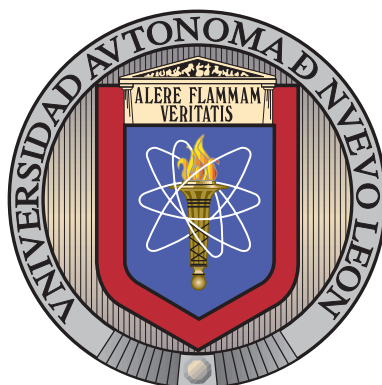


UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN

FACULTAD DE CIENCIAS FÍSICO MATEMÁTICAS

CENTRO DE INVESTIGACIÓN EN CIENCIAS FÍSICO MATEMÁTICAS



VEHICLE SCHEDULING IN PUBLIC TRANSPORTATION:
OPTIMIZATION MODELS FOR INTEGRATED PLANNING
AND ELECTROMOBILITY

POR

YULITZA YAZMÍN JACOBO ROMERO

DOCTOR EN CIENCIAS CON ORIENTACIÓN EN MATEMÁTICAS

SAN NICOLÁS DE LOS GARZA, NUEVO LEÓN

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Universidad Autónoma de Nuevo León
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Los miembros del Comité de Tesis recomendamos que la Tesis “Vehicle Scheduling in Public Transportation: Optimization Models for Integrated Planning and Electromobility”, realizada por el alumno Yulitza Yazmín Jacobo Romero, con número de matrícula 1614694, sea aceptada para su defensa como opción al grado de Doctor en Ciencias con Orientación en Matemáticas.

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Yulitza Yazmín Jacobo Romero

To Barbara Sofia, my greatest love and my greatest inspiration

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ABSTRACT

Public transportation systems require efficient operational planning to provide reliable services while effectively managing available resources. In this context, the Vehicle Scheduling Problem (VSP) plays a central role in assigning vehicles to scheduled trips while satisfying operational requirements. The transition toward electromobility and the incorporation of decisions from different planning levels introduce additional challenges that motivate extensions of traditional vehicle scheduling approaches.

This doctoral thesis develops mathematical optimization models and solution methodologies for extensions of the VSP, with particular emphasis on electric bus operations. Two main problems are addressed. The first integrates electric vehicle scheduling with on-route charging infrastructure location and timetabling decisions, while considering the impact of charging operations on passenger service. The second integrates vehicle scheduling, timetabling, and interlining within a bi-objective framework that analyzes the trade-off between fleet requirements and service coordination. Exact optimization, an epsilon-constraint approach, and a metaheuristic algorithm are employed to address the proposed problems and evaluate their operational implications.

Additionally, a generalized VSP formulation is presented in the Appendix, incorporating multiple lines, interlining, heterogeneous vehicle types, charging infrastructure location, and multiple charging technologies. Overall, this research contributes optimization approaches that extend vehicle scheduling to better represent the operational and technological characteristics of contemporary public transportation systems.

Palabras clave: Vehicle Scheduling Problem; Electric Bus Scheduling; Electromobility; Charging Infrastructure; Timetabling; Interlining

CHAPTER 1

INTRODUCTION

1.1 CONTEXT

The accelerated urbanization experienced worldwide over the last decades has considerably increased the demand for mobility within cities, making the provision of efficient, accessible, and sustainable transportation systems one of the major challenges faced by modern metropolitan areas. Population growth, urban expansion, and the increasing spatial separation between residential, commercial, industrial, and recreational areas have resulted in a substantial increase in the number of daily trips that millions of people undertake to perform their everyday activities. Consequently, many cities experience persistent problems such as traffic congestion, longer travel times, excessive energy consumption, deteriorating air quality, and reduced accessibility to economic and social opportunities, ultimately affecting both urban competitiveness and citizens' quality of life (Ibarra-Rojas et al., 2015b; ?). In this context, public transport constitutes one of the cornerstones of urban mobility by providing an efficient and affordable alternative to private automobiles. Furthermore, public transport promotes social equity by providing mobility options for people who do not have access to private vehicles, thereby strengthening social and spatial inclusion. Beyond its operational and social benefits, public transportation plays a strategic role in achieving sustainable urban development. Numerous studies have shown that increasing the share of public transport contributes to reducing greenhouse gas emissions, lowering fossil fuel consumption, and improving air quality by decreasing the number of private vehicles circulating on urban roads. Likewise, reduced traffic volumes are associated with fewer road accidents and injuries, while public transport encourages active mobility since passengers typically walk or cycle to access transit stops and stations.

The achievement of efficient and sustainable public transportation systems depends not only on infrastructure investments and technological improvements but also on the effectiveness with which planning and operational decisions are addressed. Public transport planning involves a set of highly interconnected decisions that affect different stakeholders, including transport authorities, operators, and passengers, each pursuing objectives

that may frequently conflict. (???). Traditionally, public transport planning has been structured into different decision levels according to the temporal horizon and the nature of the decisions involved. Strategic planning addresses long-term decisions, such as transit network design, infrastructure development, and fleet characteristics. Tactical planning considers medium-term decisions, including service frequencies, timetable generation, and resource allocation. Finally, operational planning focuses on short-term decisions associated with the daily execution of transit services, such as vehicle scheduling and driver scheduling (Ceder and Wilson, 1986; Desaulniers and Hickman, 2007; Ceder, 2016). This hierarchical structure has allowed transportation agencies to manage the complexity of large-scale transit systems by decomposing the overall planning process into smaller and more tractable optimization problems. The main optimization problems associated with public transportation planning include the Transit Network Design Problem, Frequency Setting Problem, Timetabling Problem, Vehicle Scheduling Problem, and Driver Scheduling Problem. Among these problems, the Vehicle Scheduling Problem has received particular attention due to its direct impact on fleet size, operational costs, and resource utilization, representing one of the most relevant operational decisions in public transportation systems (??). Although solving each planning problem independently simplifies the computational complexity, this sequential approach may neglect the interactions and dependencies among different decision processes. Consequently, solutions obtained from isolated optimization models may be locally optimal for a specific subproblem but inefficient from a global system perspective. For this reason, recent research has increasingly focused on the development of integrated optimization models capable of simultaneously considering multiple planning decisions. By incorporating the interactions among different operational components, integrated approaches can provide more realistic representations of transit systems and achieve improvements in operational efficiency, resource utilization, and service quality (Ceder, 2011; ?; Schmid and Ehmke, 2015). This evolution from sequential to integrated planning approaches has become particularly important due to the increasing complexity of modern public transportation systems. The incorporation of new technologies and operational characteristics introduces additional decision dimensions that cannot be effectively addressed through traditional isolated models.

The transition toward more sustainable transportation systems has promoted the adoption of electromobility technologies in urban public transportation networks. Since the transportation sector represents one of the main contributors to greenhouse gas emissions and urban air pollution, the electrification of bus fleets has been identified as a key strategy to reduce the environmental impact associated with urban mobility. In addition to environmental benefits, electric buses may provide long-term operational and economic advantages due to lower energy consumption and maintenance costs. However, the adop-

tion of electric buses introduces new challenges in the planning and operation of public transportation systems. Unlike conventional vehicles, whose operation mainly depends on fuel availability, electric buses are affected by battery capacity limitations, charging times, energy consumption variability, battery degradation, and charging infrastructure availability. These characteristics modify traditional vehicle scheduling decisions, since the feasibility of vehicle assignments depends not only on temporal and spatial constraints but also on energy availability throughout the operation (Hu et al., 2022).

The operation of electric bus fleets also depends on the selected charging technology, which directly influences vehicle autonomy, infrastructure requirements, operational costs, and planning decisions. In general, charging strategies can be classified into slow charging and fast charging systems. Slow charging systems are typically performed at depots during off-service periods, mainly overnight, allowing vehicles to be fully recharged before beginning their daily operation. Although this strategy reduces the impact of charging activities on passenger service, it requires larger battery capacities and may increase vehicle acquisition costs. Conversely, fast charging systems, commonly associated with opportunity charging or on-route charging, allow charging operations during service through strategically located charging stations at terminals or intermediate stops. This alternative enables the use of smaller batteries and increases operational flexibility; however, it introduces additional decisions related to charging station location, charging duration, infrastructure investment, and the potential impact of charging activities on service quality and passenger inconvenience (Alamatsaz et al., 2022; Tang et al., 2019; Hu et al., 2022). Furthermore, hybrid charging configurations that combine depot and opportunity charging strategies have been explored to adapt to the specific operational characteristics of different transit systems. Therefore, the incorporation of electric buses represents not only a technological transformation but also a new dimension in public transport planning problems. The need to simultaneously consider vehicle assignment, charging strategies, and charging infrastructure decisions has motivated the development of new optimization models, such as the Electric Vehicle Scheduling Problem (EVSP), which extends traditional vehicle scheduling models by incorporating energy-related constraints and charging decisions (?). Consequently, the electrification of public transportation requires integrated optimization approaches capable of capturing the interactions between operational decisions and energy management strategies.

The increasing complexity of public transportation systems, driven by the incorporation of new vehicle technologies, increasing operational requirements, and the need for more efficient resource utilization, has created new challenges for traditional planning models. Among the different operational decisions involved in transit planning, the Vehicle Scheduling Problem (VSP) plays a fundamental role due to its direct impact on fleet size, operational costs, and the efficient utilization of available vehicles. Tradition-

ally, vehicle scheduling models have focused on assigning vehicles to a predefined set of trips while satisfying temporal and operational constraints. However, modern transit systems require more comprehensive approaches capable of incorporating additional decisions related to energy management, service coordination, infrastructure planning, and heterogeneous vehicle technologies.

Therefore, this doctoral thesis focuses on the development of optimization models that extend traditional vehicle scheduling approaches through the incorporation of additional planning and operational decisions, with particular emphasis on the requirements introduced by electromobility. Two main optimization problems are addressed in this thesis. In addition, a generalized formulation of the VSP is developed to provide a broader modeling framework capable of simultaneously representing several of the operational and technological characteristics analyzed throughout this research.

The first problem considers the operation of a single circular electric bus line equipped with opportunity charging technology. Unlike conventional vehicle scheduling problems, electric bus operations require considering additional energy-related constraints associated with battery capacity and charging activities. In this problem, the Electric Vehicle Scheduling Problem (EVSP) is integrated with charging infrastructure location and timetabling decisions. The objective is to determine the required fleet and charging infrastructure while analyzing the effect of charging operations on passenger service. Since opportunity charging is performed during vehicle operation, charging activities may increase dwell times and affect the service experienced by passengers. Therefore, a parameter is incorporated to limit the charging time associated with each trip and control its potential impact on users. Figure 1 illustrates the interaction between the circular route, charging infrastructure, vehicle schedules, and energy-related decisions considered in this problem.

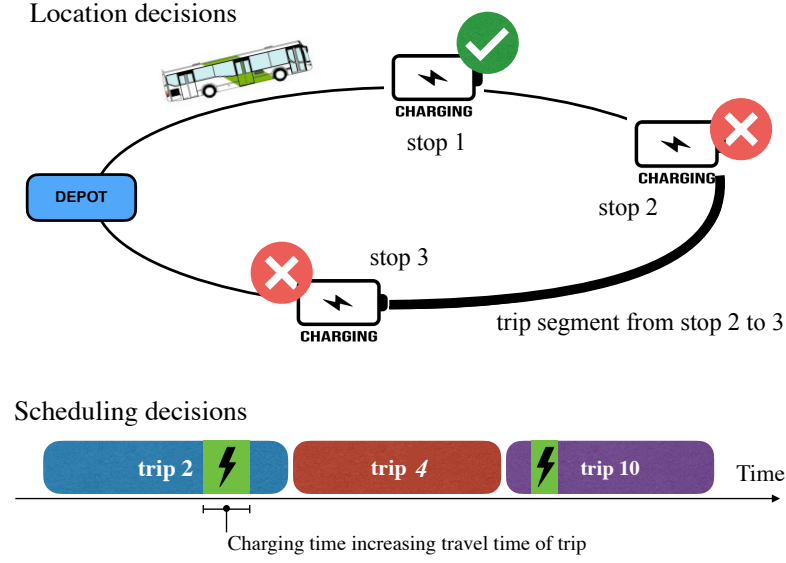


Figure 1: Partial schedule of an electric vehicle using battery charging along the route.

The second problem extends the operational perspective by considering a set of conventional bus lines with overlapping segments. In highly interconnected transit networks, planning each line independently may lead to inefficient vehicle utilization and limited coordination among services operating along common corridors. To address this issue, the Timetabling Problem and the Vehicle Scheduling Problem are jointly considered together with interlining decisions. Interlining allows a vehicle to operate consecutive trips belonging to different transit lines, providing greater flexibility in fleet utilization. However, this strategy requires simultaneously considering the temporal coordination among lines and the assignment and sequencing of trips. The proposed approach is formulated as a bi-objective optimization problem that aims to minimize both the deviation from the desired headways and the required fleet size. Figure 2 presents the conceptual representation of this problem, highlighting the interaction among overlapping lines, timetable coordination, and interlining decisions.

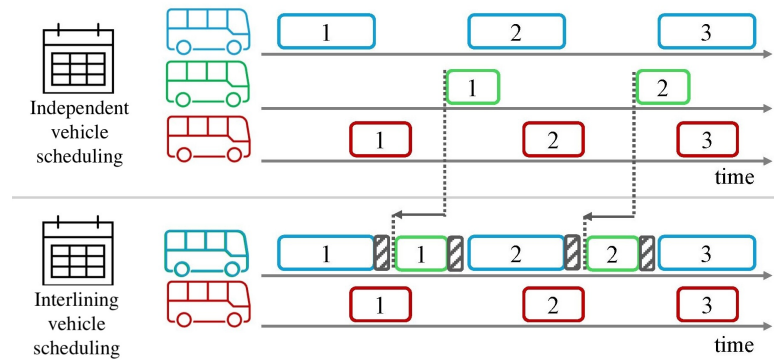


Figure 2: Improvement in vehicle scheduling assuming interlining and timetable flexibility.

As an additional contribution, this thesis develops a generalized formulation of the Vehicle Scheduling Problem, referred to as the Generalized Vehicle Scheduling Problem (G-VSP), which is presented in Appendix A. The formulation provides a broader modeling framework in which several operational and technological characteristics can be represented simultaneously. It considers multiple transit lines operated from a single depot and a heterogeneous fleet in which conventional and electric buses may coexist. Different charging technologies, including depot and on-route charging, are considered together with charging infrastructure location, predefined interlining possibilities, and deadheading movements. The G-VSP builds upon the characteristics investigated in the main problems of this thesis while extending the vehicle scheduling framework to accommodate more heterogeneous operational configurations. In this way, the generalized formulation complements the two main problems by providing a framework for analyzing the simultaneous coexistence of different vehicle technologies, charging alternatives, and operational characteristics.

The challenges discussed throughout this section demonstrate the need for optimization frameworks capable of representing the interactions involved in modern public transportation systems. In particular, the transition toward electromobility introduces additional operational and planning challenges related to the limited driving range of electric buses, charging requirements, and the availability and location of charging infrastructure. Addressing these challenges together with other operational decisions, such as timetabling and interlining, requires extending traditional vehicle scheduling approaches to better represent current transit operations.

Within this context, this doctoral thesis develops optimization models for different extensions of the Vehicle Scheduling Problem, with particular emphasis on the incorporation of electric buses and their charging requirements, while also considering other relevant planning and operational decisions. The two main problems constitute the central developments of the thesis, while the generalized formulation presented in the Appendix provides an additional framework that brings together several of the operational and technological characteristics considered throughout this research. The following sections introduce the objectives of this research, describe the methodology employed, summarize the main contributions of the thesis, and present the organization of the document.

1.2 OBJECTIVES

1.2.1 GENERAL OBJECTIVES

The general objective of this doctoral thesis is to develop mathematical optimization models and solution methodologies for the Vehicle Scheduling Problem in public transportation, incorporating planning and operational decisions as well as the technological requirements associated with electromobility.

1.2.2 SPECIFIC OBJECTIVES

To achieve the overarching objective of this doctoral thesis, several specific objectives have been established. These objectives are centered on the analysis, formulation, and evaluation of optimization models for the operational planning of public transportation systems, with a particular emphasis on vehicle scheduling. The proposed models extend the conventional Vehicle Scheduling Problem by incorporating decisions associated with different levels of transit planning and by accounting for the operational and technological requirements arising from electromobility. In this way, vehicle scheduling constitutes the core operational problem around which these additional decisions and characteristics are incorporated. The specific objectives of this research are as follows:

1. **Conduct a comprehensive literature review** on operational planning problems in public transportation, focusing on the Vehicle Scheduling Problem (VSP) to identify mathematical formulations, limitations, and research gaps regarding the integration of strategic, operational, and technological decisions.
2. **Formulate integrated optimization models** that extend the classical VSP by incorporating transit network operational characteristics, electric bus fleet constraints, and diverse charging technologies.
3. **Design a multi-objective optimization framework** to analyze the trade-off between operational efficiency (measured by fleet size) and service quality (measured by transit line synchronization along overlapping segments).
4. **Develop and implement tailored solution approaches**, employing exact mathematical programming solvers for the single-objective models, and both the ϵ -constraint method and metaheuristic algorithms for the multi-objective framework to compare their computational efficiency and solution quality.

5. **Validate the performance of the proposed models and algorithms** through computational experiments under different operational scenarios, assessing the sensitivity of key performance indicators to different charging infrastructures and operational strategies.
6. **Analyze the operational and managerial implications** of the integrated planning decisions, providing decision-support insights on how alternative charging technologies and coordination strategies contribute to more realistic and sustainable public transportation systems.

1.3 METHODOLOGY

The methodology adopted in this doctoral thesis is based on the development, mathematical formulation, and computational analysis of optimization models that extend the Vehicle Scheduling Problem (VSP) by incorporating additional planning, operational, and technological characteristics. The research begins with a review of the scientific literature on vehicle scheduling and its integration with other public transportation planning decisions, with particular attention to the operational requirements introduced by electric bus systems. Based on the research gaps identified in the literature, two main optimization problems are developed.

The first problem integrates electric vehicle scheduling with charging infrastructure location and timetabling decisions for a transit line operated with on-route charging technology. The formulation incorporates battery and charging requirements and includes a parameter that limits the charging time associated with each trip, allowing the impact of charging operations on passenger service to be analyzed. The model is evaluated through computational experiments using randomly generated instances and a real-world scenario. The second problem integrates vehicle scheduling, timetabling, and interlining for transit networks with overlapping lines. The problem is formulated as a bi-objective optimization model that analyzes the trade-off between fleet requirements and service coordination. Two solution methodologies are implemented and compared: an epsilon-constraint approach and a Biased Random-Key Genetic Algorithm (BRKGA). Their performance and the resulting trade-offs are analyzed through computational experiments. To validate the proposed models and solution methodologies, extensive computational experiments are conducted under diverse operational scenarios, assessing the sensitivity of key performance indicators (KPIs) to alternative charging strategies, infrastructure configurations, and line synchronization parameters. The resulting numerical analysis provides quantitative insights into the benefits and implications of the proposed integra-

tions, demonstrating their potential to provide robust decision-support tools for modern, sustainable public transit operations.

