batches of size n, using as a test statistic Z_i, see equation (5.10).

For the Shewhart control chart an alarm is given when $Z_i \geq \hat{\mu}^{SNS}_{i-d_i} + L_i\hat{\sigma}^{SNS}_{i-d_i}$, or $Z_i \leq \hat{\mu}^{SNS}_{i-d_i} - L_i\hat{\sigma}^{SNS}_{i-d_i}$, where $\hat{\mu}^{SNS}_{i-d_i}$ and $\hat{\sigma}^{SNS}_{i-d_i}$ are the sample mean and standard deviation of the SNS transformation at time $i - d_i$.

In the CUSUM-type control chart, with $C_0^+ = C_i^- = 0$, we use statistics

$$C_i^+ = \text{máx}\left(0, C_{i-1}^+ + \frac{Z_i - \hat{\mu}^{SNS}_{i-d_i}}{\hat{\sigma}^{SNS}_{i-d_i}} - k\right), i > 0, \tag{5.12}$$

$$C_i^- = \text{mín}\left(0, C_{i-1}^- + \frac{Z_i - \hat{\mu}^{SNS}_{i-d_i}}{\hat{\sigma}^{SNS}_{i-d_i}} + k\right), i > 0, \tag{5.13}$$

where k is a reference value, also known as the allowance. Parameters h, k are given for a target ARL_0. Alarms are triggered when $C_i^+ \geq L_i$ or $C_i^- \leq -L_i$.

In an EWMA-type control chart, the statistic used is

$$E_i = \lambda Z_i + (1 - \lambda)E_{i-1}, i > 0, \tag{5.14}$$

where $E_0 = 0$. Parameters λ, L are obtained to achieve ARL_0. An alarm is given when $E_i \geq \hat{\mu}^{SNS}_{i-d_i} + L_i\hat{\sigma}^{SNS}_{i-d_i}\sqrt{\frac{\lambda}{2-\lambda}}$, or, $E_i \leq \hat{\mu}^{SNS}_{i-d_i} - L_i\hat{\sigma}^{SNS}_{i-d_i}\sqrt{\frac{\lambda}{2-\lambda}}$.

For comparison purposes, we also evaluate these control charts by using the sequential ranks transformation statistic SRT_i, (see equation (5.11)), using the mean and standard deviations of the SRT values at time $i - d_i$, $\hat{\mu}^{SRT}_{i-d_i}$ and $\hat{\sigma}^{SRT}_{i-d_i}$, in previous equations, with the same control limits based on normal distribution to evaluate the normal approximation of these two nonparametric statistics. Alternatively, statistic $\hat{\mu}^{SNS}_{i-d_i}$ and $\hat{\sigma}^{SNS}_{i-d_i}$ can be defined as 0 and 1, respectively, for all cases, as normal scores approximate a $N(0, 1)$. Learning still happens when the reference sample is updated to get the Z_i scores.

5.1.5 Normal approximation study with guaranteed performance and cautious learning

In this section, we present a comprehensive performance analysis of three types of analysis divided by learning strategies. Learning occurs when the reference sample is updated, either to enhance parameter estimation in parametric approaches or to expand the base used for ranking in nonparametric procedures.

Guaranteed performance approaches are becoming the new norm in SPM research. However, to guarantee a minimum performance level with a prespecified degree of confidence, control limits are widened further to account for the extra variation created by estimated parameters. It follows the sense of tolerance intervals, but the correction creates a reduction in power to detect a shift in a process. Incautious updates of parameter estimates, or reference sample updates, improves a chart as information arrives, but small changes not detected quickly contaminate the reference sample, which also reduce power. Capizzi and Masarotto (2020) identified an improved state, with better power, by creating a compromise between “incautious learning” and “no learning”. They called this approach “cautious learning", as learning ocurrs only when there is evidence of an in-control state of data to incorporate. They showed the approach with Shewhart, EWMA and CUSUM schemes to monitor location changes. The approach requires numerical analysis, and the determination of control limits is a computer intense process. Traditional nonparametric approaches add computational difficulty if we wish to get exact, or close to exact, calibration to get guaranteed control limits with cautious learning. Here, the normal approximation using SRT and SNS becomes more useful. We can, probably for the first time, get access to nonparametric schemes with guaranteed performance and cautious learning by using these transformations with the already existing procedures and evaluation limits provided by Capizzi and Masarotto (2020). To evaluate this approach, we used the following guidelines:

- Initial reference sample size: $m = \{50, 100, 500\}$.
- Distributions evaluated: $N(0, 1)$, $t(4)$, and $\text{Gamma}(0.5, 1)$ distributions.
- Each scenario was simulated with 1 000 conditional ARLs, where each ARL was obtained from 10 000 RLs.
- RLs were capped at 10 000 for CUSUM and EWMA, and up to 5 000 for Shewhart. Guaranteed levels of performance, the most important aspect, were obtained with no compromise. However, extreme ARL quantiles close to these bounds were underestimated.
- We used a desired in-control performance $\text{ARL}_0 = 500$, calibrated so that:

$$p_{0.10} = P(\text{CARL} \leq (1 - 0.1)\text{ARL}_0) = 0.05,$$

- Performance was compared between methods:
    - CL: Cautious learning, parametric approach from Capizzi and Masarotto (2020).

- SNS: Sequential Normal Scores with cautious learning.
- SRT: Standardized sequential ranks transformation with cautious learning.

CUSUM and EWMA charts were evaluated with subgroups $n = 1$, and Shewhart approaches used $n = 5$. Tables 5.3 to 5.5 show the in-control metrics to compare these approaches as previously described. The most important metric to guarantee a performance level is $p_{0.10}$. As defined previously, it was defined in such a way that the probability of getting ARL values smaller than $(1 - 0.1)ARL_0$ is 0.05. We can see that the parametric CL chart gets close to this value when dealing with a normal distribution. However, this probability goes beyond 0.05 when the distribution is not normal. In Table 5.3 $p_{0.10}$ is 0.996 and 1.000 for Shewhart CL when distributions are $t(4)$ and gamma$(0.5, 1)$, respectively. Similar large values were obtained when $m = 100$ and $m = 500$ in tables 5.4 and 5.5. SNS and SRT are never above 0.05, and SNS is closer. For instance, in Table 5.3, SNS has a $p_{0.10}$ of 0.018, 0.012 and 0.024 when using the Shewhart scheme dealing with normal, $t(4)$ and gamma distributions. SRT got 0.000 for all values in the same scenarios. Both SNS and SRT are conservative, but SRT is much more conservative, which affects power as seen in the out-of-control analysis in subsequent section of the paper. In terms of the average ARL, or AARL_0, SNS is closer to the CL, and SRT shows the largest values. ARL variation, in terms of the SDARL_0, is smaller than CL when using SNS, and the lowest with the SRT.

Another way to look at the IC performance is in the boxplots shown in Figures 5.1 to 5.3. The horizontal dotted line represents the target, which is $(1-0.1)ARL = 450$, in this case. For a guaranteed performance, a boxplot, where lower and upper whiskers were created using quantiles 0.05 and 0.95, respectively, need to be above the dotted line. This is the case for the first row of subplots in these three figures where the $N(0, 1)$ distribution was used. It is evident that SNS provide a performance that is closer to what is expected by the parametric CL approach. SRT is over-conservative. When distribution is not normal, CL generates low ARL values, and performance is no longer guaranteed. Conversely, SNS and SRT approaches remain robust as expected with there distribution-free property. Finally, as the initial sample size increases, variation is reduced. This IC conclusions repeats with all three approaches, Shewhart, CUSUM and EWMA.

OC performance is presented in figures 5.4 to 5.7. Figure 5.4 deals with the normal distribution and includes all three charts in the analysis. Figure 5.5 uses a t(4) distribution. Both 5.6 and 5.7 deal with gamma distributions, where the former evaluates increments in location, and the latter goes with the decrements. All these charts analyze a situation where a change or 1 standard deviation from the mean occurs. First row is for the Shewhart schemes, EWMA goes second, and CUSUM last. First column of subfigures assesses a situation where the change-point occurs at the first monitored observation. Subsequently, the second column addresses a change-point after 150 in-control observations with no false alarms.

When the underlying distribution is normal, (Figure 5.4), CL chart performs the best, as expected from the parametric approach that assumes normality. Smaller amount of ARL values and smaller variation. CL is closely followed by SNS. SRT exhibits larger ARL values and bigger variation in most cases. By comparing the second column of subfigures it is evident that after 150 in-control observations the learning scheme improved charts performance significantly when compare with the first column, where a change happened at the start of the monitoring, leaving no room for in-control learning.

Also in Figure 5.4, all control charts using CL perform the best, as expected for the normal case, with smaller ARL and variation, being closely followed by SNS.

Figure 5.5 shows a comparison between the nonparametric methods where the IC distribution is $t(4)$. Shewhart approaches, first row of subfigures, show SNS as the method with smaller ARL values. In EWMA schemes, second and third rows, respectively, the median SNS is smaller, however, variation when the change-point occurs at the start of the monitoring is bigger, but as the change-point moves to $\tau = 150$, SNS transformation out-performs SRT in terms of lower ARL values with a significant drop in variation. With CUSUM methods, third row, median ARL of SNS is slightly bigger, but it drops as the learning happens in the second column.