1.4 CONTRIBUTION

The transition toward sustainable, electrified, and highly coordinated public transit networks demands a fundamental shift from traditional, decoupled planning methodologies to integrated optimization frameworks. This doctoral thesis addresses this challenge by developing mathematical models and solution algorithms that bridge the gap between vehicle scheduling and critical operational, technological, and strategic decisions. By capturing the complex interdependencies between fleet electrification, charging infrastructure, service synchronization, and resource management, this work provides both theoretical advancements in operations research and practical decision-support tools for modern transit agencies. The primary scientific contributions resulting from this research are summarized as follows:

1. **A novel mathematical formulation integrating electric vehicle scheduling, charging infrastructure, and timetable planning:** This contribution addresses the Electric Vehicle Scheduling Problem (E-VSP) under opportunity charging technologies. The proposed model simultaneously optimizes vehicle scheduling, charging station location, and timetable planning, capturing the complex, feedback-loop interactions between fleet operations and infrastructure deployment.
2. **An optimization-based framework to analyze the operational and service quality trade-offs of charging strategies:** By incorporating a dedicated control parameter that models the direct impact of charging actions on passengers, this formulation enables transit agencies to systematically evaluate and balance the trade-offs between fleet operational costs, infrastructure investment, and perceived passenger service quality.
3. **A multi-objective optimization model coupling timetable planning, vehicle scheduling, and interlining:** Tailored for public transportation networks with overlapping routes, this multi-objective formulation explicitly analyzes the conflict between operational efficiency (fleet size minimization) and service quality (line synchronization to facilitate passenger transfers).
4. **High-performance solution methodologies for integrated multi-objective transit planning:** This contribution delivers a rigorous computational comparison between an exact approach based on the ϵ -constraint method and a metaheuristic

algorithm. The results provide valuable algorithmic insights into the scalability, trade-offs, and applicability of exact vs. approximate methods for large-scale transit networks.

5. **A generalized Vehicle Scheduling Problem for heterogeneous fleets and complex network topologies:** This model generalizes the classical VSP to accommodate bidirectional transit lines and heterogeneous fleets, integrating interlining, deadheading, and infrastructure locations while simultaneously managing conventional, depot-charging, and opportunity-charging electric buses within a single, unified network.
6. **A unified optimization framework for modern, sustainable public transit planning:** Bridging the gap between theoretical scheduling models and the multifaceted technological constraints of modern transit, this overarching framework progressively integrates energy, technology, and operational decisions, providing a comprehensive decision-support tool for highly complex operational environments.

1.5 STRUCTURE

The remainder of this doctoral thesis is organized as follows. Chapter 2 reviews the relevant literature on the Vehicle Scheduling Problem (VSP), focusing on the incorporation of additional planning decisions and the operational characteristics associated with electric buses. Chapter 3 addresses an electric vehicle scheduling problem that incorporates on-route charging infrastructure location and timetabling decisions, while considering the impact of charging operations on passengers. Chapter 4 presents a bi-objective vehicle scheduling problem that incorporates timetabling and interlining decisions and is addressed using an ε -constraint method and a metaheuristic algorithm. Finally, Chapter 5 discusses the main findings and contributions of this research, presents the overall conclusions, and outlines directions for future research.

CHAPTER 2

STATE OF THE ART

The operational planning stage plays a fundamental role in public transportation systems, as it determines the decisions required to ensure the efficient operation of transit services. Among the different optimization problems arising at this stage, the *Vehicle Scheduling Problem* (VSP) has become one of the most extensively studied because of its direct impact on resource utilization, operating costs, and service quality. The objective of the VSP is to assign a fleet of vehicles to a set of scheduled trips by constructing feasible vehicle blocks that satisfy the operational constraints of the system while optimizing performance measures such as fleet size, operating costs, and deadheading activities (Ceder, 2016). Owing to its practical relevance and combinatorial complexity, the VSP has attracted considerable attention over the past decades, leading to the development of numerous mathematical formulations and solution methodologies for different operating environments.

The increasing complexity of modern public transportation systems has progressively expanded the scope of the classical VSP. On the one hand, researchers have incorporated decisions traditionally addressed in other planning processes, including timetable planning, service coordination, interlining, crew scheduling, and other operational decisions that directly interact with vehicle assignment. On the other hand, the transition toward electric bus fleets has introduced new operational challenges associated with battery capacity, charging strategies, charging infrastructure, and the coexistence of multiple charging technologies, giving rise to new variants of the traditional scheduling problem.

In this context, this chapter reviews the literature following these two major research directions. The first section focuses on studies that enrich the classical Vehicle Scheduling Problem by incorporating decisions from other public transportation planning processes. The second section reviews the main contributions related to electric bus scheduling, emphasizing the evolution of charging decisions and the operational characteristics introduced by electromobility. Together, these research streams provide the conceptual foundation for the optimization models developed throughout this dissertation.

2.1 INTEGRATION OF THE VEHICLE SCHEDULING PROBLEM

Research on the classical *Vehicle Scheduling Problem* has progressively evolved by incorporating operational decisions that were traditionally addressed in other stages of the public transportation planning process. Rather than optimizing these decisions independently, recent studies have proposed integrated optimization approaches that explicitly account for the interactions among different planning activities. These developments have progressively expanded the scope of the classical VSP, allowing a more realistic representation of public transportation systems while simultaneously improving operational efficiency and service quality.

The first studies aimed at extending the classical Vehicle Scheduling Problem primarily focused on incorporating timetable planning decisions. Since departure and arrival times directly determine the compatibility between consecutive trips, allowing limited timetable adjustments increases the flexibility of vehicle scheduling and can significantly improve fleet utilization without substantially affecting service quality. One of the pioneering contributions in this area was presented by Ibarra-Rojas et al. (2014), who proposed a bi-objective optimization model integrating timetable planning and vehicle scheduling while explicitly considering passenger transfer synchronization. Subsequently, Schöbel (2017) expanded this perspective by jointly optimizing line planning, timetable planning, and vehicle scheduling within a unified optimization framework. Later, Carosi et al. (2019) incorporated service regularity into the integrated planning process, whereas Schiewe (2020b) proposed one of the most comprehensive formulations by simultaneously considering line planning, timetable planning, passenger routing, and vehicle scheduling. From a passenger-oriented perspective, Schiewe (2020a) demonstrated the benefits of incorporating passenger routing decisions during timetable optimization, while Cao and Ceder (2019) balanced passenger-related and operational costs, including deadheading activities. Collectively, these studies showed that jointly optimizing timetable-related decisions provides greater flexibility and leads to more efficient vehicle schedules than the traditional sequential planning approach.

As integrated planning approaches continued to evolve, increasing attention was devoted to operational decisions aimed at improving service coordination in addition to fleet utilization. Consequently, several studies incorporated decisions related to transfer synchronization, service regularity, overlapping transit lines, and interlining, recognizing that vehicle scheduling also plays a key role in enhancing passenger experience. In this context, Heidari et al. (2020) and Chen et al. (2022) proposed optimization models that jointly

determine timetable and vehicle scheduling decisions while reducing passenger waiting times and operating costs. Similarly, Ataeian et al. (2021) developed a multi-objective formulation to maximize synchronization events while minimizing fleet size, whereas Shen et al. (2021) analyzed the trade-off between passenger-related costs and vehicle operating costs. Another important research direction incorporated interlining, allowing vehicles to operate trips belonging to different transit lines to improve fleet utilization and service coordination. Studies by Schmid and Ehmke (2015), Fonseca et al. (2018), Van Lieshout (2021), and Liu et al. (2023) further extended this concept by combining interlining with timetable planning and vehicle scheduling in increasingly integrated optimization models. Overall, these studies demonstrate a clear evolution of the literature, moving from improving fleet utilization through timetable flexibility toward formulations that simultaneously consider operational efficiency, service coordination, and passenger-oriented performance measures.

The studies reviewed in this section illustrate how the classical Vehicle Scheduling Problem has progressively evolved by incorporating planning decisions that enhance both operational efficiency and service quality. Nevertheless, these approaches still assume conventional vehicle technologies and therefore do not consider the operational constraints introduced by electric bus fleets. The transition toward electromobility has fundamentally transformed the nature of the scheduling problem by introducing battery limitations, charging requirements, charging infrastructure, and multiple charging technologies. These new challenges have motivated the development of the Electric Vehicle Scheduling Problem, which is reviewed in the following section.

2.2 VEHICLE SCHEDULING PROBLEM WITH ELECTRIC BUS OPERATIONS

The transition from conventional diesel fleets to electric bus systems has fundamentally changed the nature of the classical *Vehicle Scheduling Problem* (VSP), introducing new operational constraints that significantly increase the complexity of the scheduling process. In conventional bus operations, vehicle assignments are primarily determined according to the temporal compatibility between consecutive trips, ensuring that operational requirements such as travel times, layover times, depot capacities, and vehicle availability are satisfied. However, the incorporation of electric buses introduces an additional operational resource that must be explicitly managed throughout the planning horizon: *energy*. Since electric buses operate with limited battery capacity and driving range, vehicle assignments must not only satisfy temporal feasibility but also guarantee that each vehicle

maintains sufficient energy to complete its assigned trips. Consequently, the scheduling process must simultaneously determine vehicle assignments and a series of energy-related operational decisions that are absent in conventional fleets. These include determining *when* a vehicle should be recharged, *where* charging activities should take place, *how long* each charging operation should last, and *how* charging activities should be coordinated with scheduled services without disrupting transit operations. Furthermore, the introduction of charging infrastructure creates additional planning considerations, since the availability, location, and capacity of charging facilities directly influence vehicle circulation and scheduling decisions. The complexity of the problem is further increased by the existence of different charging technologies, each characterized by distinct operational capabilities. The most common alternatives include *slow charging*, typically performed at depots during overnight periods or extended layovers, and *fast opportunity charging*, which allows vehicles to recharge at terminal stations or strategically located charging facilities during daily operation. These technologies differ considerably in charging power, charging duration, infrastructure requirements, and operational flexibility, directly affecting fleet utilization and vehicle scheduling decisions. Consequently, selecting an appropriate charging strategy becomes an essential component of the planning process, as it influences not only charging operations but also the overall design of vehicle schedules.

Since the emergence of the Electric Vehicle Scheduling Problem, numerous studies have focused on incorporating the operational characteristics of electric buses into the vehicle scheduling process. The main contributions in this research direction are reviewed below.

One of the earliest contributions was presented by Zhang et al. (2021), who proposed an exact branch-and-price algorithm for the EVSP with partial charging, demonstrating that allowing intermediate charging operations significantly improves fleet utilization and reduces operating costs. Subsequently, Zhou et al. (2022) formulated the EVSP as a mixed-integer programming model strengthened with valid inequalities, showing the computational benefits of exploiting the mathematical structure of the problem. Seeking to better represent charging operations, Chen et al. (2023) analyzed different charging strategies through an adaptive large neighborhood search algorithm, concluding that the selection of charging policies has a significant impact on operating costs. Similarly, Zheng et al. (2024) incorporated charging power limitations into the scheduling process, explicitly considering the interaction between vehicle schedules and charging station capacities. To address large-scale instances, Xie (2023) proposed a two-stage heuristic procedure based on generation and selection mechanisms, while Tang et al. (2023) developed a genetic algorithm capable of simultaneously optimizing vehicle scheduling and charging decisions for multi-line electric bus systems. Finally, Yao et al. (2020) and Sung et al.

(2022) extended the applicability of the EVSP by considering more realistic operational settings, demonstrating that explicitly incorporating energy consumption and charging decisions substantially improves the planning of electric bus operations. Overall, these studies established the methodological foundations of the EVSP by integrating energy-related operational constraints into the vehicle scheduling process, while treating vehicle scheduling as an isolated planning problem without incorporating additional operational or strategic decisions.

Although the previous studies significantly advanced the EVSP by incorporating the operational characteristics of electric bus fleets, they generally addressed vehicle scheduling independently from other planning components. More recently, research has shifted toward the development of integrated optimization models that simultaneously consider vehicle scheduling and additional planning decisions, aiming to improve both operational efficiency and the utilization of charging resources.

One of the earliest studies in this direction was presented by ?, who jointly optimized timetable planning and vehicle scheduling for electric bus systems. Their model allowed limited timetable adjustments to improve charging feasibility and reduce fleet requirements, demonstrating the benefits of coordinating these two planning stages. Similarly, Duan et al. (2023) integrated timetable planning with the EVSP while explicitly considering charging operations, showing that timetable flexibility can substantially improve the operational performance of electric bus systems.

Another important research direction has focused on integrating charging infrastructure decisions with vehicle scheduling. In this context, Wang et al. (2022) proposed a model that simultaneously determines vehicle schedules and the location of charging stations, highlighting the strong interaction between infrastructure deployment and fleet operation. Likewise, ? jointly optimized charging station location and vehicle scheduling while considering different charging technologies, demonstrating that infrastructure planning has a significant impact on both operational costs and charging requirements. Similarly, Frieß and Pferschy (2024) incorporated charging station location into the EVSP through an exact optimization framework capable of evaluating different infrastructure configurations. More recently, ? further extended this line of research by integrating charging infrastructure planning with multiple charging technologies, providing a more comprehensive representation of electric bus operations.

Overall, these studies demonstrate a clear evolution of the EVSP toward more integrated optimization frameworks, in which vehicle scheduling is jointly addressed with other planning decisions that directly influence the operation of electric bus systems. Nevertheless, the simultaneous integration of operational decisions such as interlining,

charging infrastructure location, and multiple charging technologies remains relatively unexplored in the literature, highlighting the need for more comprehensive optimization models capable of representing the increasing complexity of modern electric public transportation systems.

CHAPTER 3

BATTERY ELECTRIC BUS SCHEDULING
AND LOCATION OF CHARGING
INFRASTRUCTURE FOR A TRANSIT LINE
WITH ON-ROUTE CHARGING
TECHNOLOGY

In this chapter, we introduce an integrated Electric Vehicle Scheduling and Location of Charging Stations framework, known as EVSP-LS, to minimize the operational costs of the fleet size for circular lines using Opportunity Charge technology (OppCharge, see OppCharge, 2017), while considering charging infrastructure and associated costs. OppCharge provides rapid charging for buses equipped with small batteries. It uses Wi-Fi-enabled chargers at some stations that connect to the vehicle's roof via a pantograph. Therefore, charging stations can be strategically located along the route, similar to the implementations in Geneva (Patey et al., 2016) and Nantes (Hampel, 2019). The motivation for considering this technology comes from its ability to reduce long charging times and prevent deadheading of electric buses returning to the depot, thus improving operational flexibility within the transit system. However, its main drawbacks are the need for sufficient charging infrastructure and the waiting time of passengers during recharging stops along the route.

The optimization problem proposed in this study determines the optimal locations for the charging infrastructure (at stops along the transit line), trip-to-vehicle assignments, and the scheduling of charging actions for electric vehicles from a homogeneous fleet taking into account the battery consumption of each segment of the trip between consecutive charging stops. The assumption of a homogeneous fleet is consistent with several real-world applications, particularly in scenarios where operators standardize vehicle types to simplify operations, reduce costs, and streamline maintenance processes.

This is especially relevant during the transition to electric fleets, where the introduction of a uniform fleet allows operators to gain efficiencies in terms of vehicle procurement, training, maintenance, and spare parts inventory. For example, public transport systems such as those in Shenzhen, have successfully adopted homogeneous electric bus fleets, where the standardization of vehicle models not only simplifies logistical challenges but also supports the large-scale deployment of electric vehicles (see Yuan et al., 2020). In addition, London and Los Angeles have also undertaken initiatives to replace their diesel buses with homogeneous electric buses fleets, underscoring the operational benefits of fleet standardization during such transitions (Li et al., 2019; Moura et al., 2019). Figure 3 provides a schematic representation of the integrated decisions in our optimization problem. The upper panel shows the location of the charging infrastructure at one of the three specific stops along a transit line and highlights an example of a trip segment from stop 2 to stop 3. The lower panel illustrates the schedule for three trips carried out by an electric bus. In the schedule, the actions highlighted in green with the lightning symbol represent the charging actions of electric buses, where these actions occur at stops along the line, as we assume the use of opportunity charging technology.

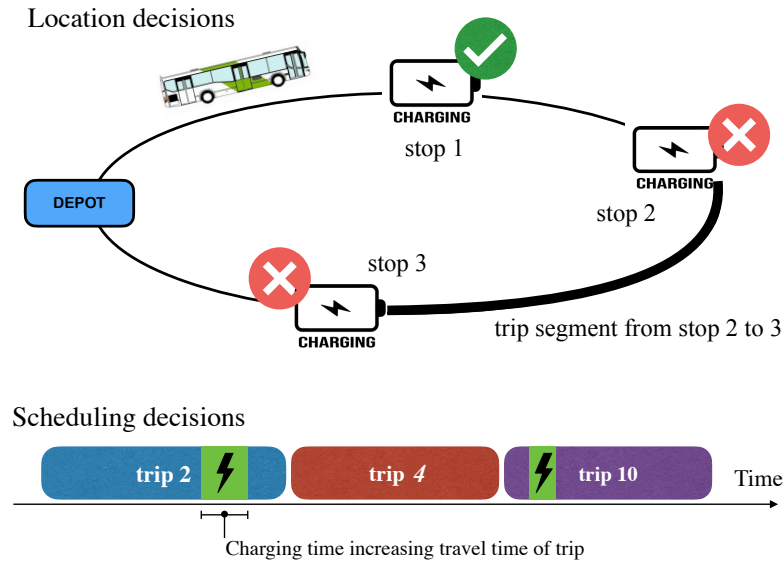


Figure 3: Partial schedule of an electric vehicle using battery charging along the route.

Our main contribution is a novel optimization problem for battery electric bus scheduling (i.e., schedule trips and charging actions for electric buses), and the location of charging infrastructure, along with its mathematical formulation, that incorporates the operational characteristics of a transport system using OppCharge technology. In particular, the decisions of charging actions that have not been addressed in the existing literature. Moreover, we propose an experimental stage that focuses on analyzing opera-

tional costs while assuming limits for the charging time of each trip to control the impact on passengers and comparing our problem with the operational version that considers only the scheduling aspect to analyze the impact of location decisions. The numerical results provide operational insights into how our approach can be implemented to generate valuable information for decision makers.

3.1 PROBLEM DEFINITION

Let us assume a circular line of a public transport system (Figure 4), where the depot is the starting and ending point for all trips. Moreover, there is a set of buses V with a state of charge of battery of 100% at the beginning of the day (the buses can be charged overnight). Let $I = \{1, \dots, |I|\}$ be a set of trips of the transit line with known departure times dt_i , $i \in I$, and overtaking is not allowed. Moreover, the set I_0 is the union of the set I and an artificial trip '0', representing the starting and ending points of the vehicles. Let $S = \{S0, S1, S2, \dots, Sn\}$ be the set of stops eligible for Opportunity Charging stations and a charging rate function f (percentage of battery capacity per minute), which is assumed to be constant, as considered in other recent studies in the literature (e.g., ?????). A travel time t_s is defined for each segment of the line from the stop s to the stop $s + 1$; a charging consumption C_s is defined per segment of the trip from s to $s + 1$; and a fixed recharge preparation time λ and a safety buffer, which is a minimum β for the state of charge of the battery during system operation, are also defined. Finally, a value α is defined to control the impact of recharging actions on users, since they wait during this procedure.

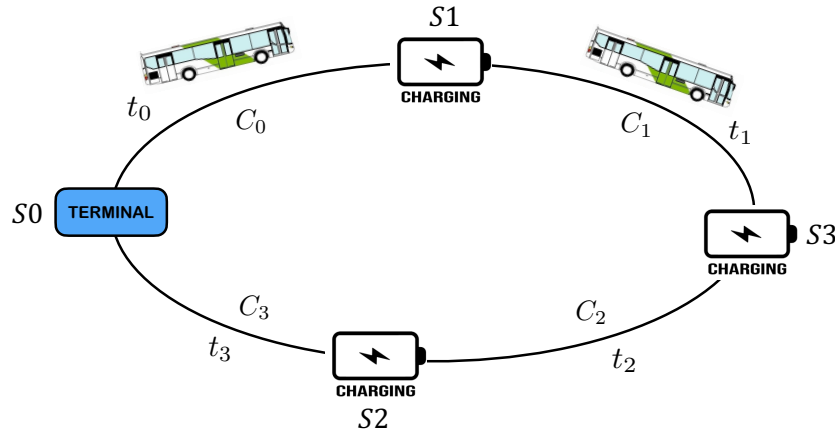


Figure 4: Circular line and potential charging stations along the route in a public transport system.

Our Electric Vehicle Scheduling Problem with Location of Charging Stations, or simply EVSP-LS, determines the location of charging stations and the sequence of trips for each bus, along with its corresponding charging schedule, in such a way that the total cost in terms of fleet size and electric infrastructure is minimized. As with other studies in the literature, we consider our optimization problem to be part of mid-term and long-term decisions, assuming that a timetable for an operational day is provided while aiming to minimize the purchase of electric buses. The following restrictions must be guaranteed: each trip must be assigned to a single bus; the number of buses required must not exceed a maximal fleet size; charging stations can be used by only one bus at a time; the charging time depends on the battery percentage of the bus when it arrives at the station; the security buffer of each bus must not be less than a specific minimum state of charge of the battery (β); the total charging time during each trip must not exceed the maximum allowed (α), which also controls the charging costs; and finally, there is a maximum number n of charging actions for each trip.

3.1.1 MATHEMATICAL MODEL

We recall that our EVSP-LS assuming fast charging determines the vehicle assigned to each trip $i \in I$ and the stations that must be installed, where battery charging actions are performed. Moreover, we summarize the parameters in Table 1.

Table 1: Summary of parameters of the EVSP-LS.

V	Set of buses.
I	Set of trips of a circular line.
I_0	Union of set I and an artificial trip ‘0’ representing the depot, that is the starting and ending point of each trip (passengers could board and alight without loss of generality).
dt_i	Departure time of trip $i \in I$.
S	Set of stops equipped with fast charging infrastructure.
f	Charging rate function, i.e., percentage of battery capacity per charging minute.
t_s	Travel time of trip segment from stop s to $s + 1$.
C_s	Charging consumption (percentage of battery) for the trip segment from s to $s + 1$.
λ	Fixed recharge preparation time.
β	Safety buffer (percentage of battery capacity) for the state of charge of electric buses during the operation.
α	Upper bound for the total recharging time for each trip.
n	Maximum number of charging actions for each trip.

Now, we propose the following decision variables to design our mathematical model.

-
- $z_s = \begin{cases} 1 & \text{if a station is installed in } s \in S \\ 0 & \text{otherwise} \end{cases}$
 - $x_{i'i} = \begin{cases} 1 & \text{if trip } i' \in I_0 \text{ is made just before } i \in I_0 \text{ by the same vehicle} \\ 0 & \text{otherwise} \end{cases}$
 - $y_{is} = \begin{cases} 1 & \text{if a charging action is performed in station } s \text{ during trip } i \\ 0 & \text{otherwise} \end{cases}$
 - $g_{i'is} = \begin{cases} 1 & \text{if trip } i' \in I_0 \text{ arrives at station } s \in S \text{ before trip } i \in I_0 \\ 0 & \text{otherwise} \end{cases}$
 - $ca_{is} = ca_{i,s-1} (1 - y_{i,s-1}) + 100y_{i,s-1} - C_{s-1}$: state of charge of the battery (percentage of total battery capacity) upon arrival at station s for trip i . This is defined in terms of the state of charge of the battery in the previous route segment $ca_{i,s-1}$, the charge consumption C_{s-1} , and the battery charging action $y_{i,s-1}$.
 - $ic_i = 100x_{0i} + \sum_{i' \in I} x_{i'i} (ca_{i',|S|} (1 - y_{i',|S|}) + 100y_{i',|S|} - C_{|S|})$: state of charge of the battery at the beginning of trip i for the assigned bus. This depends on the state of charge of the vehicle's battery at the end of the previous trip. We highlight that $ca_{i0} = ic_i$ for all $i \in I$.
 - $tc_{is} = \frac{(100 - ca_{is})}{f}$: charging time at station $s \in S$ for vehicles on trip $i \in I$ to achieve a full charge. Note that the numerator is the percentage of the battery capacity to be fulfilled. Furthermore, even when considering full charging, if reduced charging times are required for more efficient operation of the transit line, the model will generate solutions that involve more frequent charging actions, with shorter charging times on specific trips (similar to the behavior observed with low α values in the experimental section).
 - $et_i = dt_i + \sum_{s \in S} t_s + \sum_{s \in S} y_{is}(tc_{is} + \lambda)$: final time of trip $i \in I$, calculated as the starting time of the trip plus the time required to travel each segment and the battery charging time (if any).
-

- $u_{is} = dt_i + \sum_{s'}^{s-1} (t_{s'} + y_{is'}(tc_{is'} + \lambda))$: arrival time at station $s \in S$ for a vehicle assigned to trip i . It is given by the sum of the initial time of trip i , total travel times between the previous segments to the station s , and the total charging time at the previous stations to s .

Using the decision variables described above, our mathematical formulation for EVSP-LS is given below.

$$\min \zeta_1 \left(\frac{\sum_{i \in I} x_{0i}}{|V|} \right) + \zeta_2 \left(\frac{\sum_{s \in S} z_s}{|S|} \right) \quad (3.1)$$

subject to:

$$\sum_{i \in I} y_{is} \leq |I| z_s \quad \forall s \in S \quad (3.2)$$

$$\sum_{i' \in I_0 - \{i\}} x_{i'i} = 1 \quad \forall i \in I \quad (3.3)$$

$$\sum_{i \in I} x_{0i} \leq |V| \quad (3.4)$$

$$g_{ii's} + g_{i's} \leq 1 \quad \forall i, i' \in I, s \in S \quad (3.5)$$

$$u_{i's} + y_{i's}(tc_{i's} + \lambda) \leq u_{is} + M(3 - y_{is} - y_{i's} - g_{i's}) \quad \forall i, i' \in I, s \in S \quad (3.6)$$

$$et_{i'} \leq dt_i + M(1 - x_{i'i}) \quad \forall i', i \in I \quad (3.7)$$

$$x_{ii'} + x_{i'i} \leq 1 \quad \forall i', i \in I \quad (3.8)$$

$$(1 - y_{is})ca_{is} + 100y_{is} - C_s \geq \beta \quad \forall s \in S, i \in I \quad (3.9)$$

$$\sum_{i' \in I_0 - \{i\}} x_{i'i} - \sum_{i' \in I_0 - \{i\}} x_{ii'} = 0 \quad \forall i \in I, \quad (3.10)$$

$$\sum_{s \in S} (tc_{is} + \lambda)y_{is} \leq \alpha \quad \forall i \in I \quad (3.11)$$

$$\sum_{s \in S} y_{is} \leq n \quad \forall i \in I \quad (3.12)$$

$$z_s, x_{i'i}, y_{is}, g_{ii's} \in \{0, 1\} \quad \forall i' \in I_0, i \in I, s \in S$$

$$tc_{is}, et_i, u_{is}, ic_i, ca_{is} \in \mathbb{R}_0^+ \quad \forall i \in I, s \in S$$

The objective function (3.1) represents the total cost in terms of the fleet size of electric buses and charging infrastructure. Inequalities (3.2) ensure that charging actions are performed at station s only if that location s is selected for charging infrastructure

installation. Constraints (3.3) ensure that all scheduled trips are assigned to a single vehicle. The constraints (3.4) define an upper bound for the size of the fleet. Inequalities (3.5) guarantee that, given two trips, only one can arrive before the other at a specific station. Constraints (3.6) avoid overlapping charging actions at the same station at a particular time. Inequalities (3.7) guarantee that two consecutively scheduled trips in the same vehicle do not overlap in time. Constraints (3.8) ensure that if two trips are assigned to the same vehicle, one is assigned before the other. Inequalities (3.9) ensure that the vehicle has sufficient battery charge to make the segment from s to $s + 1$ during the trip i . Restrictions (3.10) guarantee that each trip i is part of the sequence of trips for some vehicle. Note that we consider a homogeneous fleet, and the decision variables for vehicle scheduling do not include the vehicle index. In fact, the number of vehicles is calculated using the number of active variables x_{0i} , and the trips assigned to each vehicle are described by a sequence of trips starting and ending at the dummy trip ‘0’. Constraints (3.11) limit the total battery recharging time for each trip to control the impact on passengers in the system (and also charging costs). Finally, restrictions (3.12) limit the number of recharges for the same trip by the parameter n . We highlight that considering battery charging on-route requires different mathematical models compared to depot charging, as the ending times of trips are unknown and depend on charging actions that can be decided during the operation of transit lines. This characteristic also leads to the possibility of computing the effect on passenger travel times.

We highlight that the location of the charging infrastructure is performed for a single line, as we are specifically assuming the use of OppCharge at selected stops along each line. This assumption allows us to focus on the operational characteristics of the technology and the corresponding impact on the scheduling of electric buses. However, it is important to note that our mathematical model utilizes decision variables that are indexed by trips, rather than by specific transit lines. This means that the model does not directly assume or require that each trip be tied to a single line. Instead, the model operates on a trip-by-trip basis, with travel times and charging requirements defined for each individual trip.

Therefore, without loss of generality, we can easily extend our model to accommodate scenarios where trips belong to different lines and share common charging stops. This approach would allow us to optimize the location and usage of the charging infrastructure across multiple lines simultaneously. In fact, this flexibility is a key strength of the model, as it can be adapted to more complex network scenarios where several lines intersect at common stations or terminals, sharing charging facilities.

We observe that the formulation described by (3.1)–(3.12) involves the multiplication of variables in both variables and constraints. Thus, the experimentation was implemented

based on the linearized version of EVSP-LS (see Appendix A). Moreover, we initiate our experimental stage using the operational version of the EVSP-LS, which only considers scheduling decisions, to analyze the impact of optimizing strategic decisions while solving our EVSP-LS.

3.2 EXPERIMENTAL STAGE

In this section, we present the experimental results of our optimization approach. The computational experiments were conducted on a MacBook Pro computer with a chip Apple M3 Pro and 18GB of RAM. The proposed models were implemented in C++ using the ILOG Concert Technology (CPLEX 20.1), with a stopping criterion of 3 hours of computational time and default parameters, except for the relative gap that was set to 0%.

First, Section 3.2.1 details the instance generation process. In Section 3.2.2, we analyze the solutions obtained by solving only the operational decisions of our EVSP-LS. This includes the scheduling problem referred to as EVSP and its impact on the level of service for passengers. Finally, Section 3.2.3 presents the results of our EVSP-LS, which illustrate the impact of integrating decisions related to infrastructure location and vehicle scheduling problems.

3.2.1 INSTANCE GENERATION

We designed six types of instances to simulate circular public transport lines (10 instances per type): A1, A2, A3, B1, B2 and B3 with a number of trips within the set $\{30, 40, 50\}$. For all the parameters, we use a uniform random generation procedure given reference values. First, the departure time for the first trip on the line is chosen randomly from the set $\{15, 30\}$ minutes. Then, for each pair of consecutive trips i and $i+1$ ($i \in \{1, 2, \dots, |I| - 1\}$), the headway $h_{i,i+1}$ between those trips is generated randomly from $\{15, 30\}$ minutes; therefore, each instance has a specific duration of the planning period given by $\sum_{i=1}^{|I|-1} h_{i,i+1}$ plus the departure time of the first trip. We consider the set $S = \{0, 1, 2, 3, 4\}$ that corresponds to four stations and the depot (denoted as station $s = 0$) where buses can perform charging actions. Instances A simulate short lines, where the length of each segment of the trip is a random value between $[3, 8]$. In addition, instances type B simulate lines with higher mileage, where the length of each segment of the trip is generated randomly within $[9, 17]$. We define the maximum size of the fleet $|V|$ as the maximum

travel time (assuming charging actions at all charging stops S) divided by the minimum frequency, while the maximum number of charging actions n for a trip is fixed at $\lceil \frac{|S|}{2} \rceil$.

Regarding technical details about electric vehicle operation, we consider an average speed of 16.47 km/h. The buses are equipped with batteries with a capacity of 70 kWh and a battery consumption rate of 1.2 kWh/km (which leads to a bus autonomy of 58.333 km). The battery charging rate is defined as $f = 10\%$ of the battery capacity per minute, leading to a full battery charging time of 10 minutes starting from a 0% of the battery capacity. In addition, we incorporate a safety buffer of $\beta = 20\%$ for battery capacity.

Furthermore, we generate ten instances of type C based on the case study by ?. This case study involves a circular line of 28 km in Changchun City, China (see Figure 5). The timetable comprises 68 trips where the headway between consecutive trips is within $[4, 21]$ (the schedule is available for reference at ?). The only variable characteristic among the ten instances of type C is the locations of potential stations $|S|$, which are generated as follows. The length of the first $|S| - 1$ travel segments were randomly generated within the interval $[1, 11]$, and the last travel segment from $|S| - 1$ to 0 is calculated as $28 - \sum_{s=0}^{|S|-2} km_s$, where km_s is the length of the trip segment the trip from station s to $s + 1$.



Figure 5: Circular line of case study in ?.

It is important to note that limiting the number of recharge actions for a bus during the same trip is desirable to minimize the impact on passengers' waiting time inside the vehicle. We recall the definition of the parameter α as an upper bound of the charging time. This parameter is expressed as a percentage of the maximum travel time (MTT) assuming charging at all stations but without considering the fixed times of charging actions, and our experiments consider the values $\alpha = 0.06MTT$ and $\alpha = 0.10MTT$ in the 70 instances to analyze the impact of recharging times on the system's operational

performance. We decide considering a small fraction of travel time to avoid long waiting times for users (a sensitivity analysis is shown in Appendix B).

Table 2 describes the types of instances and their parameter details, totaling 70 instances.

Table 2: Instance characteristics.

Instance type	Trips	Headway (min)	Stations	Locations	Line length (km)	MTT (min)
A1	30					
A2	40	[15,30]	4		[20,30]	[76.28, 114.43]
A3	50					
B1	30					
B2	40	[15,30]	4		[50,70]	[190.72, 267.01]
B3	50					
C	68	[4,21]	4		28	106.8

In the following section, we present the computational experiments conducted to evaluate the performance of the proposed solution approaches.

3.2.2 EVSP PERFORMANCE

As mentioned earlier, the EVSP is defined as a vehicle scheduling problem with a given infrastructure for charging actions. Specifically, we assume that all locations in the set S are used (that is, $z_s = 1$ for all $s \in S$), and we set parameters $\zeta_1 = |V|$ and $\zeta_2 = 0$ to reduce the size of the fleet in the EVSP. Table 3 displays the average GAP and the computing time (in seconds) achieved for all instance types.

Table 3: Average GAP and computing times using $\alpha = 0.06MTT$ and $\alpha = 0.10MTT$ for the EVSP.

Type	$\alpha = 0.10MTT$		$\alpha = 0.06MTT$	
	GAP %	CPU_time (s)	GAP %	CPU_time (s)
A1	0	1.17	0	4.48
A2	0	5.12	0	19.22
A3	0	7.62	0	346.47
B1	0	0.96	0	0.79
B2	0	4.34	0.83 ^(*1)	7.01
B3	0.71 ^(*1)	11.57	0	13.81
C	0	34.73	0	114.09

It is evident that the solver achieves optimal solutions for most instances, regardless of the value of the parameter α or the type of instance. We highlight that symbol ‘*’ indicates a solver inconsistency, as there is one instance where the solver halted prematurely, failing to meet the stopping criterion. In particular, we observe longer computational times when we employ a smaller value for the parameter α and as the size of the instances increases.

Figure 6 illustrates the Gantt chart of the solutions obtained for the first instance of type C, using $\alpha = 0.06MTT$ (see Figure 6-(a)) and $\alpha = 0.10MTT$ (see Figure 6-(b)). The horizontal axis represents time, the intervals on each line correspond to trips assigned to the same vehicle, and the small green intervals depict recharging actions.