Figures 5.6 and 5.7 show OC performance when dealing with a gamma distribution. Figure 5.7 shows SNS outperforming SRT in every scenario. Alternatively, Figure 5.6 presents mixed results. Shewhart schemes put SNS as the best candidate, however, EWMA and CUSUM procedures perform with lower ARL values when using SRT scores. SRT scores are spreaded wider between scores than SNS close around the mean, while SNS values a more spread near the tails. This subtle difference makes both approaches behave differently over tails, light or heavy, in a distribution. In both figures, with $\tau = 150$, SNS variation of ARL values get significantly reduced, even more than SRT.

Shewhart									
	Normal			t-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	857.08	851.53	2 951.14	193.51	841.10	2 946.18	123.43	832.51	2937.22
$SDARL_0$	189.90	370.42	452.19	167.44	167.44	423.09	73.00	175.74	429.65
$p_{0.10}$	0.058	0.018	0.000	0.996	0.012	0.000	1.000	0.024	0.000
EWMA									
	Normal			t-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	873.57	801.13	2 154.01	249.29	805.75	2 138.07	172.71	798.75	2163.09
$SDARL_0$	187.50	139.79	310.23	85.77	134.18	326.18	92.96	128.40	339.43
$p_{0.10}$	0.049	0.007	0.000	0.981	0.005	0.000	0.996	0.006	0.000
CUSUM									
	Normal			t-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	966.40	846.02	1 257.82	272.10	839.81	1257.73	242.14	849.42	1257.31
$SDARL_0$	228.88	151.37	196.83	89.70	143.78	205.75	114.59	158.25	199.94
$p_{0.10}$	0.055	0.011	0.000	0.971	0.013	0.000	0.965	0.017	0.000

TABLA 5.3: Cautious Learning IC performance for $m = 50$

Shewhart									
	Normal			*t*-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	755.90	755.12	2 679.08	172.18	750.73	2 678.54	113.97	748.97	2 670.41
$SDARL_0$	163.34	79.76	206.18	72.01	74.10	186.68	62.58	76.69	180.54
$p_{0.10}$	0.045	0.011	0.000	0.996	0.008	0.000	0.999	0.009	0.000
EWMA									
	Normal			*t*-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	743.00	715.32	2 024.62	249.29	713.36	2 020.53	161.71	710.52	2027.25
$SDARL_0$	159.05	67.05	198.36	85.77	63.71	201.33	79.66	65.25	206.39
$p_{0.10}$	0.051	0.006	0.000	0.981	0.005	0.000	0.994	0.007	0.000
CUSUM									
	Normal			*t*-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	770.90	709.46	1 053.10	335.70	713.09	1 053.70	196.14	709.8	1 048.34
$SDARL_0$	169.70	64.91	86.79	120.70	66.39	84.37	95.21	56.07	88.64
$p_{0.10}$	0.045	0.012	0.000	0.864	0.012	0.000	0.986	0.004	0.000

TABLA 5.4: Cautious Learning IC performance for $m = 100$

Shewhart									
	Normal			*t*-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	668.51	657.7	2 370.00	160.54	655.98	2 370.83	101.95	659.93	2 366.12
$SDARL_0$	128.88	41.55	47.27	51.37	43.28	64.97	37.53	42.36	69.51
$p_{0.10}$	0.042	0.000	0.000	0.997	0.001	0.00	1.000	0.000	0.000
EWMA									
	Normal			*t*-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	643.66	624.64	1 771.74	240.91	626.39	1 772.72	156.97	624.44	1 769.41
$SDARL_0$	110.10	68.64	40.99	68.18	65.18	40.86	51.02	66.81	41.42
$p_{0.10}$	0.039	0.009	0.000	0.981	0.007	0.000	1.000	0.006	0.000
CUSUM									
	Normal			*t*-Student			Gamma		
Metric	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**	**CL**	**SNS**	**SRT**
$AARL_0$	642.45	628.67	922.69	303.13	625.49	923.26	175.45	623.9	923.74
$SDARL_0$	117.17	63.87	42.37	265.63	65.96	42.59	56.84	66.78	43.92
$p_{0.10}$	0.029	0.005	0.000	0.950	0.004	0.000	0.997	0.010	0.000

TABLA 5.5: Cautious Learning IC performance for $m = 500$

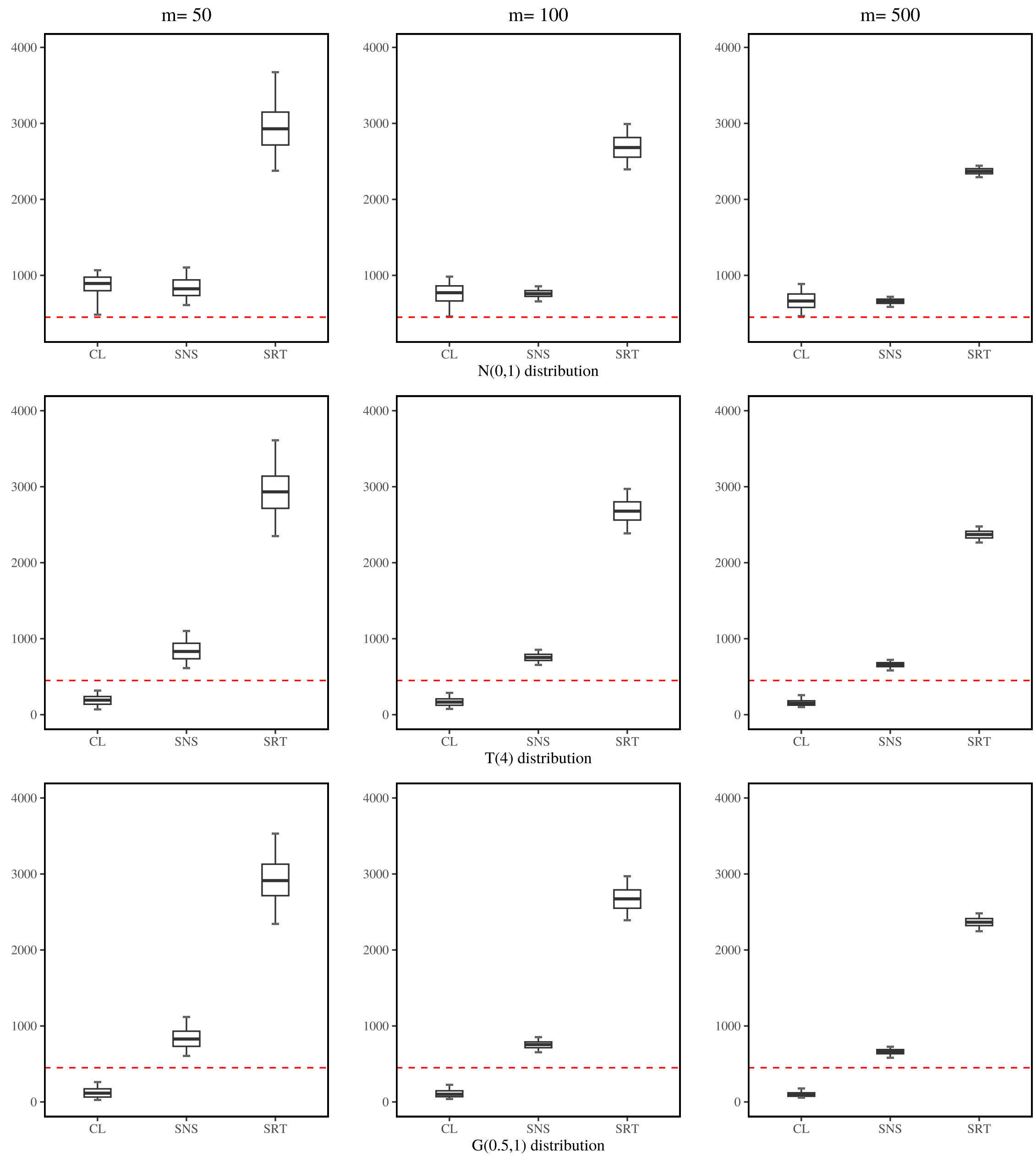

FIGURA 5.1: In-control performances from Shewhart ($n = 5$). Boxplots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