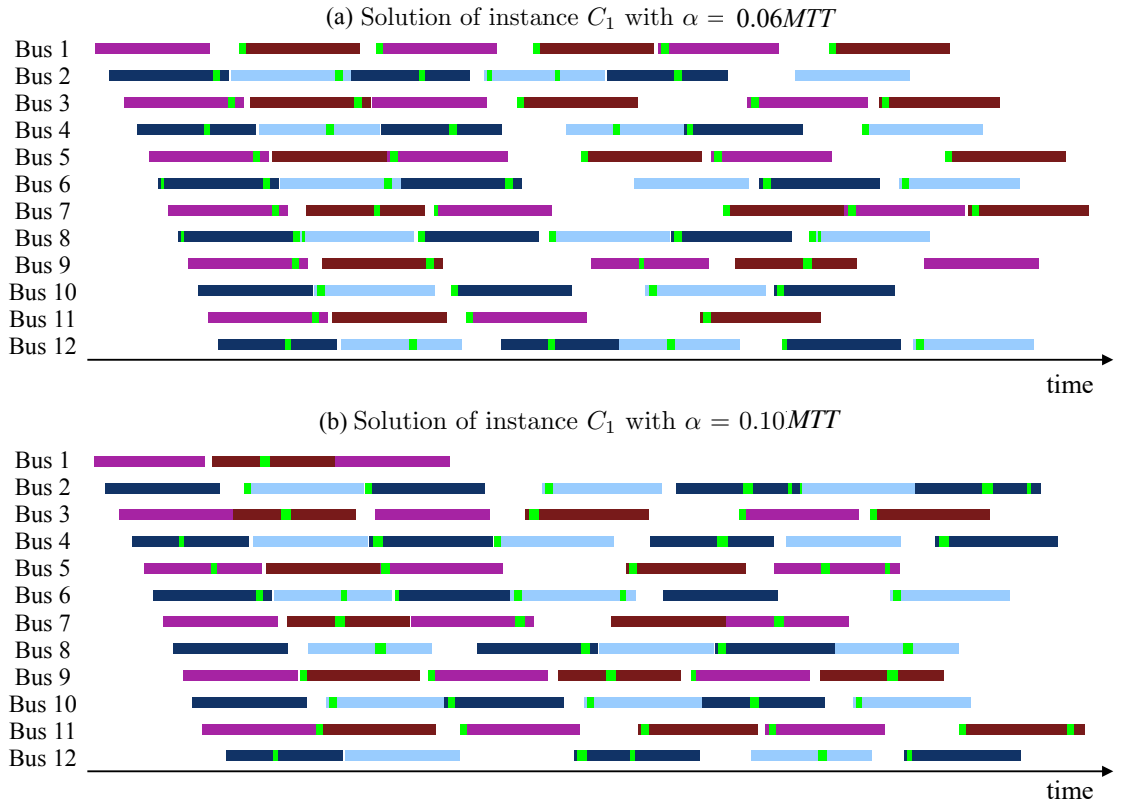


Figure 6: Solutions of instance C_1 (Changchun City) using both α values.

Note that buses recharge the battery along the route; It is evident that, for most trips, there is only one recharge action for instances with tighter bounds when $\alpha = 0.06MTT$ (see Figure 6-(a)). In contrast, with greater flexibility in charging time ($\alpha = 0.10MTT$ in Figure 6-(b)), we observe some trips with two charging actions.

Regarding the solver’s efficiency on the EVSP, the experimental results demonstrate that the proposed methodology is an effective tool for supporting decision making in pub-

lic transport planning. Optimal solutions are obtained within a reasonable computational time, even for real-size instances in operational strategies (decisions determined day by day). Furthermore, the parameter α plays a crucial role, allowing the planner to “control” the charging time that impacts operational costs, as well as the level of service for transport systems that employ charging technologies on the route.

Next, we propose some key performance indicators to analyze our generated solutions, so we can better understand how different assumptions in the model and parameters values affect the overall performance of the system.

ANALYSIS OF KEY PERFORMANCE INDICATORS

The first indicator of the level of service is the average recharging time per trip in the system, which directly impacts the travel time of the passengers in the line. Figure 7 illustrates the behavior of this indicator (Charging Time) for all instances using the values $\alpha = 0.06MTT$ (solid line) and $\alpha = 0.10MTT$ (dotted line).

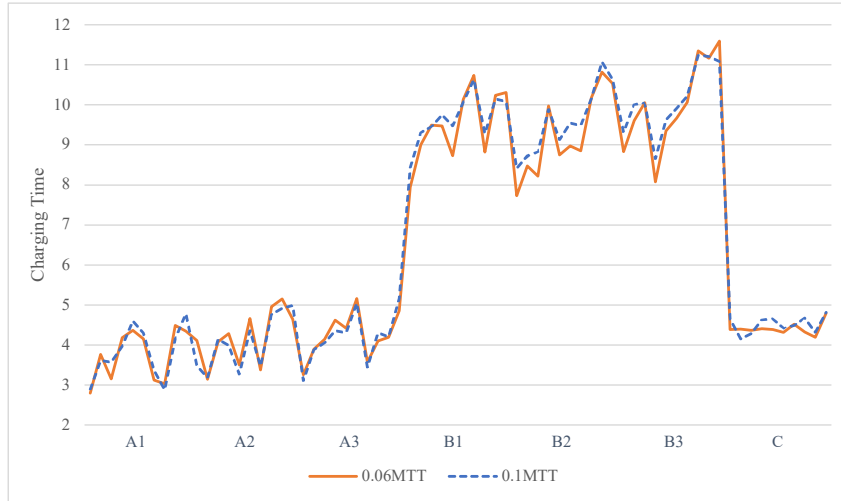


Figure 7: Average charging time per trip for all instances.

We observe that the average charging time per trip is very similar for both values of the parameter α in most instances. However, significant differences emerge, particularly in type A3 and B2 (noted by the solid line curve below the dotted one). This implies that for most cases, a specific battery charging time is inevitable, even with a strict upper bound on the maximum charging time per trip (smaller value of the parameter α).

In addition to revealing the time spent on recharging actions per trip, we are interested in analyzing the distribution of battery charging actions among different charging

stations. Therefore, we compare the average number of charging actions for a bus (average recharging), as depicted in Figure 8 for all instances.

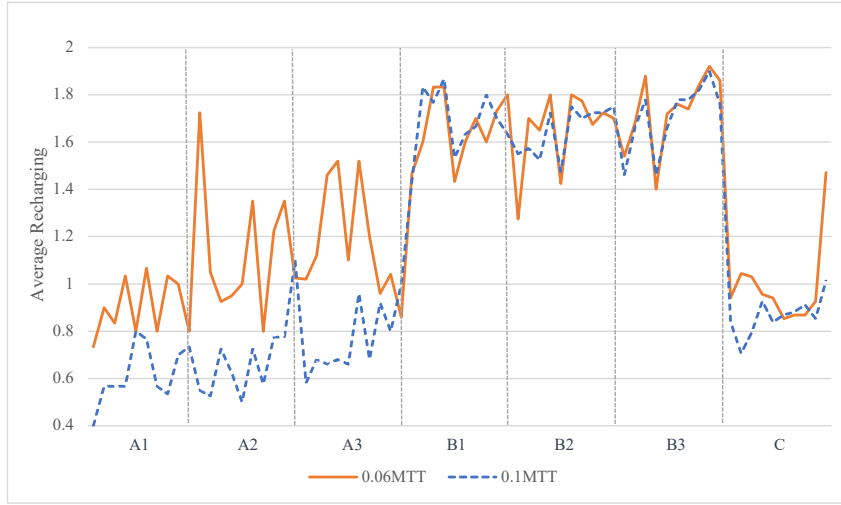


Figure 8: Indicator of average recharging actions per trip for all instances.

In particular, significant differences arise when varying the value of α , particularly for instances with short lines (A1–A3 and C). Despite efforts to control the impact of charging time in long-line instances, a certain number of recharging actions must be performed due to the limited number of available electric buses. On the other hand, for instances with short lines (A1–A3 and C), a smaller number of recharging actions is obtained by using a high value of α . This implies that the time spent on each recharging action is longer with a larger value α , since the average recharging time remains similar for different parameters α .

Finally, we propose an analysis of the level of utilization of charging stations in $S = \{S0, S1, S2, S3, S4\}$. For this purpose, we introduce an indicator that represents the percentage of station usage for each instance, using $\alpha = 0.06MTT$ (Fig. 9) and $\alpha = 0.10MTT$ (Fig. 10). It is important to note that there is a significant difference between the percentage of use of stations, which is more evident for the types of instance A1–A3. This emphasizes that the length of the line influences the percentage of station usage. Another characteristic of the instances affecting this indicator is the location of the stations, as it is the only varied characteristic while generating the ten instances of type C.

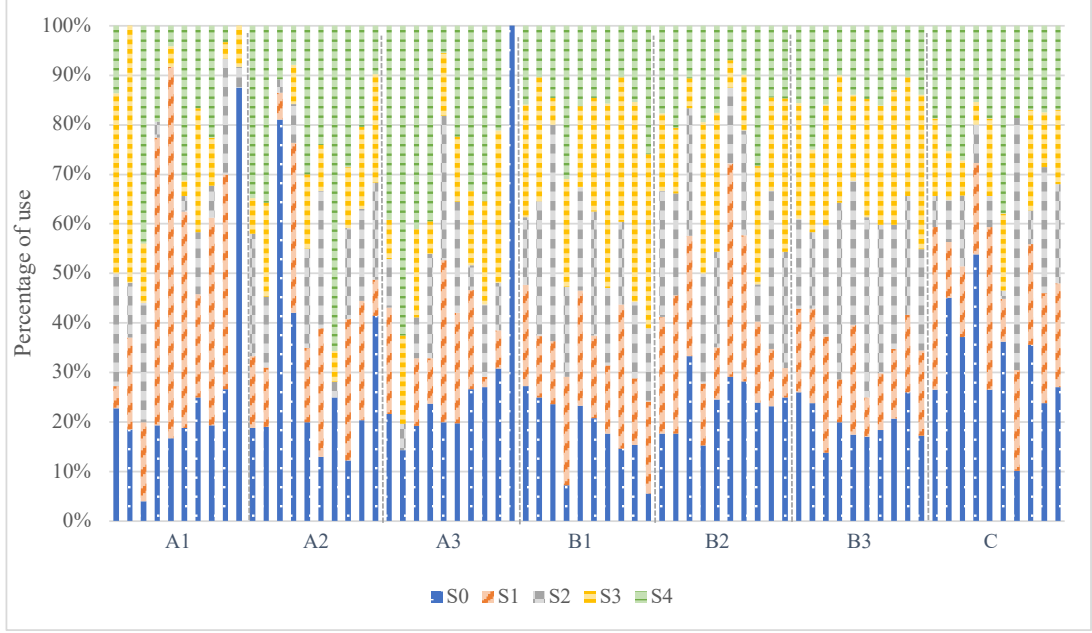


Figure 9: Percentage of station usage for all instances using a value of $\alpha = 0.06MTT$.

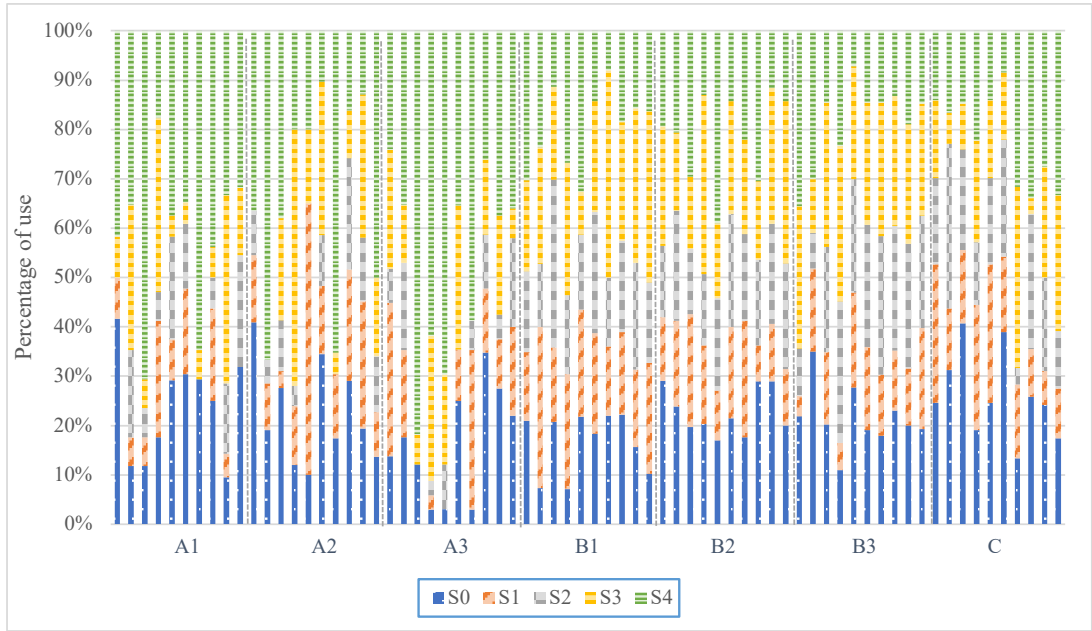


Figure 10: Percentage of station usage for all instances using a value of $\alpha = 0.10MTT$.

Fig. 9 and 10 illustrate that a higher value of the parameter α leads to a greater variation in the percentage of station usage. Specifically, we observe that station S_0 is used more on average in all instances when using $\alpha = 0.06MTT$ (see the blue bars in Fig. 9), while S_4 is the most used when $\alpha = 0.10MTT$ (see green bars in Fig. 10).

In general, charging stations are utilized at varying levels, resulting in the same operating cost. The results indicate that this level of utilization is sensitive to different characteristics of the instances. Therefore, it is reasonable to analyze how effectively the stations in our EVSP-LS are used.

3.2.3 EVSP-LS PERFORMANCE

In this section, we analyze the performance of the proposed MILP for the EVSP-LS, focusing on how the location of the charging infrastructure impacts the system. This analysis highlights the value of using an optimization approach as a decision support tool in the planning process of the transport network. In particular, we are now interested in also optimizing costs of the charging infrastructure; thus, we use parameters such as $\zeta_1 = 1$ and $\zeta_2 = 0.1$, since we consider the charging infrastructure as a secondary objective to optimize.

We highlight that weight parameters should be defined according to the types of technology and the operational characteristics relevant to the specific case study. The importance of each objective may vary depending on the operational context. For this purpose, further research can focus on the implementation of Multi-Criteria Decision Methods (Tsiakis et al., 2009), which would enable a more systematic approach to determining the values of these weights using information and preferences of users, operators or stakeholders, for example, Analytic Hierarchy Process (see Bremmers and Westerhof, 2010) or Technique for Order of Preference by Similarity to Ideal Solution (e.g., Chakraborty and Yilmaz, 2013). These methods would allow for a more transparent process in selecting the weight values, ensuring that they reflect the priorities and trade-offs that are most relevant to each specific transportation system.

It is important to note that while the weights may vary depending on the case study or the method used to determine them, the overall methodological approach and the underlying analytical framework would remain the same. The model would still provide valuable information on the optimization of battery electric bus scheduling and charging infrastructure, regardless of the specific weight assignments. Therefore, the core structure of the model is robust enough to adapt to different operational priorities without compromising the validity of the analysis.

Table 4 shows the average relative improvement¹ for the total charging time (column “CT Impr”) and its standard deviation (column “CT_sd”) for the solutions of the

¹The relative improvement is calculated as $\frac{EVSP_{sol} - EVSP-LS_{sol}}{EVSP_{sol}} * 100\%$, where $EVSP_{sol}$ is the solution obtained with the EVSP and $EVSP-LS_{sol}$ is the solution resulting from solving the EVSP-LS.

EVSP-LS compared to the EVSP on the same set of instances, and for both values of α . Similarly, the number of stations used and their standard deviation in columns “NS.impr” and “NS.sd” are provided, respectively. We highlight that symbol “*” indicate a solver inconsistency, as there are some instances where the solver was prematurely stopped, failing to meet the stop criterion.

In general, the results show that computation times are longer when solving the EVSP-LS formulation. The total time required to solve all instances with $\alpha = 0.06MTT$, for the EVSP and the EVSP-LS, increased from 1.40 to 3.88 hours, while for instances with $\alpha = 0.10MTT$, it increased from 0.18 to 1.21 hours. The most challenging instances of EVSP-LS using $\alpha = 0.06MTT$ were type A3 cases, corresponding to scenarios with short transit lines.

Table 4: EVSP-LS vs EVSP comparison on the improvement of charging time and number of stations.

Instance Type	$\alpha = 0.06MTT$				$\alpha = 0.10MTT$			
	CT.impr	CT.sd	NS.impr	NS.sd	CT.impr	CT.sd	NS.impr	NS.sd
A1	2.99% ^(*)	8.93	77.17%	4.38	11.07% ^(*)	9.38	77.67%	4.39
A2	4.67%	9.50	77%	6.32	3.27% ^(*)	6.99	76%	6.15
A3	5.77%	2.10	69.5%	25.22	2.40%	7.74	77.17%	4.38
B1	0.95%	5.95	60%	0	6.18%	8.52	60%	0
B2	-0.46%	7.62	60%	0	6.01%	5.66	60%	0
B3	-0.56%	5.10	60%	0	1.49%	5.29	60%	0
C	-0.04%	3.58	59%	3.16	2.94%	4.57	60%	0

Note that there is a positive relative improvement (savings) in the charging time for all instances, except instances B2–B3 and C with $\alpha = 0.06MTT$. It is important to note that as α decreases, the passenger travel time has less impact. Furthermore, the maximum average relative improvement was obtained for instances A1 with $\alpha = 0.10MTT$. Furthermore, we also observe that in instances with long lines (B1–B3) and real scenario C, greater savings on charging times are possible when assuming a value of $\alpha = 0.10MTT$ compared to using $\alpha = 0.06MTT$. Therefore, EVSP-LS does not significantly improve charging times for instances with long lines when the impact on the passenger is already controlled using low values of the parameter α .

Regarding the use of stations, it should be noted that EVSP solutions commonly use 4 or 5 charging stations in most instances. When using a parameter of $\alpha = 0.06MTT$, we observe that the EVSP-LS leads to savings of at least 59% for all instances, showing the largest savings of 77.17% in instances of type A1. In instances with a value of $\alpha = 0.10MTT$, the EVSP-LS obtains an average saving of at least 60% on the stations

used for all instances. In particular, an improvement of at least 76% is obtained for all instances that simulate short lines. Finally, the number of stations was reduced by 60% for instances B1-B3 (longer lines) and real scenario C. Generally, the reduction in the number of charging stations with EVSP-LS is more significant in scenarios where the impact on the passenger is less restricted (i.e., a higher value of α).

Based on the previous results, the EVSP-LS solutions allow for the implementation of a similar amount of total charging time (and even longer) than in the EVSP but using fewer charging stations. Consequently, savings in electrical infrastructure are achieved using the EVSP-LS problem, providing a good compromise/option to support strategic decisions.

We want to emphasize that the experimental results in Table 4 only present the average savings when using the EVSP-LS compared to the EVSP without providing details on the station usage for both problems. To address this, we offer a more detailed analysis of the potential information that EVSP-LS can generate in a real-world scenario. Specifically, Figure 11 displays a graph for each location $s \in S$, where the horizontal axis represents the instances, and we use a binary indicator taking the value of 1 if the station is used, and 0 otherwise. This indicator is defined for the solutions of the EVSP with circles and those of the EVSP-LS with triangles. We calculate these indicators only for instances with $\alpha = 0.10MTT$.

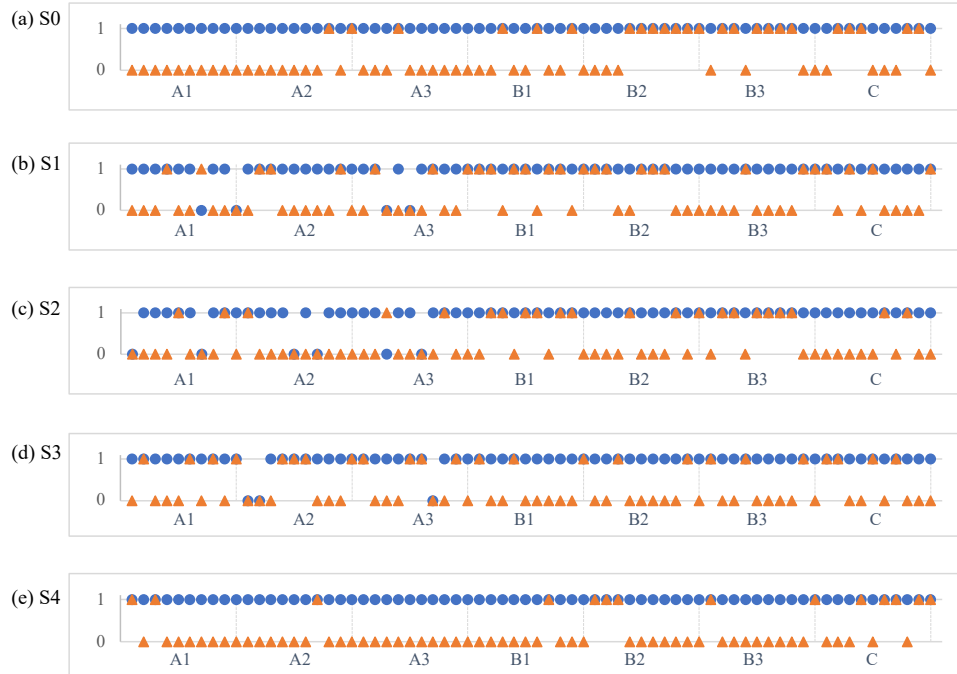


Figure 11: Indicators of station usage among solutions obtained through the EVSP and the EVSP-LS.

Note that in most solutions of the EVSP, all stations are used (circles take a value of 1 in almost all graphs). However, by minimizing the number of charging stations using the EVSP-LS, we clearly achieve operational cost savings by using fewer battery charging stations. Specifically, station S4 can be removed in most instances (most triangles align with the horizontal axis in Figure 11-(e)). However, the most used stations were S3 for short-line instances A1–A3, stations S0–S2 for long-line instances B1–B3, and station S4 for C-type instances of the Changchun City line.

Although vehicle scheduling problems are typically seen to support operational decision making, this type of approach can be used to evaluate different types of electric bus technologies, as well as the layout and design of the charging infrastructure network. Simulations can complement these evaluations as a tool for making strategic decisions regarding the adoption of electromobility (e.g., Janovec and Kohani, 2019, in the context of electric taxis).

In summary, the proposed EVSP-LS can identify optimal locations of charging infrastructure for lines with specific characteristics, which proves highly useful when dealing with a transport network that commonly exhibits a heterogeneous topology throughout the system, such as short turning lines, limited stop lines, and express lines.

3.2.4 EVSP WITH TIMETABLE FLEXIBILITY

This section presents an extension of the mathematical model for the EVSP that considers the flexibility of the timetable as a key feature to maximize fleet utilization and minimize operational costs. We use this model because it is the one that guarantees the highest solver efficiency. The main difference lies in the fact that the starting time of the trips is a decision, denoted by variable w_i . This variable can take values within the interval $w_i \in [dt_i - \delta, dt_i + \delta]$, where dt_i represents the start time parameter of the trip in the original EVSP model, and δ is a parameter defining the flexibility of the timetable. In addition, the constraints involving the parameter dt_i are modified by replacing it with the variable w_i . In other words, these modifications affect the constraints of the original model that depend on dt_i .

An experimental stage was conducted using the set of 70 instances, previously described in Section 3.2.1, and using the value $\alpha = 0.1MTT$. The experiments were carried out using three different values for δ : $\delta = 1$ minutes, $\delta = 3$ minutes, and $\delta = 5$ minutes. Table 5 presents a detailed analysis of the efficiency of these models compared to the original EVSP. The column “Flexibility” shows the values δ , with $\delta = 0$ representing the original EVSP. The “% Optimality” column displays the percentage of cases where

the solver achieves optimality. Cases marked with ‘*’ indicate a solver inconsistency, as there is one instance where the solver stopped prematurely, failing to meet the stopping criterion. Column “% Instances Impr” shows the percentage of instances that achieve an improvement in the objective function compared to the EVSP. As the timetable flexibility increases, the percentage of instances with improvements increases, indicating a higher number of cases where the fleet size is reduced. Meanwhile, column “Avrg Impr” reports the average relative improvement in the objective function (considering only the instances where there is improvement). The results indicate that this average remains relatively constant when different values of δ are considered. Finally, it should be noted that increasing flexibility, represented by higher values of δ , also results in a significant increase in computation times (column “Avrg Time” denotes the average computational time). This highlights the trade-off between operational efficiency and computational cost in such models.

Table 5: EVSP vs EVSP-TF comparision for different level of flexibility.

Flexibility	% Optimality	% Instances Impr	Avrg Impr	Avrg Time
0 min	98.57% ^(*)	0%	0%	9.36.
1 min	98.57% ^(*)	14.29%	13.75%	11.31s.
3 min	98.57% ^(*)	28.57%	13.77%	16.64s.
5 min	100%	40%	14.24%	17.17s.

It is essential to consider the impact that the timetable could have on the usage of charging stations. For this reason, we analyze the effect of the flexibility parameter on station usage. Figure 12 shows that the flexibility of the timetable slightly reduce the percentage of usage of station S3. It should be noted that changes in usage percentages under flexible conditions are very similar to each other, regardless of the value δ considered.

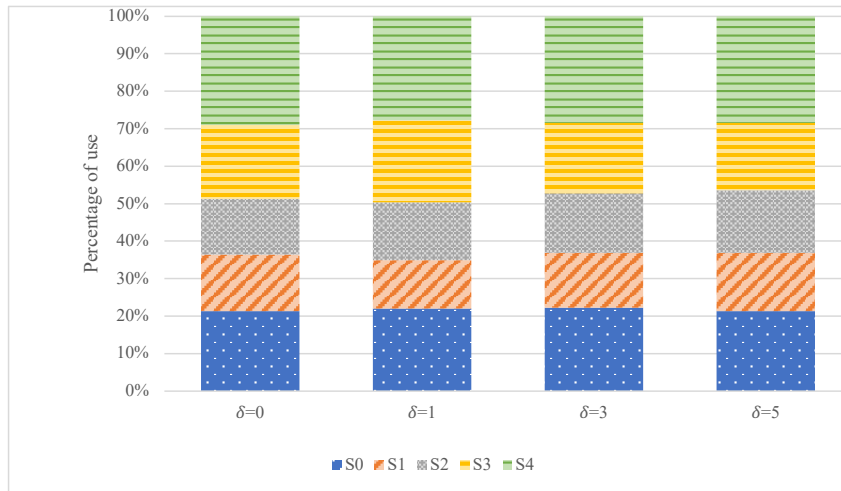


Figure 12: Percentage of station usage with different δ values.

Evident from prior findings, adopting flexible departure times offers significant advantages. However, service quality is often evaluated on the basis of the timeline, suggesting that including alternative objective functions could lead to a more robust optimization strategy in future studies (beyond the scope of this article). To conclude, the preceding section outlines the foundational steps in the creation of an advanced method that provides novel information to decision makers involved in planning public transport.

This chapter addresses an integrated optimization problem for electric bus scheduling and charging station location under on-route charging technology. The results indicate that optimal solutions can be obtained within reasonable computational times for the operational version of the problem, while the incorporation of strategic decisions allows the identification of a reduced subset of charging stations that significantly lowers infrastructure costs. In addition, the maximum charging time per trip is shown to have a substantial impact on station utilization, and increased flexibility in bus departure times contributes to a reduction in fleet size, albeit at the expense of greater computational effort. These findings suggest several directions for future research, including the analysis of alternative objective functions, the incorporation of non-linear charging processes, heterogeneous fleets, and multi-line systems, thereby enabling a more realistic representation of electromobility systems. A more comprehensive analysis and discussion of these aspects are provided in the Chapter 5.

CHAPTER 4

INTEGRATION OF TIMETABLING TO COORDINATE OVERLAPPING LINES AND VEHICLE SCHEDULING WITH INTERLINING

This study integrates two of the most relevant subproblems of transit network planning: bus timetabling and vehicle scheduling. In general, the timetabling problem determines the departure times of the trips for each line. In addition, the term ‘timetable synchronisation’ refers to the coordination of different transit lines to optimize specific objective functions, such as enhancing the level of service or reducing operational costs. The vehicle scheduling problem is part of the operational process of the public bus system and determines which vehicle will cover each trip at the time scheduled by the timetable. Very few studies in the literature deal with these two subproblems in an integrated approach (Liu and Ceder, 2018; Schöbel, 2017; ?).

A novel concept we consider in this work is to consider *overlapping lines*. It involves a set of transit lines, each with a specific frequency, that share significant route segments and stops. Thus, passengers can board multiple lines to reach the same destination. Moreover, if the lines are operated by the same agency, resources such as vehicles and drivers can be assigned to different lines, enabling interline operations. For example, a vehicle can perform trips for different lines, which improves resource use but results in a more complex operation, as it introduces greater flexibility in decisions and requires deadheading actions to perform consecutive trips of different transit lines with the same vehicle. Figure 13 illustrates a real example of four overlapping lines in the public transport system of Santiago, Chile, with these characteristics.

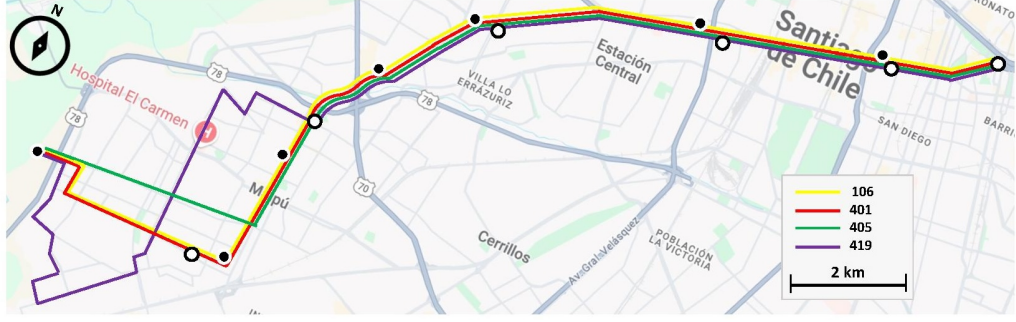


Figure 13: Four overlapping lines of the public transport system RED in Santiago, Chile.

More particularly, the first subproblem we address is the *Coordination of the Timetabling of Overlapping Lines (CTOL)* aiming for a coordinated operation of overlapping lines, ensuring that passengers benefit from an optimized joint interval in shared route segments. By increasing the regularity of the service for the overlapping lines, planners aim to reduce waiting times for passengers arriving at a constant rate at stops in the shared route segment. The second subproblem is the *Vehicle Scheduling with Interlining (VSI)*, to minimize operational costs related to fleet size.

The advantage of integrating CTOL and VSI problems denoted CTOL-VSI is shown in Figures 14 and 15 for transit lines that share a significant route segment as described by the example in Figure 1. The first three Gantt vehicle sequences in Figure 14 correspond to the case when lines 1, 2, and 3 are treated independently. There are three trips for lines 1 and 3, whereas line 2 has two trips, and each vehicle performs trips for a single line. An example solution for the CTOL-VSI bi-objective problem is shown in the last two Gantt vehicle sequences, revealing that only two vehicles are needed when assuming interlining and timetable flexibility, where dashed arrows represent trips shifted in order to remove one vehicle from the solution, and deadheading actions are needed to connect consecutive trips in the first vehicle.

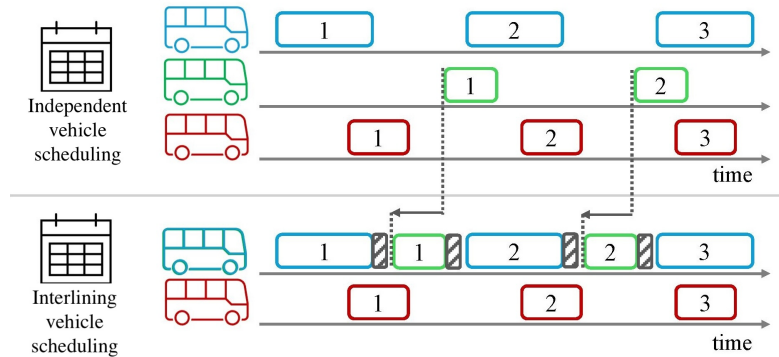


Figure 14: Improvement in vehicle scheduling assuming interlining and timetable flexibility.

Moreover, Figure 15 shows that the timetable can promote service regularity in shared route segments. In particular, we present a time versus space plot for trips of three overlapping lines represented with different colors along the shared route segment (coordinated on the left side and not coordinated operation on the right side). In particular, the arrival times on the coordinated timetables are staggered to increase the regularity of the service that can help reduce the waiting times of passengers boarding at those stops with a constant arrival rate (see Ibarra-Rojas et al., 2015a).

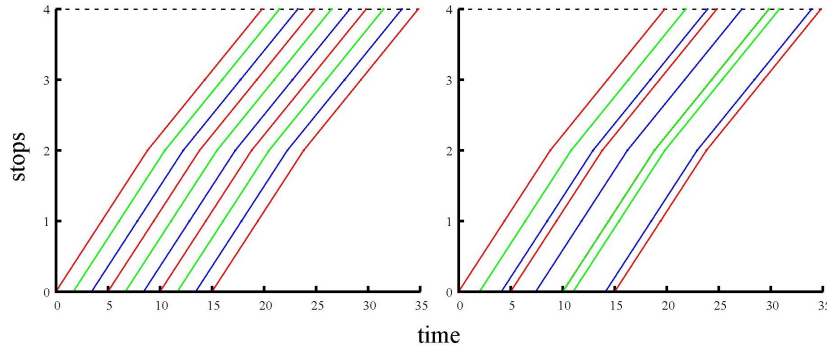


Figure 15: Effect of coordination of three overlapping lines.

Thus, the timetable significantly impacts vehicle schedules, operational costs, and the level of service, which are two conflicting objectives; therefore, our bi-objective integrated approach will offer valuable insights into the trade-off of optimizing both measures. In this way, the CTOL-VSI problem is a handy tool for a well-coordinated, reliable, and resource-efficient public transportation system. We highlight that our optimization problem could be implemented across multiple corridors or an entire transit network. However, we emphasize that drivers operate vehicles, which makes it challenging to cover completely different lines with the same person. In general, we propose decomposing the transit network into segments with overlapping lines operated by the same agency to effectively address overlapping coordination and interlining. The main reason for not addressing our problem with different agencies is that we do not assume that resources such as drivers and vehicles can be shared. However, under the assumption of integrated operations, where a regulatory entity oversees operations and there is no competition for demand, coordination could indeed be optimized for a network segment covered by lines from different agencies. Therefore, to achieve greater efficiency, our optimization approach promotes a holistic perspective for planning and operating public transport systems. A transit corridor, as presented in Figure 1, serves as an example of practical implementation. The latter situation is also common in cities where the transit network is divided to be operated by different agencies, such as the transit system in Santiago, Chile.

This work significantly contributes to the field by introducing a novel optimization problem that integrates coordination timetabling and interlining vehicle scheduling of overlapping lines, which has not been previously addressed in the existing literature (as shown in Section ??). Our study introduces a novel bi-objective mathematical formulation in terms of scheduling variables, aiming to enhance service regularity for overlapping lines by minimizing the total deviation of headways concerning the joint headway at coordination stops while simultaneously minimizing the fleet size. To solve the CTOL-VSI bi-objective model, we propose an ε -constraint algorithm and a Bi-objective Biased Random-Key Genetic Algorithm capable of finding Pareto optimal solutions to analyze the trade-off between optimizing the level of service and operational costs. The experimental results show the significance of the data obtained for decision makers. Indeed, quantifiable information on the trade-off between reducing the size of the fleet and optimizing the level of service is a valuable tool for strategic decision making, empowering operators to assess the implications of various operational scenarios to make more measurable decisions. (

4.1 THE BI-OBJECTIVE CTOL-VSI PROBLEM

This section describes the integrated CTOL-VSI problem that addresses coordination timetabling and interlining vehicle scheduling of overlapping lines. We present a bi-objective MILP to minimize the total deviation from optimal separation times at coordination stops and the operational cost of the fleet size of overlapping lines. Specifically, our multifaceted problem involves determining timetables for a set of overlapping lines and scheduling vehicles, assuming interlining by considering bounded departure times and known travel times for all trips.

To define our problem, we consider the following input. Let L be the set of overlapping and interlining transit lines. S is the set of coordination stops where some transit lines must be coordinated. Since at each stop s , there may be different sets of lines to be coordinated due to the destination of passengers traveling from s , we define K^s as the collection of indexes $\{1, 2, \dots, |K^s|\}$ to enumerate the different sets of lines L_k^s to be coordinated at stop $s \in S$.

We define $F^l = \{1, 2, \dots, |F^l|\}$ as the set of trips for each line $l \in L$, leading to an ideal headway $headway^l$ in terms of the number of trips $|F^l|$. Since the frequency of the lines is fixed, we can compute the *ideal joint headway* Δ_k^s for overlapping lines L_k^s at stop s , where $k \in K^s$. In the example of Figure 15, assume that the frequencies of lines 1, 2, and 3 are $|F^1| = 3$, $|F^2| = 4$, and $|F^3| = 3$ departures per hour, respectively. Additionally, we assume that there is a single coordination stop s where the only one set

of lines to be coordinated ($K^s = \{1\}$) correspond to the three transit lines. Therefore, the joint headway can be computed as $\Delta_1^s = \frac{60}{(3+4+3)} = 6$ minutes. Ideally, a bus from lines 1, 2, or 3 should arrive at stop s every 6 minutes.

Moreover, with these ideal headways for each line l , we generate departure time windows D_p^l for every trip $p \in F^l$ of the line $l \in L$, providing flexibility to the CTOL-VSI problem. In particular, we compute feasible departure time windows for trips of a planning period t starting in $begin_t$ as Ibarra-Rojas and Muñoz (2016), that is:

$$D_p^l = \left[begin_t + \frac{headway_t^l}{2} + (p-1)headway_t^l - \beta, \frac{headway_t^l}{2} + (p-1)headway_t^l + \beta \right]$$

We highlight that the β parameter represents the flexibility of the timetable (in minutes). We use the formula from Ibarra-Rojas and Muñoz (2016) to define the flexibility of the timetable based on even headways for each sub-period of the operating day, ensuring an average headway between the last trip of one sub-period and the first trip of the next. However, different reference timetables and objective functions can be used to define variations of our optimization problem. Figure 16 represents an example of feasible departure time windows for a transit line with four trips in a planning period of 40 minutes and a flexibility of $\beta = 1$ minute. Notice that if $\beta = 0$ instead of one minute, the first trip $p = 1$ would depart at half the headway $headway_t^l$, and we would have a timetable with evenly spaced departure times throughout the planning period $[begin, end]$.