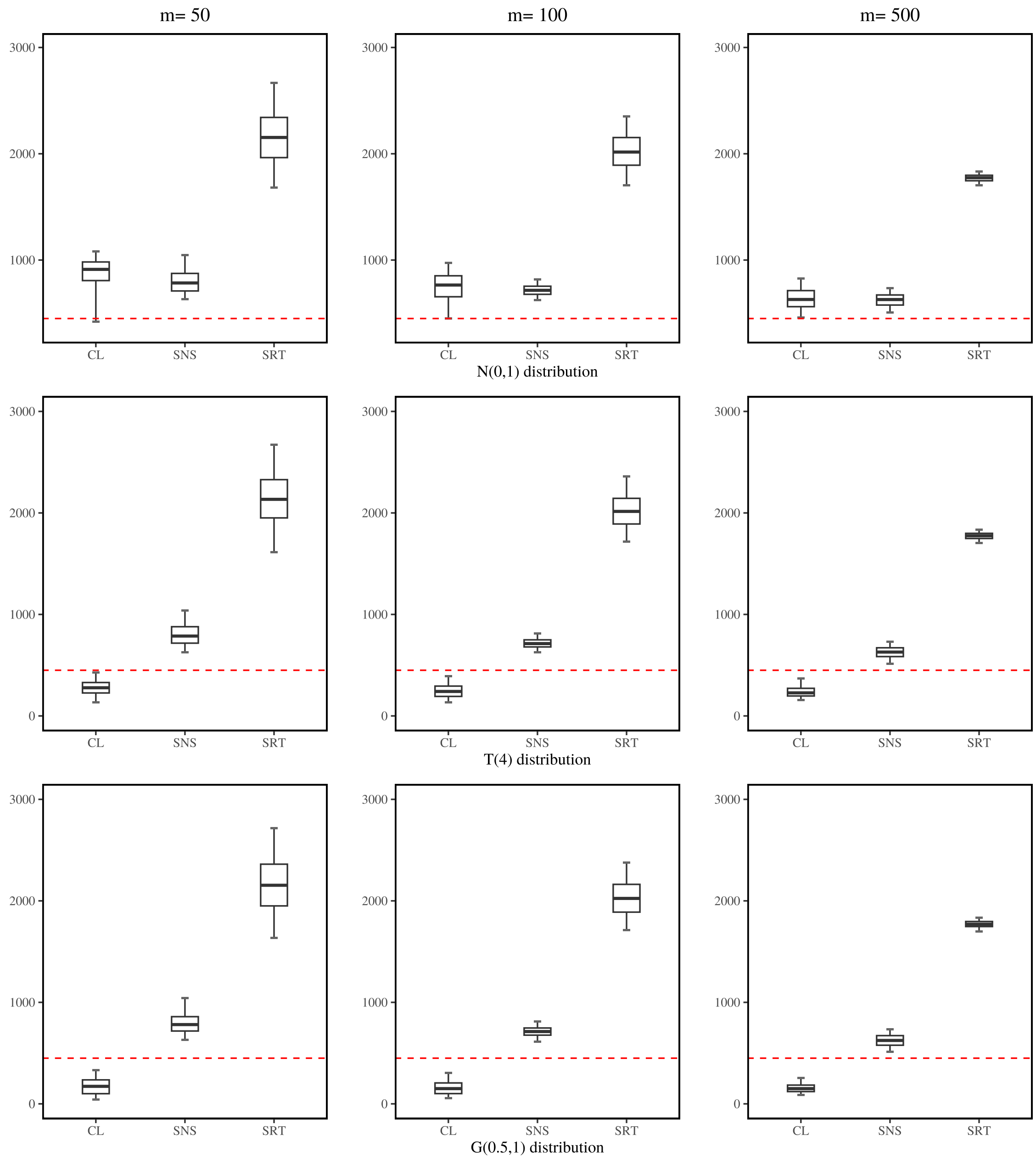

FIGURA 5.2: In-control performances from EWMA (0.2). Boxplots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

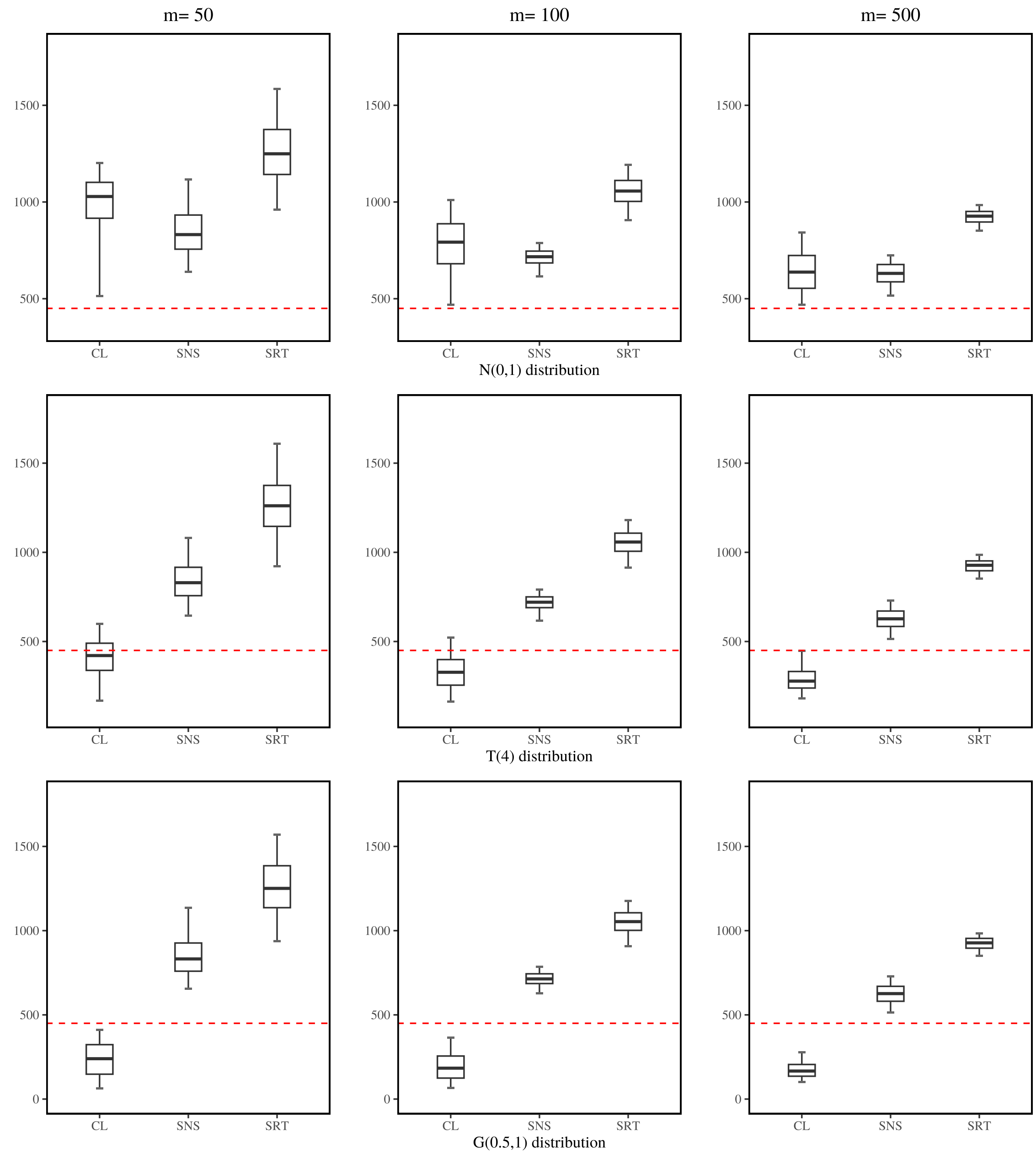

FIGURA 5.3: In-control performances from CUSUM (0.5). Boxplots represent the distribution of ARL conditional values. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

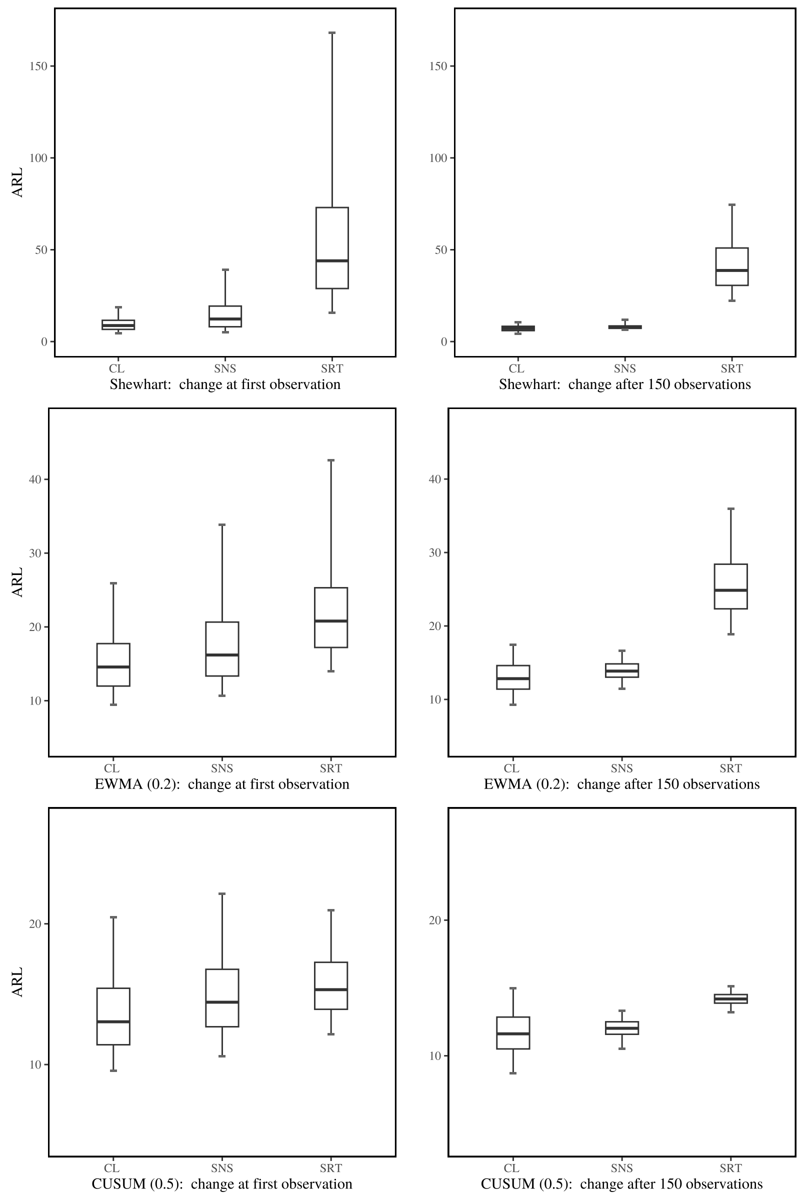

Figura 5.4: Normal: Out-of-control performance for Shewhart, EWMA (0.2) and CUSUM (0.5) with $\delta = 1$. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

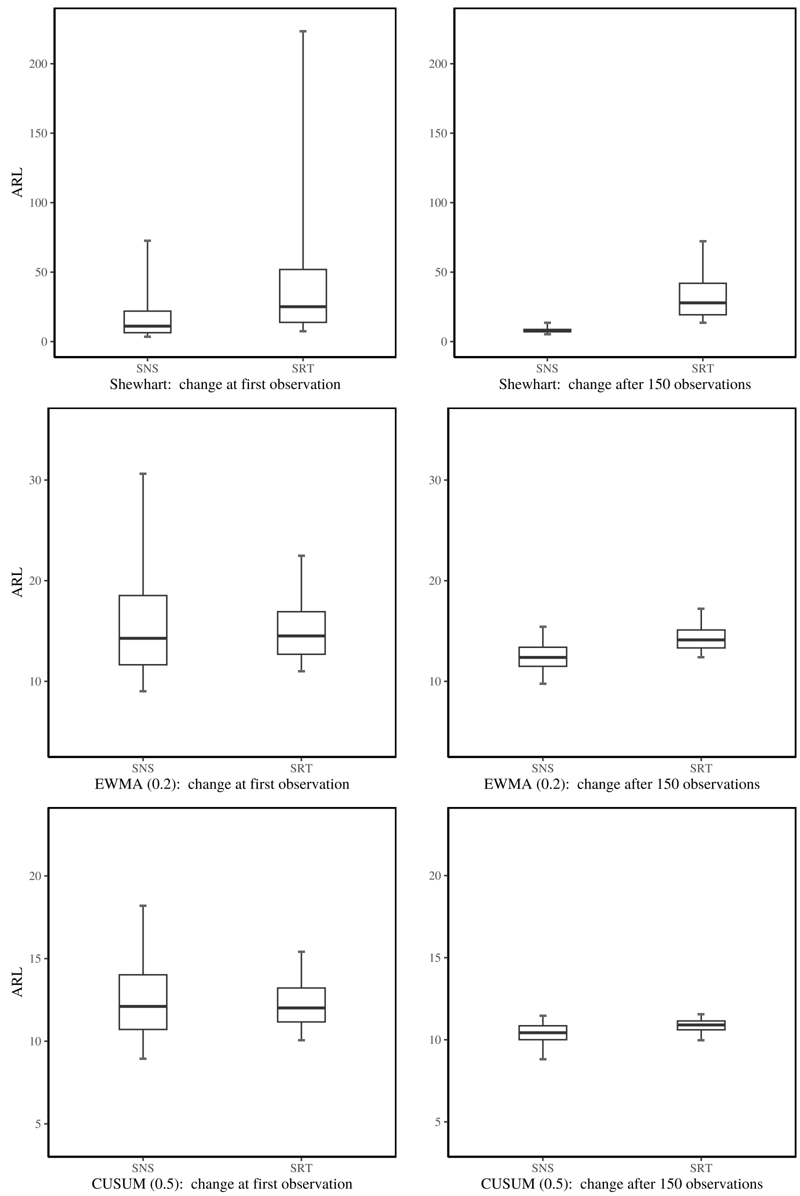

FIGURA 5.5: t-Student: Out-of-control performance for Shewhart, EWMA (0.2) and CUSUM (0.5) with $\delta = 1$. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

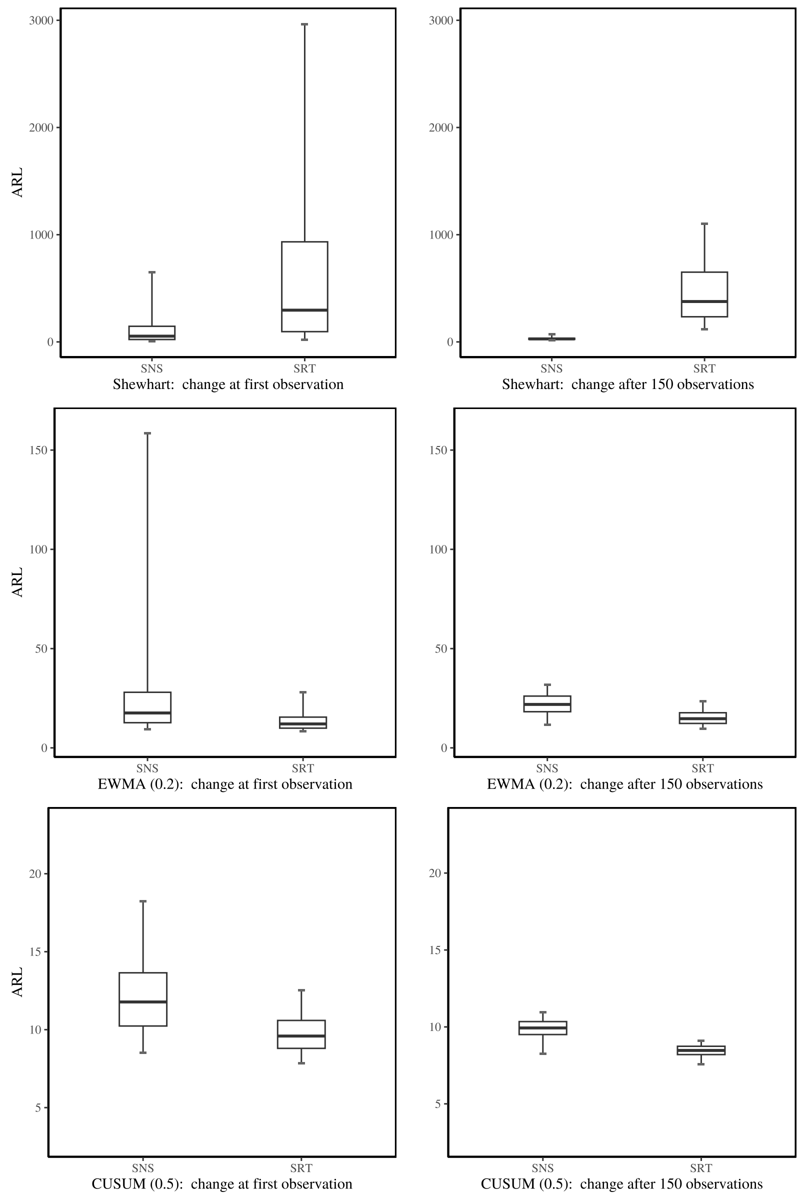

Figura 5.6: Gamma: Out-of-control performance for Shewhart, EWMA (0.2) and CUSUM (0.5) with $\delta = 1$. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