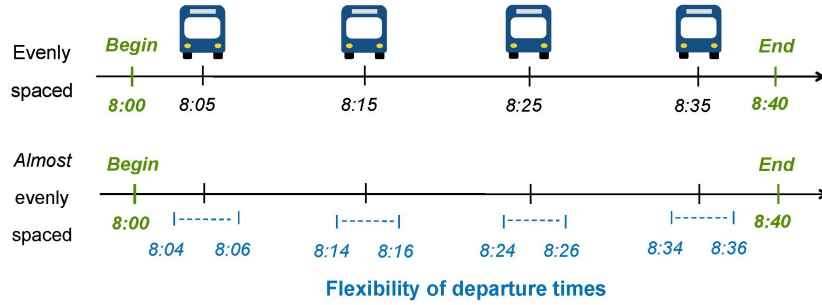


Figure 16: Example of feasible departure time windows for a transit line (Ibarra-Rojas and Muñoz, 2016).

Let us denote by (l, p) the p -th trip of the line $l \in L$, $p \in F^l$, where a dummy trip 0 denotes the depot, and it is not necessary to use two indices. The travel time of the trip (l, p) from its starting point to the coordination stop s is τ_p^{ls} , while its total travel time is t_p^l , for $s \in S, l \in L, p \in F^l$. Indeed, assuming deterministic travel times is one of the strongest assumptions when defining optimization problems related to planning and controlling public transport systems. We acknowledge that travel times can vary throughout the planning period. However, our definition of travel time is trip-dependent,

meaning it is based on bounded departure times, which allows us to represent certain behaviors of travel times during peak and non-peak periods. When a vehicle is assigned to (l, p) and its next trip is from another line (l', p') , then it may incur a deadheading time $h_{pp'}^{ll'}$ to be able to relocate at the starting stop of line l' , for $l, l' \in L, p \in F^l, p' \in F^{l'}$. Finally, let n be the upper bound for the number of vehicles in the homogeneous fleet.

Thus, CTOL-VSI can be formally described as the bi-objective problem of minimizing the total deviation from the ideal headways at the coordination stops $s \in S$ of overlapping lines and minimizing the fleet size needed to cover all the trips by determining the departure time within feasible windows of each trip (l, p) , $l \in L, p \in F^l$. Our optimization approach aims to coordinate different overlapping lines by assuming a timetable flexibility parameter β that bounds the service regularity for each line independently. This parameter allows us to control the impact on passengers who do not use overlapping lines.

To develop our mathematical formulation, we define the following integer and binary variables to represent the decisions in our CTOL-VSI MILP. Let x_p^l be integer variables that represent the departure time in minutes of trip (l, p) , $l \in L, p \in F^l$. The flexibility in the timetable is represented by the deviation from the ideal joint headway times Δ_k^s . Thus, let the integer variables $\bar{\delta}_{kpp'}^{sll'} \left(\underline{\delta}_{kpp'}^{sll'} \right)$ be the shortage (excess) of Δ_k^s for trips (l, p) and (l', p') at stop $s \in S$, $l, l' \in L_k^s, p \in F^l, p' \in F^{l'}, k \in K^s$. To track the sequence of the trips of different overlapping lines, we define the next set of binary variables

$$w_{kpp'}^{sll'} = \begin{cases} 1 & \text{if trips } (l, p) \text{ and } (l', p') \text{ of lines in } L_k^s \text{ are consecutive at stop } s, \\ 0 & \text{otherwise.} \end{cases}$$

Finally, the following binary variables are needed to count the number of vehicles needed to cover consecutive trips:

$$v_{pp'}^{ll'} = \begin{cases} 1 & \text{if trips } (l, p) \text{ and } (l', p') \text{ are consecutively covered by the same vehicle} \\ 0 & \text{otherwise.} \end{cases}$$

We summarize the above notation for the input and proposed decision variables with Table 6.

Table 6: Notation for the CTOL-VSI MILP.

Parameters	Description
L	set of transit lines
S	set of coordination stops
K^s	collection of indexes k to enumerate the different sets of lines to be coordinated at stop $s \in S$
L_k^s	k -th set of lines to coordinate at $s \in S, k \in K^s$
F^l	set of trips of line $l \in L$
D_p^l	departure time window for $(l, p), l \in L, p \in F^l$
Δ_k^s	ideal joint headway at stop $s \in S$ for the set of lines L_k^s to be coordinated, $k \in K^s$
τ_p^{ls}	travel time of trip (l, p) to coordination stop $s \in S, l \in L, p \in F^l$
t_p^l	total travel time of trip $(l, p), l \in L, p \in F^l$
$dh_{pp'}^{ll'}$	deadheading time between (l, p) and $(l', p'), l, l' \in L, p \in F^l, p' \in F^{l'}$
n	number of vehicles of the homogeneous fleet
Variables	Description
x_p^l	integer departure time of trip $(l, p), l \in L, p \in F^l$
$\bar{\delta}_{kpp'}^{sll'}$	shortage of the arrival separation time between (l, p) and (l', p') at $s \in S$, compared to Δ_k^s , for all $l, l' \in L_k^s, p \in F^l, p' \in F^{l'}, k \in K^s$
$\underline{\delta}_{kpp'}^{sll'}$	excess of the arrival separation time between (l, p) and (l', p') at $s \in S$, compared to Δ_k^s , for all $l, l' \in L_k^s, p \in F^l, p' \in F^{l'}, k \in K^s$
$w_{kpp'}^{sll'}$	binary variable taking the value of 1 if trips (l, p) and (l', p') are consecutive at stop s , and 0 otherwise, where variable $w_{k0p}^{sl} = 1$ ($w_{kp0}^{sl} = 1$, resp) denotes that trip (l, p) is the first (last, resp) arrival of lines L_k^s at stop s
$v_{pp'}^{ll'}$	binary variable taking the value of 1 if trips (l, p) and (l', p') are consecutively performed by the same vehicle, and 0 otherwise, where variable $v_{0p}^l = 1$ ($v_{p0}^l = 1$, resp) represent that (l, p) is the first (last, resp) trip performed by a vehicle

Now, we model the CTOL-VSI problem by the following bi-objective mixed-integer linear program.

$$\min \left[F_{CTOL} = \sum_{s \in S'} \sum_{k \in K^s} \sum_{l \in L_k^s} \sum_{l' \in L_k^s} \sum_{p \in F^l} \sum_{p' \in F^{l'}} \left(\bar{\delta}_{kpp'}^{sll'} + \underline{\delta}_{kpp'}^{sll'} \right), F_{VSI} = \sum_{l \in L} \sum_{p \in F^l} v_{0p}^l \right] \quad (4.1)$$

$$x_p^l \in D_p^l \quad l \in L, p \in F^l \quad (4.2)$$

$$\sum_{l \in L_k^s} w_{k01}^{sl} = 1 \quad s \in S, k \in K^s \quad (4.3)$$

$$\sum_{l \in L_k^s} w_{k|F^l|0}^{sl} = 1 \quad s \in S, k \in K^s \quad (4.4)$$

$$w_{k0p}^{sl} + \sum_{l' \in L_k^s} \sum_{p' \in F^{l'}} w_{kp'p}^{sll'} = 1 \quad s \in S, k \in K^s, l \in L_k^s, p \in F^l \quad (4.5)$$

$$w_{kp0}^{sl} + \sum_{l' \in L_k^s} \sum_{p' \in F^{l'}} w_{kpp'}^{sl'} = 1 \quad s \in S, k \in K^s, l \in L_k^s, p \in F^l \quad (4.6)$$

$$(x_p^l + \tau_p^{ls}) \leq (x_{p'}^{l'} + \tau_{p'}^{l's}) + M(1 - w_{kpp'}^{sl'}) \quad s \in S, k \in K^s, l, l' \in L_k^s, p \in F^l, p' \in F^{l'} \quad (4.7)$$

$$x_{p'}^{l'} + \tau_{p'}^{l's} - x_p^l - \tau_p^{ls} + \bar{\delta}_{kpp'}^{sl'} - \underline{\delta}_{kpp'}^{sl'} \geq \Delta_k^s - M(1 - w_{kpp'}^{sl'}) \quad s \in S, k \in K^s, l, l' \in L_k^s, p \in F^l, p' \in F^{l'} \quad (4.8)$$

$$x_{p'}^{l'} + \tau_{p'}^{l's} - x_p^l - \tau_p^{ls} + \bar{\delta}_{kpp'}^{sl'} - \underline{\delta}_{kpp'}^{sl'} \leq \Delta_k^s + M(1 - w_{kpp'}^{sl'}) \quad s \in S, k \in K^s, l, l' \in L_k^s, p \in F^l, p' \in F^{l'} \quad (4.9)$$

$$\sum_{l \in L} \sum_{p \in F^l} v_{0p}^l \leq n \quad (4.10)$$

$$v_{0p}^l + \sum_{l' \in L} \sum_{p' \in F^{l'}} v_{p'p}^{l'l} = 1 \quad l \in L, p \in F^l \quad (4.11)$$

$$v_{0p}^l + \sum_{l' \in L} \sum_{p' \in F^{l'}} v_{p'p}^{l'l} = v_{p0}^l + \sum_{l' \in L} \sum_{p' \in F^{l'}} v_{pp'}^{ll'} \quad l \in L, p \in F^l \quad (4.12)$$

$$(x_p^l + t_p^l) + dh_{pp'}^{ll'} \leq x_{p'}^{l'} + M(1 - v_{pp'}^{ll'}) \quad l \in L, p \in F^l, l' \in L, p' \in F^{l'} \quad (4.13)$$

The function F_{CTOL} in (4.1) represents the total deviation from the optimal separation of arrivals at coordination stops, since each term $\bar{\delta}_{kpp'}^{sl'} + \underline{\delta}_{kpp'}^{sl'}$ in an optimal solution represents the deficit or surplus between the headway of consecutive trips (l, p) and (l', p') at stop s compared to the desired joint headway. The second term F_{VSI} in the objective function is the size of the fleet, where the variables v_{0p}^l are activated if the trip (l, p) is the first trip for a vehicle. The bi-objective function is subject to constraints (4.2) that bound the departure times for all trips. Equations (4.3) and (4.4) define the first and last arrivals at stop $s \in S$ for each set of lines L_k^s , respectively. Notice that the first arrival at stop s should correspond to the first trip of some line (we only use $w_{k0p}^{s'l}$ variables). In contrast, the last arrival should correspond to the last trip of some line (we only use variables $w_{k|F^l|0}^{sl}$). Meanwhile, equations (4.5) and (4.6) define the sequence of arrivals of trips of lines L_k^s at stop $s \in S$. In particular, each trip (l, p) with $l \in L_k^s$ must arrive at stop s , therefore, equations (4.5) guarantee that trip (l, p) defines the first arrival, or there is only one previous arrival (l', p') , and analogously for (4.6). Inequalities (4.7) allow the definition of consecutive arrivals in terms of departure and travel times, where M is a

big number. The constraints (4.8) and (4.9) define the deviation variables for consecutive arrivals. Specifically, when the variable $w_{kpp'}^{sl'}$ takes the value of 1, constraints (4.8) and (4.9) enforce an equality, where the δ variables model the deviation of the actual headway of trips (l, p) and (l', p') from the desired joint headway. Inequality (4.10) establishes the bound for the fleet size. The constraints (4.11) guarantee that every trip (l, p) is assigned to a vehicle. This assignment is either as the initial trip immediately following the dummy trip 0 in a vehicle's sequence ($v_{0p}^l = 1$), or it occurs directly before another trip (l', p') in the sequence (expressed mathematically as $\sum_{l' \in L} \sum_{p' \in F^{l'}} v_{p'p}^{l'l} = 1$). Equations (4.12) ensure that the vehicle assigned to one trip must be consecutively assigned to another trip or return to the depot represented with the dummy trip '0', where variables v_{0p}^l and v_{p0}^l denote the beginning and ending of a vehicle schedule, respectively. Finally, inequalities (4.13) ensure that two trips are consecutive only if the departure, ending, and deadhead times allow that event. We highlight that our proposed formulation for the CTOL-VSI is not present in the literature, the major advantage is the definition of scheduling variables w , which allows us to define the sequence of arrivals at each stop s of different lines, since other timetabling problems for overlapping lines usually define time-indexed variables, increasing the size of the mathematical models.

When integrating timetabling and vehicle scheduling subproblems, there is a conflict between optimizing the level of service and operational costs. Obtaining a minimal value for both objectives is impossible, leading to implementing algorithms to search for incomparable solutions that allow us to analyze the conflict between both objectives, as we present in the rest of the article.

4.2 SOLUTION METHODOLOGY

4.2.1 ε -CONSTRAINT ALGORITHM FOR THE CTOL-VSI PROBLEM

Since we have proposed a bi-objective model for the CTOL-VSI problem, we implement a ε -constraint algorithm (Mavrotas, 2009) to find Pareto optimal solutions that allow us to analyze the trade-off between both objective functions F_{CTOL} and F_{VSI} . The ε -constraint algorithm is a powerful technique for multi-objective optimization problems (Mavrotas and Florios, 2012). It operates by systematically relaxing one of the objectives and adding a constraint to bound the value of that objective function with an acceptable value (represented by the ε parameter), allowing the optimization algorithm to focus on exploring solutions along the Pareto front for the remaining objectives (also known as the set of non-dominated, Pareto optimal, Pareto efficient solutions). None of the objective functions can be improved in value without degrading some of the other objective values

for the solutions in the Pareto front. As a result, the algorithm produces a set of solutions representing the trade-off between the two conflicting objectives, offering decision-makers a range of options based on their preferences. Figure 17 shows a representation of the ε -constraint algorithm to minimize two conflicting objectives f_1 and f_2 , where the ‘ideal point’ represents the optimal value of both objective functions. Notice that a value ε defines an upper bound of the objective f_2 (preventing the extremely worst values of f_2) while optimizing the objective function f_1 . Without additional subjective preference information, an infinite number of Pareto optimal solutions may exist, all considered equally good. In our implementation, ε is linked to the vehicle fleet, which has the advantage of being integer and bounded.

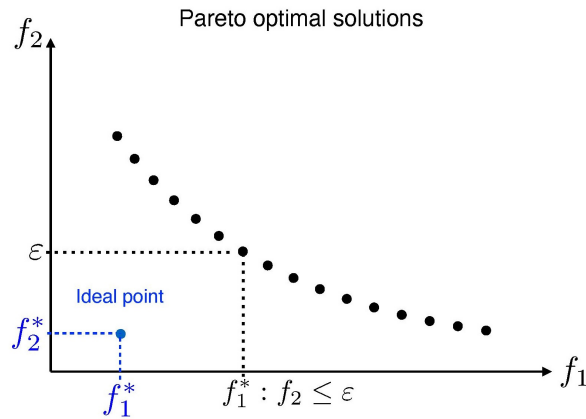


Figure 17: Pareto optimal solutions obtained through variation of ε .

The details of our ε -constraint method for CTOL-VSI are provided by Algorithm 1. First, the Pareto frontier is initialized as empty ($ListPareto = \emptyset$). Step 1 determines the optimal vehicle schedule that minimizes the size of the fleet. Analogously, Step 1 obtains the optimal timetable that minimizes the total deviation of actual headways compared to the ideal one in the timetable sub-problem. Then, Step 1 determines an extreme point of the Pareto front by solving the timetabling problem subject to the constraints of the CTOL-VSI and $F_{VSI} \leq VSI^*$. The other extreme point is obtained similarly in Step 1. Steps 1 to 1 implement iterations for different values of the parameter ε . Taking into account the objective function of the vehicle scheduling problem as a constraint, we optimize F_{CTOL} in Step 1. Step 1 updates the Pareto front if solving P_ε yields a Pareto optimal point, that is, we verify whether $F_{VSI} = \varepsilon$.

Algorithm 1 : ε -constraint for the CTOL-VSI

$ListPareto = \emptyset$ Solve $VSI^* = \{\min F_{VSI} : (4.3)-(4.13)\}$
 Solve $CTOL^* = \{\min F_{CTOL} : (4.3)-(4.13)\}$ Solve $P_1^* = \{\min F_{CTOL} : (4.3)-(4.13), F_{VSI} \leq VSI^*\}$ Solve $P_2^* = \{\min F_{VSI} : (4.3)-(4.13), F_{CTOL} \leq CTOL^*\}$ $ListPareto = ListPareto \cup \{(CTOL^*, P_2^*), (P_1^*, VSI^*)\}$ Let $\varepsilon = P_2^* - 1$ ($\varepsilon > VSI^*$) Solve $P_\varepsilon^* = \{\min F_{CTOL} : (4.3)-(4.13), F_{VSI} \leq \varepsilon\}$ Update $ListPareto$ considering $(P_\varepsilon^*, \varepsilon)$ $\varepsilon = \varepsilon - 1$

As mentioned, the objective function constrained in Algorithm 1 is the size of the fleet required to cover trips. Therefore, we can ensure finding the complete and exact Pareto frontier if we reach optimal solutions during each iteration of our ε -constraint method. The computational experimentation in Section 4.3 will demonstrate the performance of our optimization approach.

4.2.2 BIASED RANDOM-KEY GENETIC ALGORITHM FOR THE CTOL-VSI PROBLEM

To evaluate the efficiency of our ε -constraint algorithm, we compare using the multi-objective version of the BRKGA proposed by Silva-Soto and Ibarra-Rojas (2021).

The BRKGA is a metaheuristic optimization method that uses random keys (vectors of real numbers within $[0,1]$) to encode solutions to an optimization problem. These keys are decoded into feasible solutions using a deterministic algorithm called a decoder. The biased aspect of the algorithm comes from the crossover operator, which favors genes from the better performing parent, thereby guiding the search to more promising areas of the solution space (see Gonçalves and Resende, 2011). This approach balances exploration and exploitation effectively, making it well-suited for solving complex combination optimization problems. Figure 18 represents the BRKGA, where we include a Non-Dominated Sorting Algorithm to identify promising solutions for a multi-objective optimization problem.

A Non-Dominated Sorting Algorithm is generally used in multi-objective optimization to classify solutions based on Pareto dominance. It identifies non-dominated solutions, ranking them first. The algorithm then iteratively removes these solutions and identifies subsequent non-dominated solutions from the remaining ones, creating ranked Pareto front approximations that guide the selection process throughout the algorithm's iterations (see details in Deb et al., 2002). This approach ensures that non-dominated solutions are prioritized for further optimization.