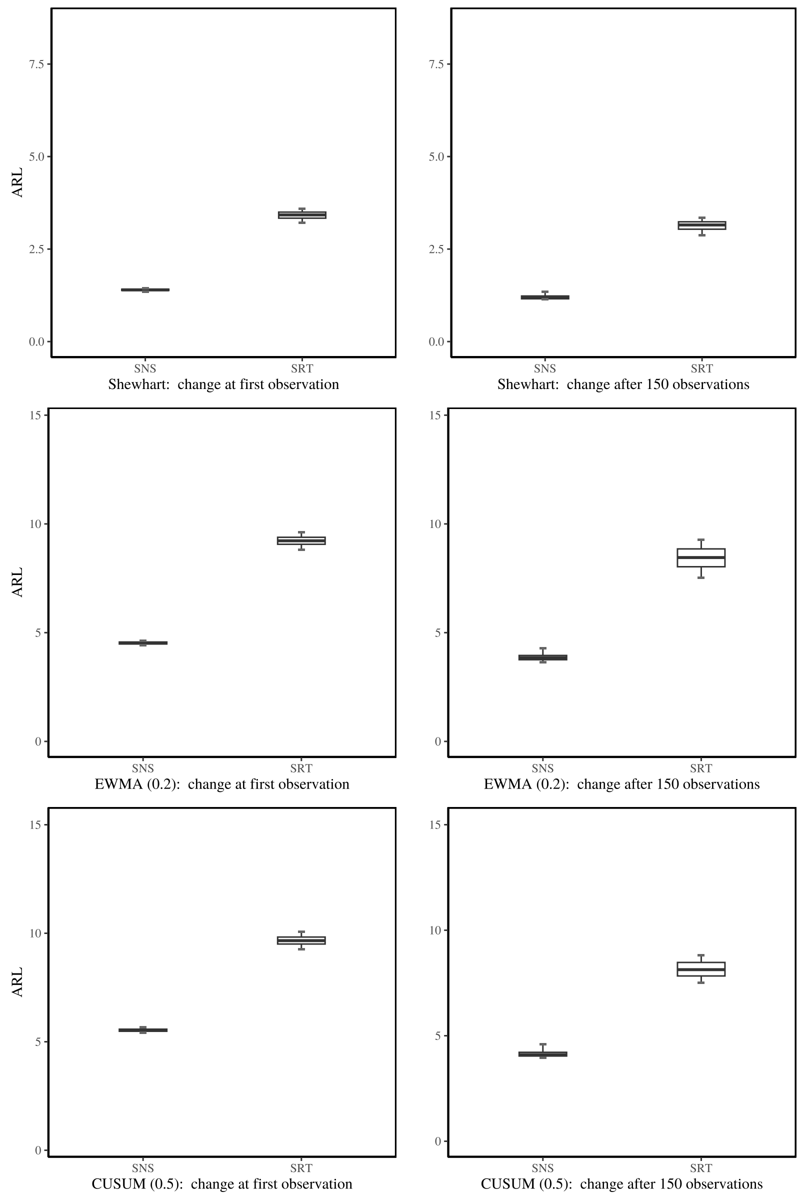

FIGURA 5.7: Gamma: Out-of-control performance for Shewhart, EWMA (0.2) and CUSUM (0.5) with $\delta = -1$. Lower and upper whiskers were created with quantiles 0.05 and 0.95, respectively.

5.2 DETAILED ANALYSIS OF DETECTION IN CHANGES OF DIFFERENT MAGNITUDE

The following is an analysis of the detection of very small changes, i.e. $\delta \in \{0.1, 0.2, ..., 0.9, 1.0\}$. Only the SNS statistic with its conditional variants, the CSNSK and the CSNSU, is evaluated.