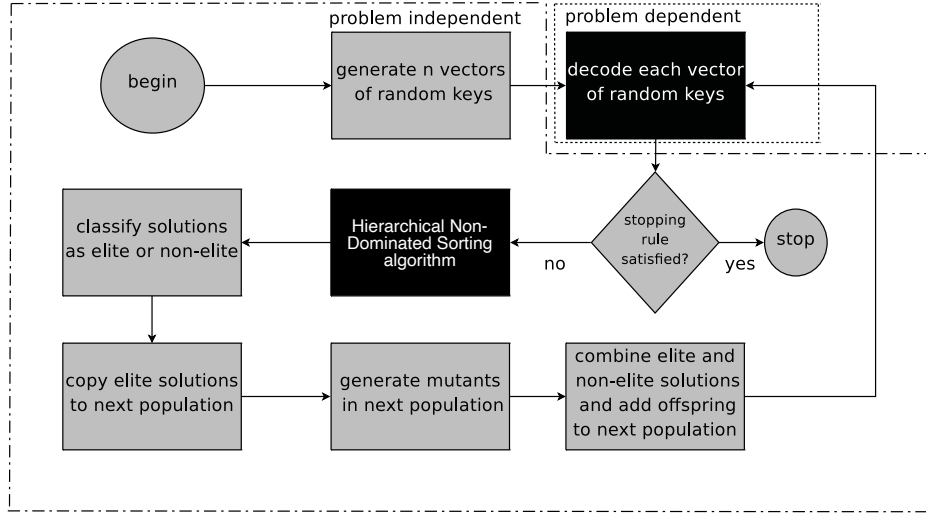


Figure 18: HNDS-BRKGA proposed in Silva-Soto and Ibarra-Rojas (2021).

Note that we use the Hierarchical Non-Dominated Sorting algorithm recently proposed by Bao et al. (2017), whose main advantage lies in its ability to efficiently handle large populations and many-objective optimization problems by reducing computational complexity. Unlike traditional non-dominated sorting methods, which become increasingly resource-intensive as the number of objectives and solutions grows, HNDS employs a hierarchical structure that organizes the sorting process into multiple levels. This approach significantly improves scalability, distinguishing between solutions faster and more effectively, particularly in complex, high-dimensional objective spaces.

To implement our HNDS-BRKGA for CTOL-VSI (detailed in Algorithm 2), the proposed decoder processes a random-key $\mathbf{r}$ of size twice the number of trips in the instance, where the first half of $\mathbf{r}$ is related to timetable decisions and the second half determines the schedule of buses. It starts with an initial position and then decodes the vector by determining the departure times based on the value of the random-key in the current position. To define the vehicle scheduling decisions, we create a list of trips ordered according to the values in the random-key. Finally, the algorithm iterates, attempting to define the schedule of each vehicle by assigning trips in the ordered list. We recall that it is necessary to compute the related objective functions to rank solutions using the HNDS algorithm.

Algorithm 2 *Decoder(r)* for the CTOL-VSI

$position = 1$ $l \in L$ $p \in F^l$ $x_p^l = \underline{D}_p^l + \left[\left(\overline{D}_p^l - \underline{D}_p^l \right) r[position] \right] + 0.5$
 $position = position + 1$ Define $List = \{(l, p)\}$ ordered increasingly in
 terms of $r \left[\sum_{l \in L} |F^l| + \sum_{l'=1}^{l-1} |F^{l'}| + p \right]$ $v = 1$ to $\sum_{l \in L} |F^l|$ each trip (l, p) in
 ordered $List$ Insert (l, p) in schedule of v . If possible, remove (l, p) from
 $List$ Compute objective functions

The following section shows the numerical experimentation implementing both of our algorithms, and we analyse the trade-off between the level of service and the fleet size.

4.3 EXPERIMENTAL RESULTS

To evaluate the effectiveness of our solution methodology for the CTOL-VSI problem, we generated random instances that represent various scenarios within a single corridor with multiple operational lines. Subsequently, we implemented our ε -constraint algorithm for the CTOL-VSI MILP model to analyze the trade-off between the level of service and operational costs. In particular, we will analyze the impact of fleet size reduction on service regularity by examining the Pareto front of the generated instances. Experimental results will demonstrate that there is indeed a conflict between these two performance measures, providing new insights for decision makers in urban transport planning.

4.3.1 RANDOMLY GENERATED INSTANCES

Scenarios are based on a 12 km corridor featuring 15 stops spaced 800 metres apart. Three bidirectional lines are being considered: one regular line and two alternative lines with 75% frequency of the regular line, and feasible departure time windows D_p^l defined using a flexibility parameter of $\beta^l = 2$ minutes for each line $l \in L$. For the first alternative line, the initial stop is generated between stops 2 and 4, while the final stop is between 10 and 12. The initial stop of alternative line 2 is between 0 and 5, with a variable number of stops in the range of [5,8]. At each bus stop where more than one transit line passes, we define two sets of lines, K_1^s and K_2^s , to be coordinated. These sets comprise every line that includes that stop along their routes in both directions. Figure 19 illustrates an example of lines with these characteristics. Note that passengers who can board more than one line to reach their destinations benefit from the coordination of multiple lines at common stops. The latter is because the regularity of service among overlapping lines (as a joint frequency) leads to shorter waiting times for passengers with a constant arrival

rate. Finally, we note that we duplicate the lines to consider both directions along the transit corridor.

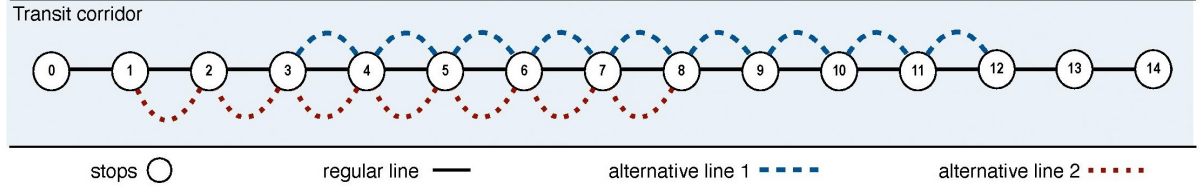


Figure 19: Example of transit lines in the instance generation.

We consider an operational day of 19 hours, divided into 7 periods that simulate peak and off-peak periods of the day. The average speed is 12 km/hr, and in particular, the base travel times for each segment of consecutive stops are generated between three and five minutes. These base travel times are multiplied by a factor $FactorPerTravel$ that depends on the planning period to simulate traffic congestion. A base frequency is generated within $[MinFreq, MaxFreq]$ trips for each planning period. Table 7 shows the details associated with the planning periods.

Period	Duration (min)	$MinFreq$	$MaxFreq$	$FactorPerTravel$
1	120	3	4	1.1
2	120	5	8	1.3
3	300	8	10	1
4	120	4	6	1.1
5	120	6	8	1.3
6	120	4	8	1.2
7	240	6	8	1.1

Table 7: Details of planning periods for the generated instances.

To generate different types of instances, the frequency of the regular line is generated by multiplying the base frequency by the following factor: 0.75 for *Small* instances, 1 for *Medium* instances, and 1.5 for *Large* instances.

4.3.2 REAL SCENARIO

This case study analyses the coordination of four lines of the RED public transportation system in Santiago, Chile, operated by the same agency and exhibits a high degree of overlap along several routes. Figure 13 shows the segments of the routes of these four lines, where there are coordination stops (black markers for the east-to-west direction and white markers for the opposite direction) to promote a better joint frequency. The stops

and lines selected for coordination were based on shared origin-destination pairs. The proposed strategy aims to optimize the joint frequency at these shared stops to improve transport service for users in the overlapping segments of different routes. This approach seeks to improve operational efficiency and better allocate resources by leveraging route overlap.

Data for this case study are based on lines 106, 401, 405, and 419 from the RED transit system in Santiago. The analysis assumes circular lines and covers a planning period of 17 hours, considering the frequency of each line corresponding to night services to simulate long waiting times for users. Moreover, we used an average speed and an average cycle time for each line corresponding to a regular day. These parameters are shown in Table 8.

Parameters	Transit Lines			
	106	401	405	419
Frequency (trips)	34	26	34	34
Average Speed (km/h)	17.29	17.24	16.87	19.13
Average Cycle Time (min)	172	172	130	186

Table 8: Parameters used to design the case study based on a real transit system in Chile.

4.3.3 EXPERIMENTAL ENVIRONMENT

The experimental stage involves implementing our ε -constraint algorithm in each instance, using a MacBook Pro with an Apple M3 Pro processor and 18 GB of RAM. We employed the commercial solver CPLEX 20.1 (IBM, 2021), with a stopping criterion set to one hour of computational time (except for the real case where we use two hours) or 3% of relative gap¹. For our HNDS-BRKGA, we conducted preliminary experiments with different parameter settings to balance intensification and diversification. Specifically, we solved the smallest instances using the parameter values listed in Table 9, with a stopping criterion of one minute.

¹the relative gap is computed as $\frac{UpperBound - LowerBound}{UpperBound} \times 100\%$

Table 9: Tested parameters for the HNDS-BRKGA.

Population size	Fraction of elite solutions	Fraction of mutants	Inheritance probability
100	20	10	0.75
100	20	10	0.85
100	10	5	0.75
100	10	5	0.85
100	30	15	0.75
100	30	15	0.85
200	20	10	0.75
200	20	10	0.85
200	10	5	0.75
200	10	5	0.85
200	30	15	0.75
200	30	15	0.85

The numerical results of the preliminary experiments show that the best combination of parameters is (200, 20, 10, 0.75), as it outperforms the other configurations in 80% of the tested instances. Finally, to ensure a fair comparison between the metaheuristic and the ε -constraint algorithm, we used a stopping criterion of 500 iterations for our HNDS-BRKGA. As a result, larger instances require more computational time to generate Pareto front approximations. The next section demonstrates the efficiency of both algorithms in terms of computational time and solution quality.

4.3.4 ANALYSING THE EFFICIENCY OF THE ε -CONSTRAINT ALGORITHM AND BRKGA

We obtained optimal Pareto solutions with our multi-objective algorithms since we modelled the CTOL-VSI problem as a bi-objective optimization approach. Then, we analyse the trade-off between optimizing the coordination of overlapping lines and the fleet size considering interlining.

Our proposed ε -constraint algorithm performs iterations that solve mathematical programs to find Pareto optimal solutions, and we are interested in efficient optimization approaches to support decision making in transportation systems. Then, we analyse the computational time and the gap for each iteration of our solution method.

Table 10 shows the number iterations (ε Iter), the size of the Pareto front approximation (SizePFA), the average gap (AvgGap), the standard deviation of gap (StdGap), and the total computational time in seconds (CPUTime) of implementing our ε -constraint

algorithm for each instance in our experimental stage. Moreover, we also report the size of the Pareto front approximation and computational times of implementing our BRKGA.

Instance	ε -Constraint					HNDS-BRKGA	
	ε Iter	SizePFA	AvgGap	StdGap	CPUTime	SizePFA	CPUTime
Small_1	11	3	2.210%	0.770	96.080	1	131.971
Small_2	12	5	2.723%	0.818	4711.770	1	120.761
Small_3	7	2	2.331%	0.961	65.550	1	144.305
Small_4	10	3	2.614%	0.873	2957.327	1	157.737
Small_5	10	4	2.647%	0.883	242.032	1	137.529
Small_6	11	2	2.373%	0.807	126.450	2	151.073
Small_7	12	3	2.729%	0.820	2778.951	1	149.179
Small_8	12	2	2.600%	0.808	268.326	1	151.428
Small_9	8	2	2.389%	0.933	41.719	1	103.074
Small_10	9	3	2.385%	0.892	90.305	1	121.426
Medium_1	16	6	2.314%	0.804	666.339	1	253.485
Medium_2	18	4	2.810%	0.683	34006.691	2	221.140
Medium_3	21	4	2.553%	0.605	1038.008	1	258.506
Medium_4	15	4	3.004%	0.851	50149.848	2	266.967
Medium_5	15	4	2.300%	0.691	823.768	1	247.097
Medium_6	16	2	2.465%	0.750	1216.971	1	254.965
Medium_7	15	7	2.710%	0.730	36225.431	1	247.564
Medium_8	12	6	2.528%	0.771	1225.073	2	255.882
Medium_9	12	5	2.528%	0.771	1311.99	3	255.183
Medium_10	18	7	2.581%	0.711	1095.794	1	232.097
Large_1	15	5	2.492%	0.735	5151.404	2	314.950
Large_2	13	4	2.931%	0.865	42788.107	1	285.919
Large_3	19	2	2.515%	0.740	8725.307	5	324.011
Large_4	20	4	4.841%	1.448	68937.803	1	350.046
Large_5	16	7	2.583%	0.685	5037.503	2	306.864
Large_6	16	7	2.580%	0.692	5053.416	3	325.483
Large_7	18	4	4.690%	1.829	61824.71	1	321.549
Large_8	12	5	2.648%	0.801	4446.052	1	323.358
Large_9	12	3	7.175%	2.281	43270.796	1	239.618
Large_10	17	6	2.740%	0.691	3185.135	2	290.432
Real scenario	12	3	2.62%	0.79	23409.93	1	193.42

Table 10: Numerical results of implementing our ε -constraint algorithm for the bi-objective timetabling and VSP.

We observe that the average number of iterations for Small instances is 10.2, resulting in Pareto fronts with an average size of 2.9 points. For medium instances, we note an average of 16.2 iterations, leading to Pareto fronts with an average size of 5 solutions. In the case of Large instances, the average number of iterations is 15.8, yielding Pareto fronts with an average size of 4.7 points. The latter indicates that, as the size of instances increases, more iterations are required to find non-dominated solutions, aligning with the

challenging nature of multi-objective problems. Regarding the real case, we obtained only three points using our ε -constraint method, with no significant differences in the objective function representing the level of service. The latter exhibits a scenario without conflict between optimizing both objective functions.

Regarding computational times, the total time for implementing all iterations of the ε -constraint is presented in column ‘CPUTime’ of Table 10. We highlight that the instance requiring the longest time to obtain a Pareto front approximation was Large_4, demanding more than 19 hours. Nevertheless, this computational time is acceptable given our focus on strategic-operational decisions. Generally, the average computational times to implement our ε -constraint algorithm on small, medium, and large instances are 113.78, 1271.08, and 2484.14 seconds, respectively. On the other hand, the HNDS-BRKGA generally results in shorter times, with reductions of 87.97%, 98.04%, and 90.88% for small, medium, and large instances, respectively. However, our metaheuristic algorithm has the disadvantage of finding fewer non-dominated solutions than the ε -constraint method.

Additionally, Figure 20 shows a graphical representation of the time required for each iteration of our ε -constraint for each instance. We did not observe any particular behaviour of the computational times concerning the parameter ε in all instances, but we would like to highlight some specific observations. In Figure 20(a) it is observed that in certain instances, such as Small_2, Small_4, and Small_7, the smaller the value of ε , the longer the computational times, but in general, our algorithm reaches high-quality solutions in short times for most small instances. In the panel Figure 20(b), it can also be noticed short computational times for 70% of the medium-size instances. Furthermore, Figure 20(c) highlights that in most iterations of instances Large_2, Large_3, Large_7, and Large_8 the solver reaches the time limit. Finally, Figure 20(d) shows short computational times for the real scenario, where we emphasise that only one iteration of our ε -constraint method results in computational times of more than one hour. The efficiency of these computational times highlights the algorithm’s advantage for real-world applications where time limitations directly impact decision-making processes.

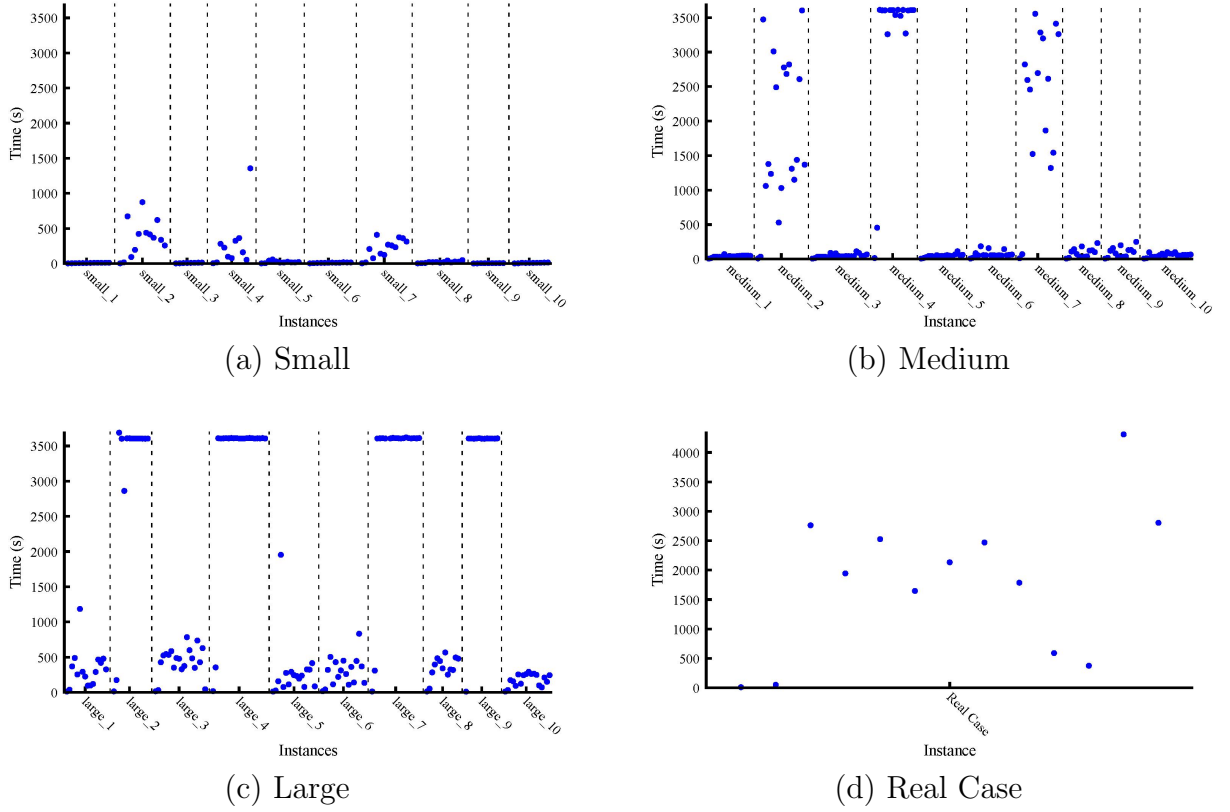


Figure 20: Computational times along the iterations of our ε -constraint algorithm.

The numerical results exhibited the algorithms' ability to find Pareto front approximations within acceptable computational times, but we note that solutions generated by one algorithm may be dominated by those generated by the other. A detailed comparison will be presented in Section 4.3.5.