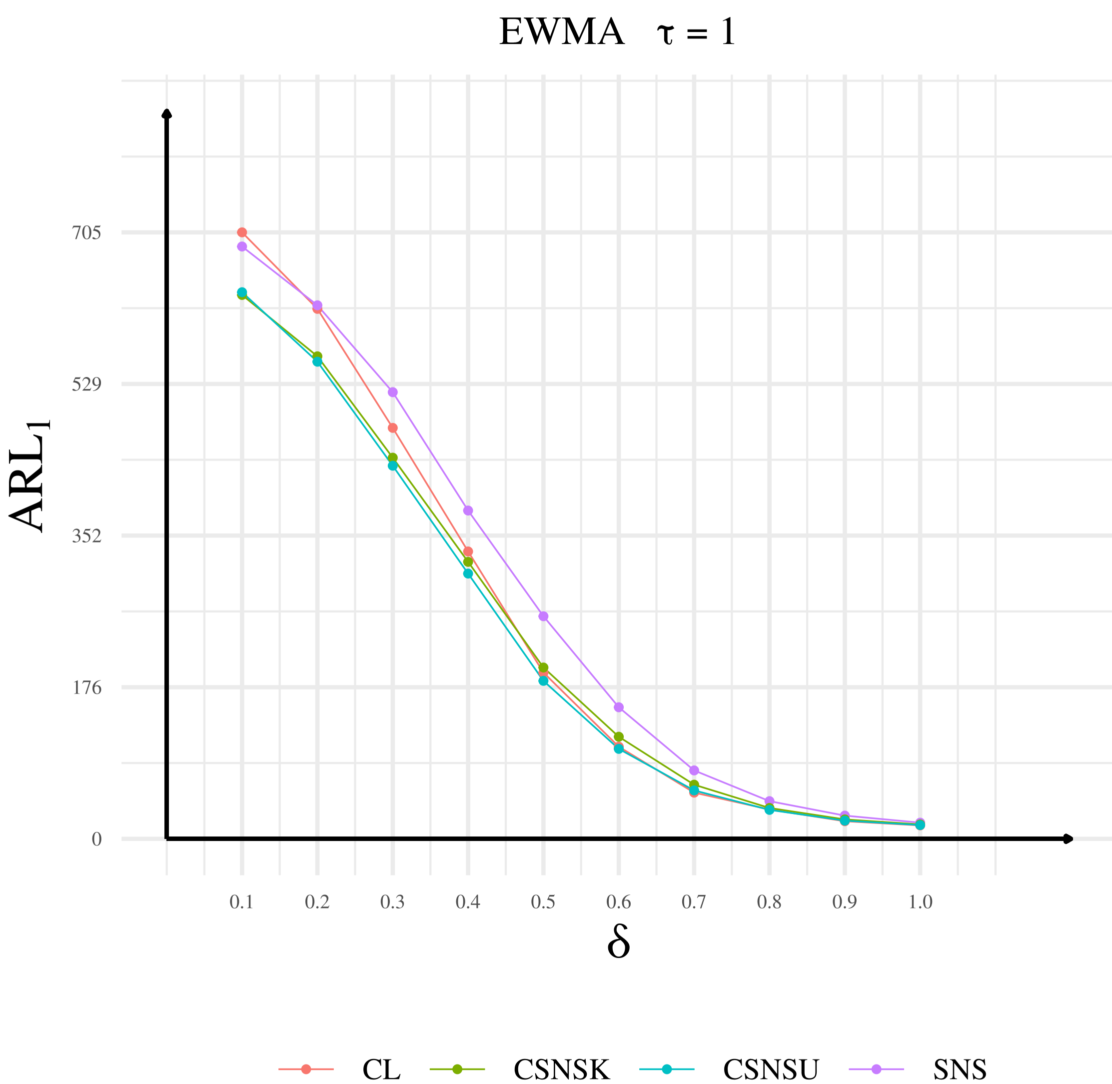

Figura 5.8: EWMA: Out-of-control performances for $N(0, 1)$ distribution

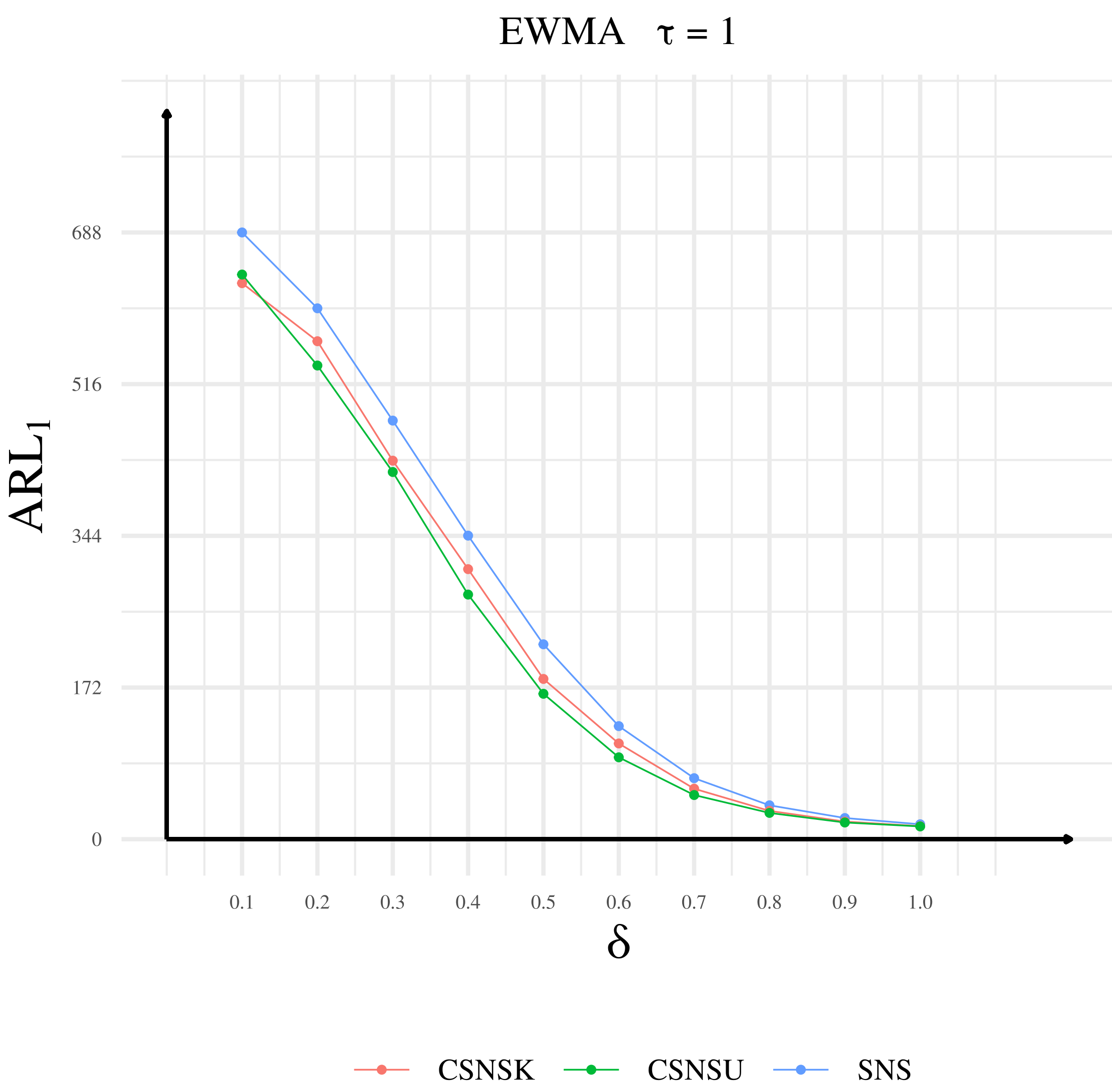

Figura 5.9: EWMA: Out-of-control performances for T_4 distribution

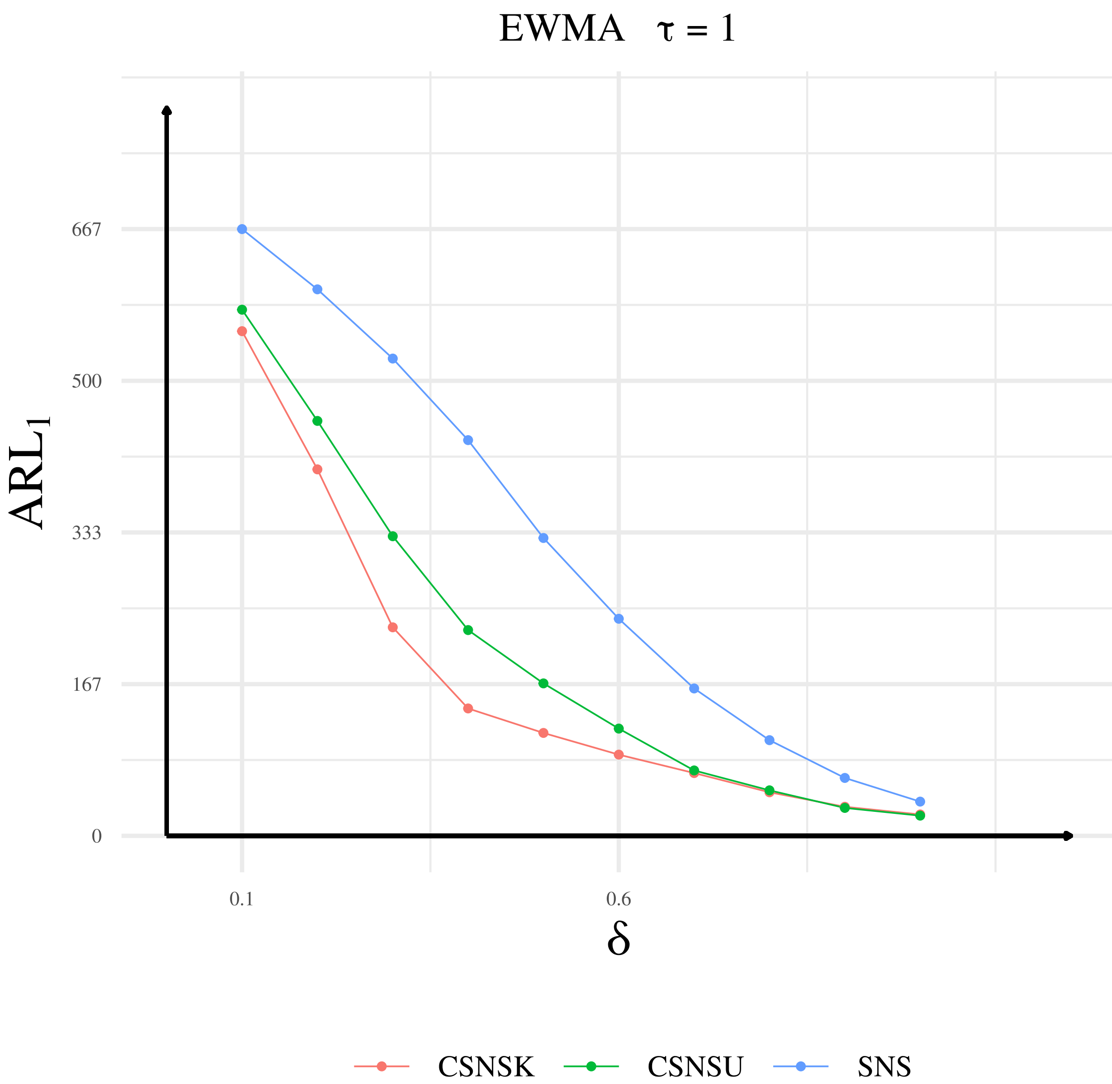

FIGURA 5.10: EWMA: Out-of-control performances for $G(0.5, 1)$

In normal observations, the SNS and its conditional variants match the parametric CL, even exceeding it for changes $0.1 \leq \delta \leq 0.3$.

In non-normal process monitoring, change detection is very similar between the SNS and its variants, with the exception of the Gamma distribution, where knowledge or estimation of a quantile of the reference sample allows even faster detection of small changes.

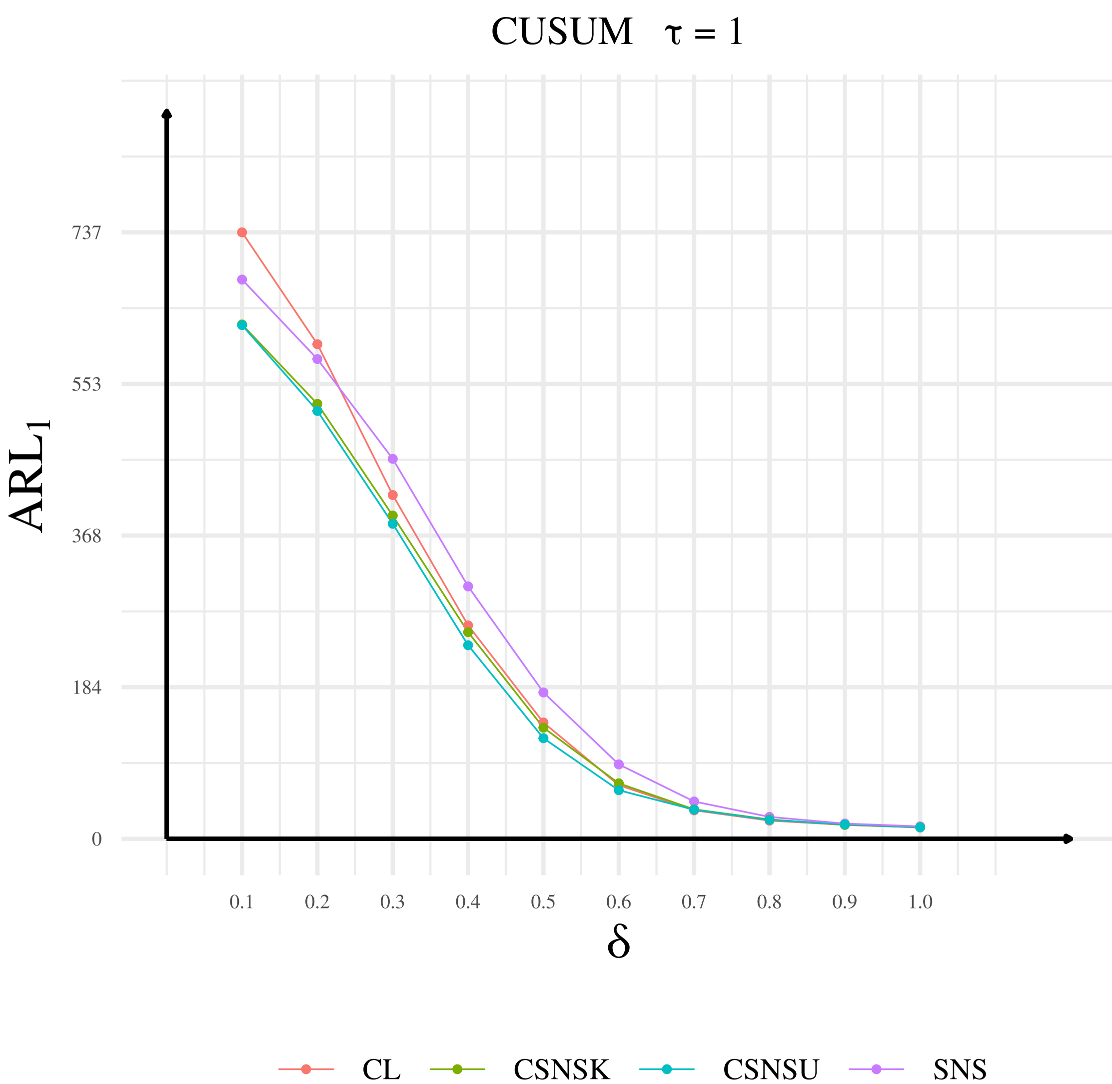

Figura 5.11: CUSUM: Out-of-control performances for $N(0, 1)$ distribution

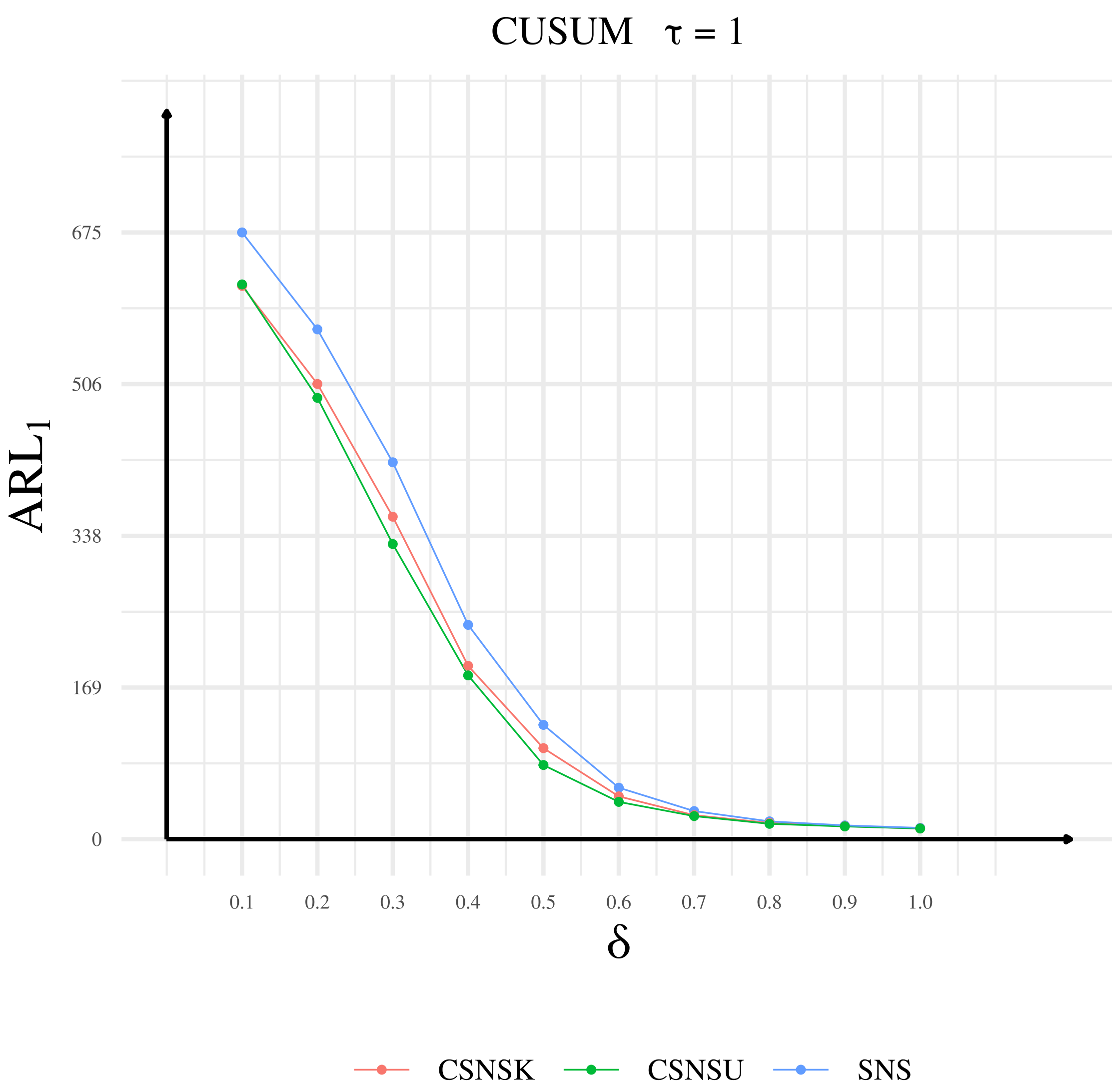

FIGURA 5.12: CUSUM: Out-of-control performances for T_4 distribution

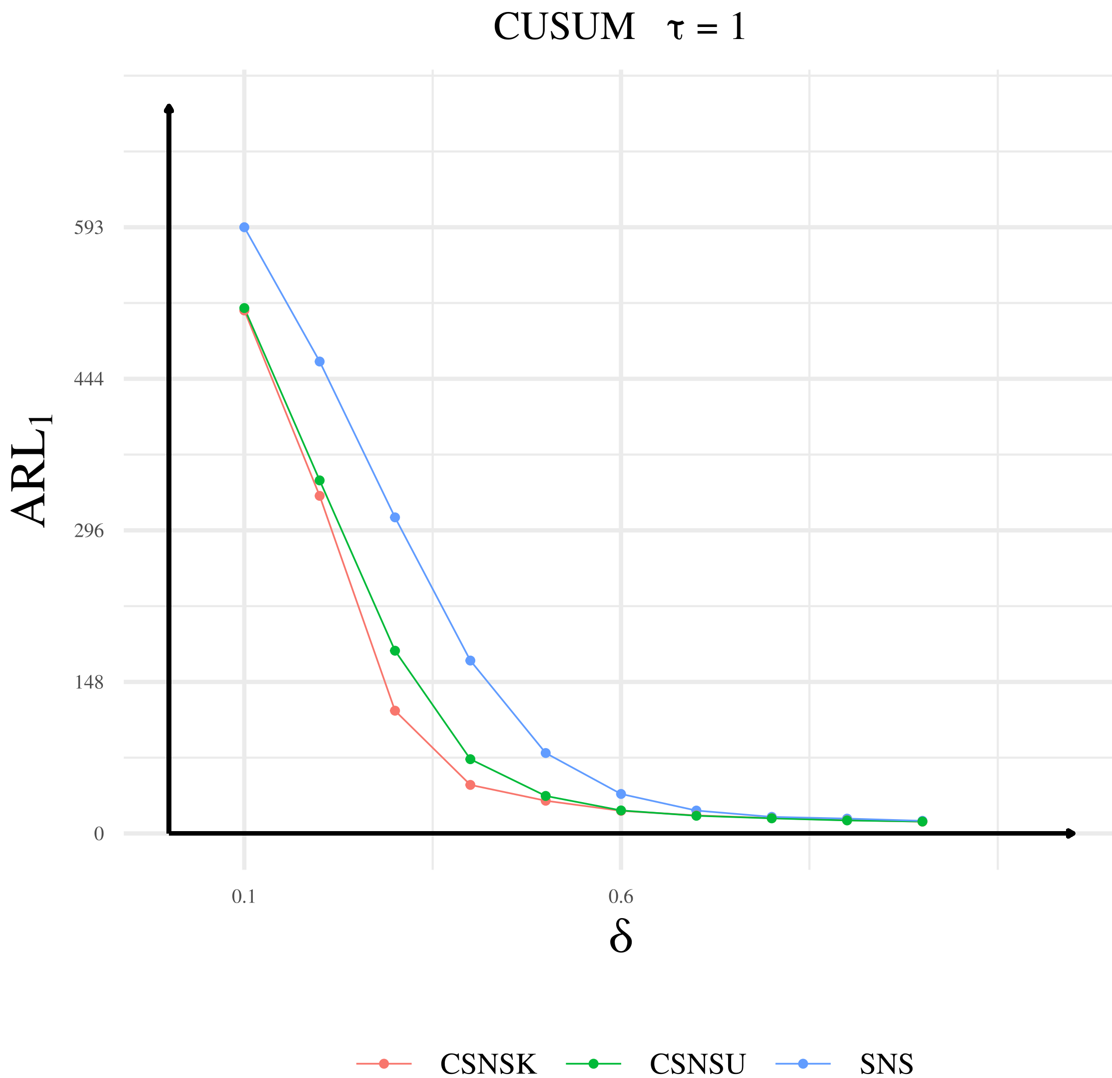

FIGURA 5.13: CUSUM: Out-of-control performances for $G(0.5, 1)$ distribution

Similarly to under the EWMA scheme, under normal conditions, non-parametric statistics match the parametric CL, with conditional SNS outperforming the rest, mainly in changes $0.1 \leq \delta \leq 0.6$.

In non-normal processes, estimating a sample quantile from the reference population is sufficient to outperform the rest of the statistics when the process originates from a T-distribution population. Conversely, for processes from a Gamma population, SNS with known quantile have an advantage over the rest, especially in changes of 0.3 or 0.4.

Finally, an analysis of the detection of both positive and negative changes will be made. Taking Normal and Gamma processes, with $\delta \in \{-1.0, -0.9, \ldots, -0.1, 0.0, 0.1, \ldots, 0.9, 1.0\}$ it will be seen how a key feature of the distribution such as skewness can represent a difference in the power of the chart.

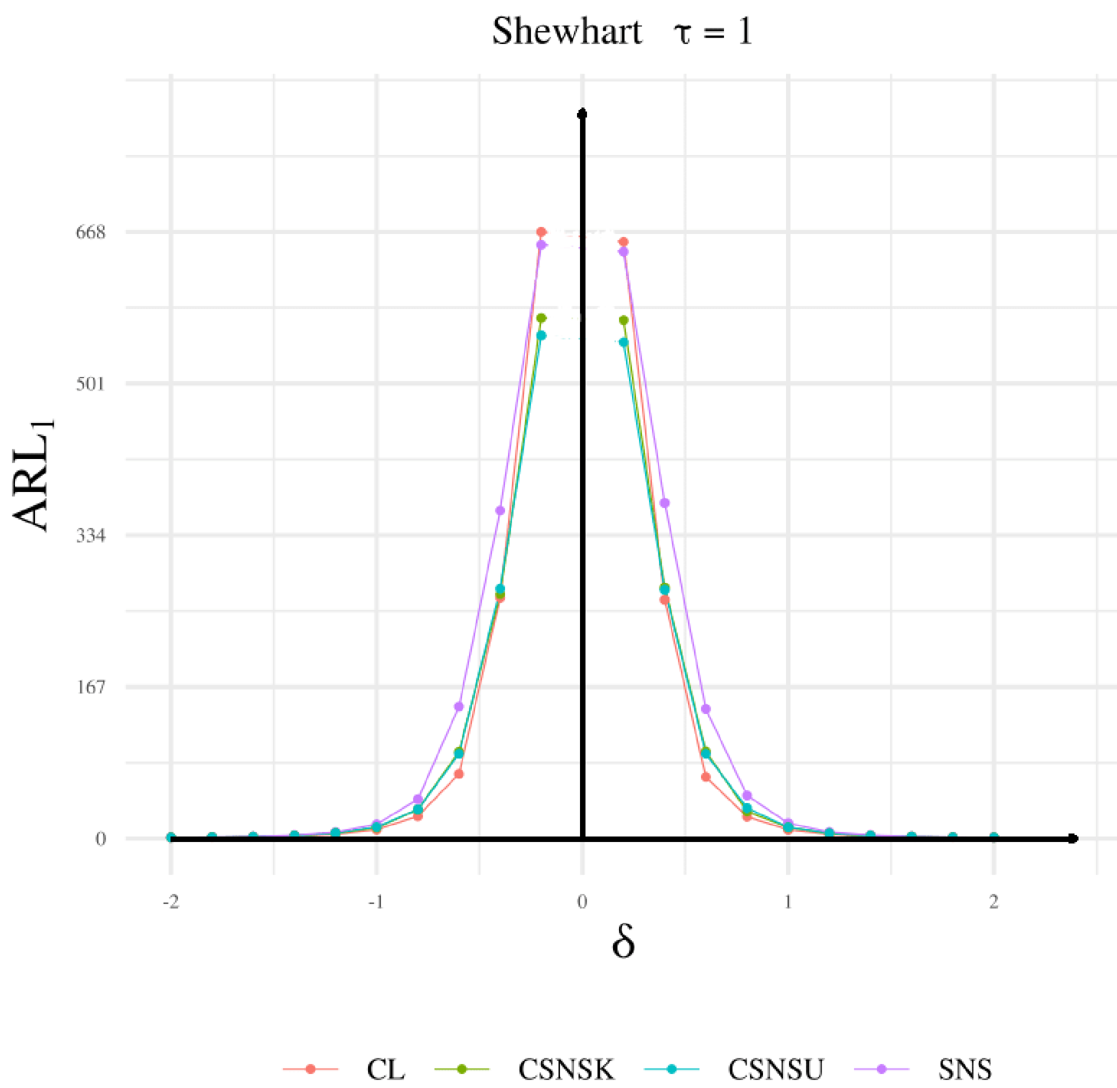

Figura 5.14: Shewhart: Out-of-control performances for $N(0, 1)$ distribution for positive and negative changes

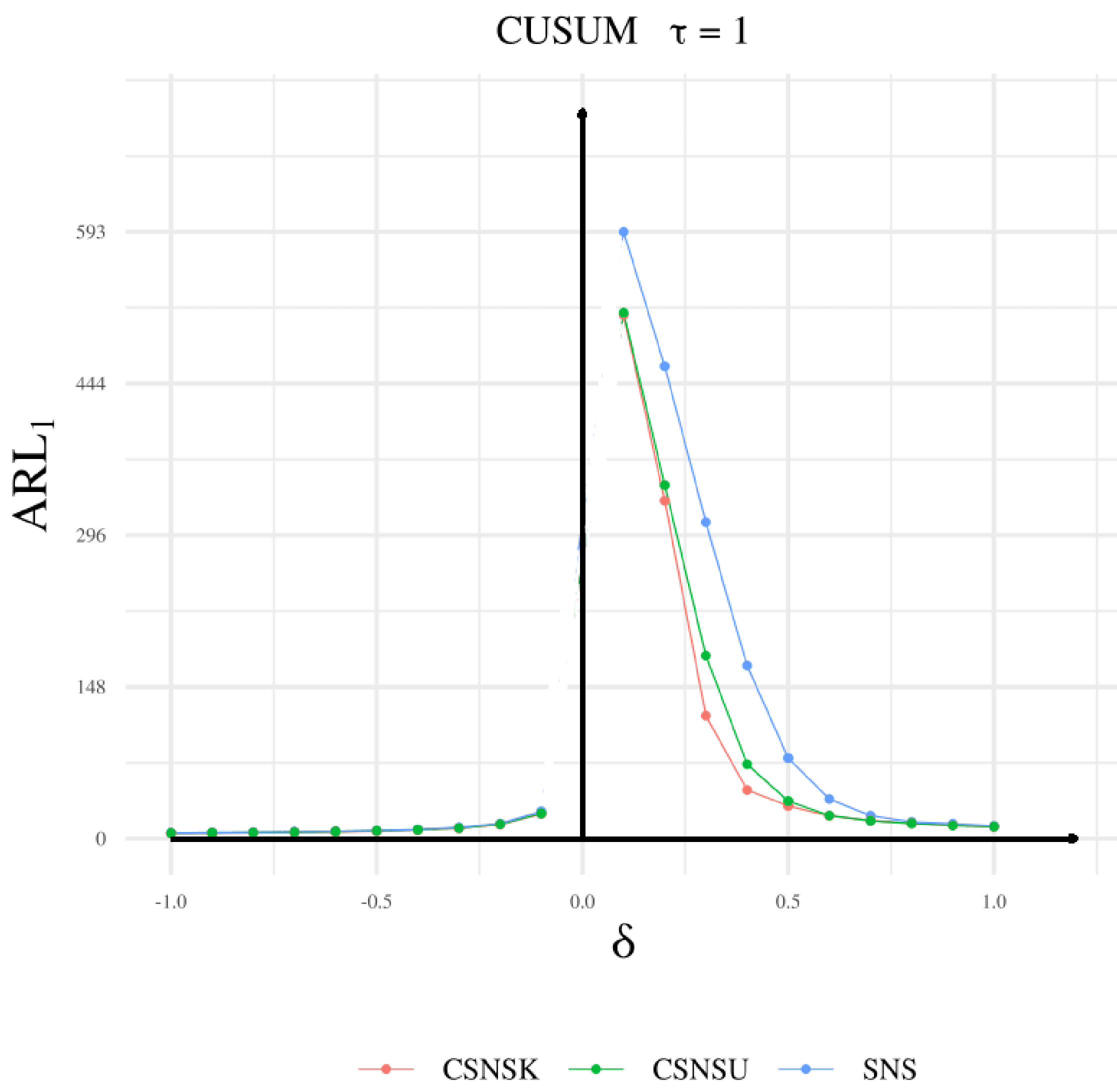

Figura 5.15: CUSUM: Out-of-control performances for $G(0.5, 1)$ distribution for positive changes

5.3 LOGISTIC REGRESSION MODEL

A logistic regression model describes the relationship between a variable that takes only two possible values 0 and 1[1] and one or more continuous variables.(Sperandei, 2014)

$$log\left(\frac{P(Y=1)}{P(Y=0)}\right) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 ... + \beta_n x_n ; n > 1 \tag{5.15}$$

Its name comes from the fact that a simple logistic function can be used to obtain the probabilities of each category of the dependent variable, as follows:

$$\hat{P}(Y=1) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 ... + \beta_n x_n)}} \tag{5.16}$$

Logistic regression is widely used in machine learning as a classification model. Its performance is measured based on its sensitivity and sensitivity.

$$P(\hat{P}(Y=0)|Y=0) \tag{5.17}$$

Depending on the context of the variable Y the equation is interpreted, if Y is a negative event such as a disease then (5.17) represents the probability that the model correctly detects healthy people, being the sensitivity:

$$P(\hat{P}(Y=1)|Y=1) \tag{5.18}$$

The probability that the model correctly detects sick people.

[1]These values can represent any event with two possible outcomes, life-death, health-disease, success-failure, etc.

In Rstudio, the function **glm** can be used to obtain a logistic regression model from a data set as specified.

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