4.3.5 COMPARISON OF ε -CONSTRAINT AND HNDS-BRKGA

Our optimization approach is based on implementing two multi-objective optimization algorithms on random and real instances of the CTOL-VSI problem. In response, we analyse the solutions obtained by both algorithms for all instances. First, we recall that different algorithms yield different approximations of the Pareto front, and we did not reach the exact front due to the stop criteria of 3% of the relative gap in the ε -algorithm. Figure 21 exhibits the percentage of solutions found by each algorithm that are dominated by the union of the two Pareto front approximations, called the 'Union Approximation'. A value of 0% indicates that all solutions obtained through a particular algorithm remain in the Union Approximation, while a value of 100% represents a case where the algorithm is useless.

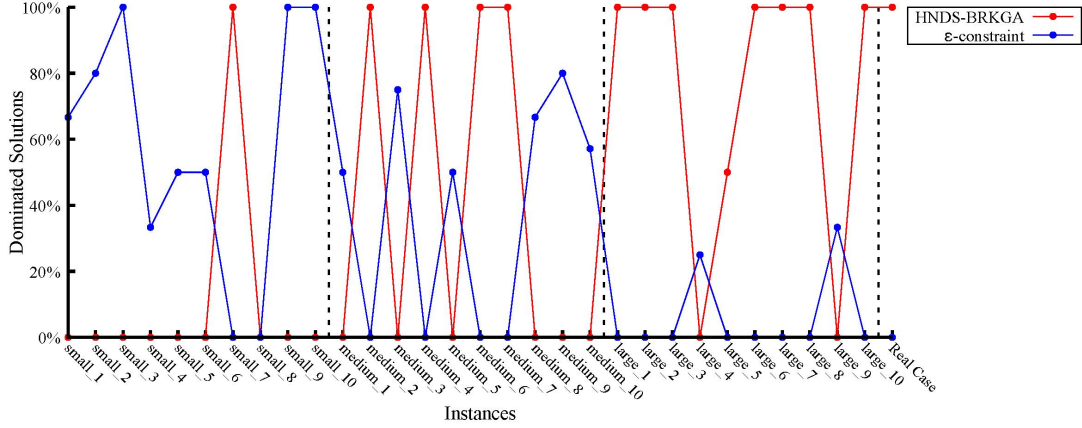


Figure 21: Percentage of dominated solutions of ε -constraint and HNDS-BRKGA compared with the union of both Pareto front approximations for each instance.

Notice that for most small instances, the majority of solutions found by the HNDS-BRKGA algorithm remain in the Union Approximation (represented by the red points at 0%), indicating that the HNDS-BRKGA is particularly effective at identifying non-dominated solutions in smaller problem sizes, contributing significantly to the overall quality of the approximation. As we move to the medium-sized instances, it is worth noting that the Union Approximation is composed of solutions generated by both algorithms, showing a complementary relationship between the two methods in mid-range problem sizes, where each contributes a unique set of non-dominated solutions, enhancing the diversity and robustness of the Pareto front approximation. For large instances and the real scenario, we observe that the solutions produced by the ε -constraint algorithm dominate the Union Approximation, as indicated by the blue points at 0%.

Based on the latter numerical results, the ε -constraint algorithm becomes more effective as the size of the problem increases, likely due to its ability to explore the solution space more thoroughly under larger and more complex scenarios. This behaviour highlights the scalability of the ε -constraint algorithm in handling more intricate optimization problems, where it consistently identifies high-quality, non-dominated solutions.

The previous comparison shows that neither of the proposed algorithms dominates the other in generating Pareto optimal solutions. Furthermore, additional analysis of the Union Approximation can be conducted to provide more comprehensive information to decision-makers, as detailed below.

4.3.6 ANALYSING THE PARETO FRONTS

The Pareto fronts represent non-dominated solutions, emphasising the inherent trade-offs between conflicting objectives and offering various solutions, granting decision-makers the ability to explore options that align with various preferences. In this regard, conducting a detailed analysis of these fronts is significant as it facilitates an informed and strategic decision-making process. Using our ε -constraint algorithm and our HNDS-BRKGa for the CTOL-VSI problem, we successfully identified approximations to the Pareto fronts in the tested instances by using the union of solutions found by both algorithms (the Union Approximation), where we denote with blue dots and red dots the solutions found by the ε -constraint and the HNDS-BRKGa, respectively.

We highlight that a detailed analysis of the shape and size of Pareto fronts allows us to understand how the objectives are compromised. However, the specific characteristics of the shape and size of the front vary significantly among the instances, emphasising that this behaviour depends not only on the inherent properties of the conflicting objectives. Figure 22, shows the approximations obtained for the Pareto fronts of instances (a) Small_7, (b) Medium_2, (c) Medium_3, (d) Medium_5, (e) Large_2, and (f) Large_5 that represent the different shapes for the generated Pareto fronts approximation among all instances. Moreover, we highlight in red font the relative improvement in each objective function for the extreme point on the right side of the Pareto front approximation compared to the extreme point on the left side.

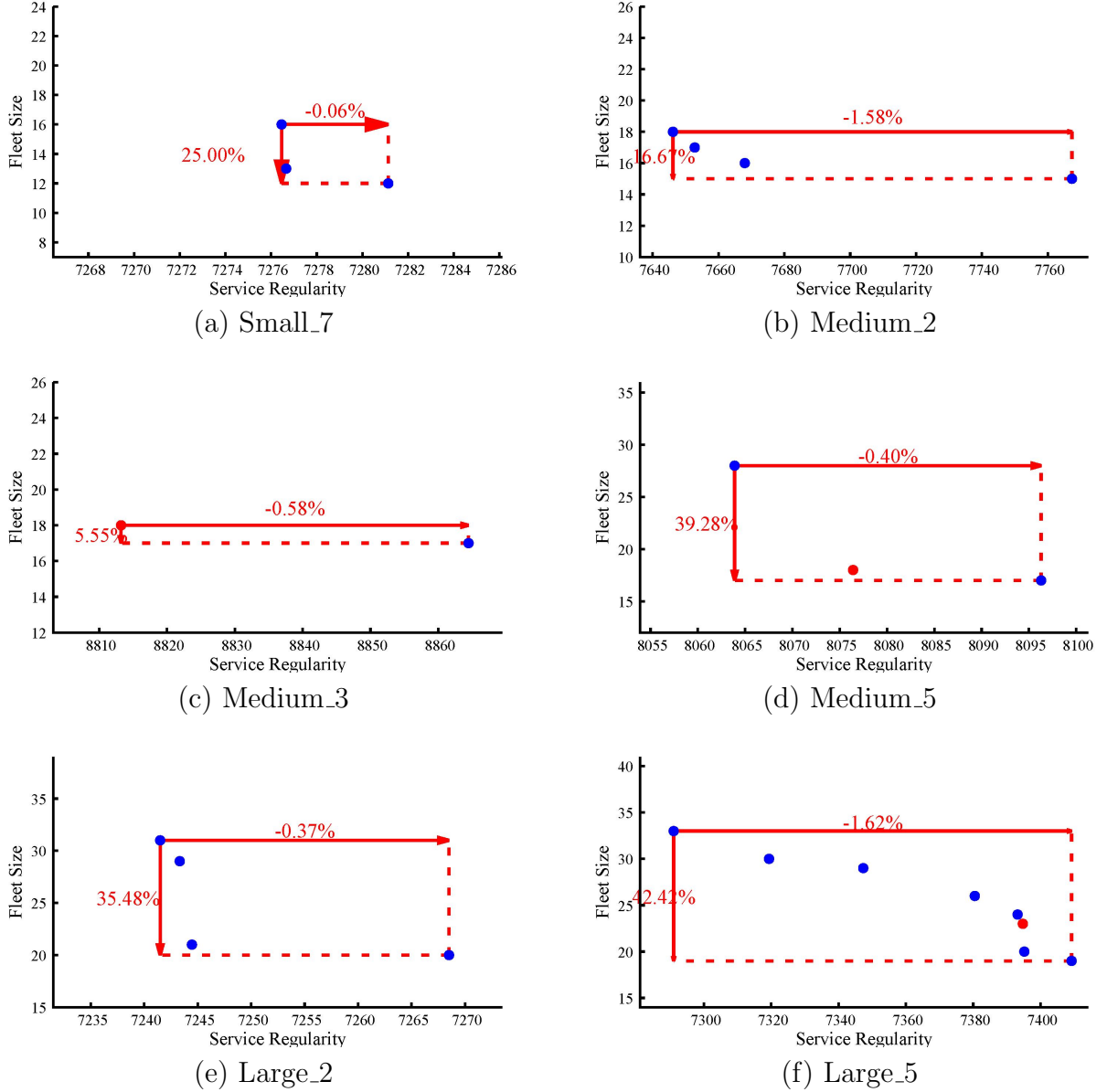


Figure 22: Examples of union approximations for different instance sizes.

Notice that in Figure 22(a), it is observed that the fleet size objective F_{VSI} improves by 25% if we worsen the service regularity objective F_{ST} by 0.06% compared to its left extreme optimal value. Similarly, in Figures 22(b), 22(c), 22(d), and 22(e) it is noted that for the Medium_2 instance, reducing the fleet from 18 to 15 vehicles only has a minimal impact on the F_{ST} objective, with an increment of 1.58% on the optimal value. For Medium_3, reducing the fleet from 18 to 15 vehicles results in an increment of only 0.58% for the function F_{ST} . In instance Medium_5, there is a larger improvement of the fleet size (39.28%) when allowing only a worsening of 0.4% in the level of service. For Large_2, reducing the fleet from 32 to 20 vehicles results in an increment of only

0.37% for objective F_{ST} . Finally, instance Large_5 is one of the scenarios with a more evident trade-off between both objectives, since it is possible to achieve an improvement of 42.42% for the fleet size, resulting in an increment of only 1.62% for the objective F_{ST} . We highlight with blue and red the points found by the ε -constraint method and HNDS-BRKGA, respectively. Then, it is evident that both of our solutions algorithms contribute to better approximating the Pareto front, as previously discussed in Section 4.3.5.

Although the variation in the F_{ST} objective appears small, the following section discusses the impact of the service regularity measure for overlapping lines.

4.3.7 DETAILS OF THE TRADE-OFF AMONG CONFLICTING OBJECTIVES

The previous section highlights the conflict in the values of the two objectives optimized in the CTOL-VSI. In particular, there is little variability in the relative improvement for the service regularity objective. In response, this section analyses the implications of different values of our service regularity measure with respect to system operation to assess the impact of a variation in function F_{ST} . This can be illustrated with a time vs. space graph for the trips of the transportation lines, as shown in Figure 23 for a non-dominated point in the instance Large_4. We use different colours to identify the three generated transportation lines (red for line 1, green for line 2, and blue for line 3). Quasi-equidistant lines along common stops represent an excellent joint operation of overlapping lines; for example, observe from stops 5 to 12 where all three transportation lines operate.

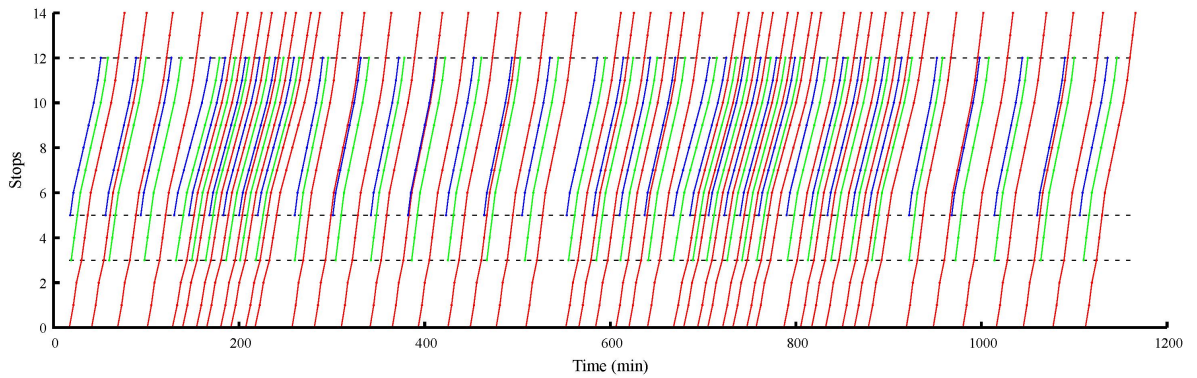


Figure 23: Example of a time vs. space graph depicting the operation of trips for overlapping lines of instance Large_4.s

To illustrate the trade-off details of the optimized objectives, Figure 24 shows a time vs. space graph for the instance Large_4 during operation period 5, where the difference between the extremes of the Pareto front approximation is more evident. Furthermore,

the graph in the left panel corresponds to the leftmost extreme point (9279.5, 45) in the Pareto front approximation, which is associated with the best value for the level of service. In contrast, the right panel represents the rightmost extreme point (9723.7, 23) of the same approximation, corresponding to the best value for operational costs.

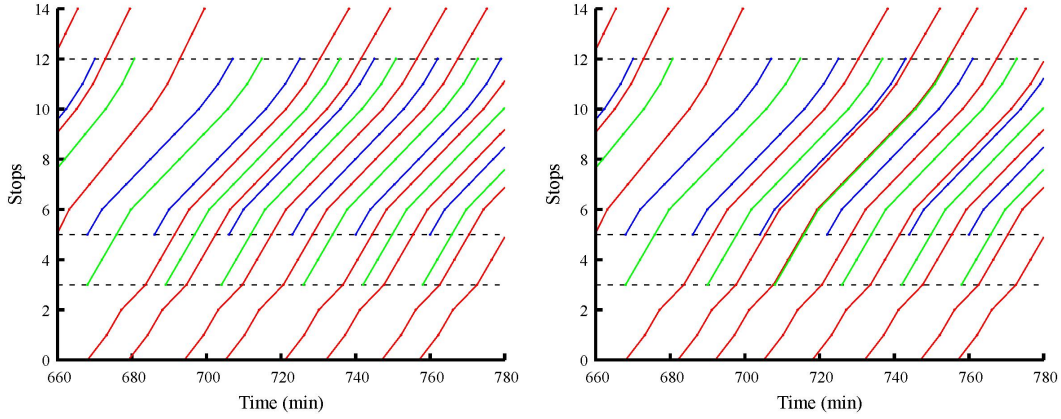


Figure 24: Example of a time vs. space graph, for instance Large_4 in period 5.

Observe that optimizing service regularity (refer to the left panel in Figure 24) leads to a more uniform joint interval for overlapping lines within the shared segments, compared to the solution obtained by optimizing fleet size, where lines begin to overlap on their routes and generate congestion at these common stops. As mentioned in the problem description, this is a tangible benefit for passengers who can board any overlapping lines to reach their destination, reducing waiting time. Moreover, we emphasize that the limited timetable flexibility helps maintain the regularity of non-overlapping segments. It can be observed that not only do we achieve coordinated operation along the shared route segment of different lines, but the red line also operates with nearly evenly spaced arrivals at stops 1, 2, 13, and 14, where it is the only line serving these stops.

We emphasize that period 5 corresponds to the afternoon peak hour, associated with a higher influx of passengers in the system. It is important to note that our model does not incorporate passenger demand, leading to a lack of weight in passenger demand for each planning period. The impact on service regularity would be more pronounced if demand is considered. In particular, each passenger who benefits from the coordination of overlapping lines will experience a more regular operation in shared route segments, amplifying the impact of our deviation metric regarding the joint headway.

We note that the feasible space of our integrated problem depends on the parameter β (representing the flexibility of the timetable), which we set to 2 min during our experimental stage. A sensitivity analysis of this parameter provides the following managerial insights. Specifically, we solved small instances using values of $\beta \in 1, 3$, and the numerical

results show an average relative improvement² of -11.63% and 10.15% for the minimum level of service using $\beta = 1$ and $\beta = 3$, respectively, compared to the base scenario of $\beta = 2$ minutes. It should be noted that no feasible solution was obtained for the instance *Small_2* with $\beta = 3$. Regarding the fleet size, the numerical results show an average relative improvement of -2.44% and 2.43% for the minimum level of service using $\beta = 1$ and $\beta = 3$, respectively, compared to the base scenario of $\beta = 2$. We did not use medium or large instances due to the intractability of $\beta = 3$, as the problem could not be solved for all instances within a reasonable time frame.

Through our sensitivity analysis, we assess the effects of modifying the flexibility of the timetable by increments of one minute. It is important to note that allowing a flexibility of 3 minutes creates a feasible time window of 6 minutes for the departure time of each trip. Therefore, we select values between 1 and 3 minutes to avoid deviating excessively from the reference timetable and an almost even headway policy. Specifically, reducing the flexibility of the timetable by one minute decreases the feasible space, leading to solutions that degrade the values of the objective function. In contrast, increasing the flexibility of the timetable by one minute improves service regularity by 10.15% and reduces operational costs by 2.43%.

Summarizing our numerical results, we generated different kinds of information for decision makers, providing detailed insights into the trade-off between reducing the fleet size and optimizing the level of service. The understanding provided by our optimization approach becomes a valuable tool for strategic decision making, empowering operators to evaluate the implications of various operational scenarios effectively and propose quantifiable and measurable decisions.

This chapter examines a bi-objective optimization framework that jointly addresses timetabling and vehicle scheduling in bus systems under interlining considerations. The proposed formulation, solved through a combination of an ε -constraint method and a metaheuristic procedure, enables the exploration of trade-offs between service regularity and operational efficiency. The computational results suggest that coordinating these decisions leads to improvements in system performance, particularly through reductions in fleet requirements and service variability. These outcomes highlight the value of integrated modeling approaches in the analysis of complex transit operations. Additional insights and a deeper discussion are provided in the subsequent sections.

²The average relative improvement is computed as $\frac{ObjectiveFunction_{(\beta=2)} - ObjectiveFunction_{(\beta \in \{1,3\})}}{ObjectiveFunction_{(\beta=2)}} \times 100\%$

DISCUSSION AND CONCLUSIONS

This chapter presents the main conclusions derived from the research developed throughout this thesis. The research has focused on extending the traditional Vehicle Scheduling Problem (VSP) by incorporating additional planning decisions and operational characteristics that are increasingly relevant in modern public transportation systems. Two main optimization problems were investigated. The first addresses electric bus scheduling together with charging infrastructure location and timetabling decisions, while the second integrates vehicle scheduling, timetabling, and interlining within a bi-objective framework. In addition, a generalized formulation of the VSP is presented as an additional contribution, bringing together several operational and technological characteristics within a unified modeling framework. The following sections summarize the main findings of these developments, discuss their overall contribution, and identify directions for future research.

5.1 ELECTRIC VEHICLE SCHEDULING PROBLEM WITH LOCATION OF CHARGING STATIONS

The first problem addressed in this thesis considers an integrated optimization problem related to battery electric bus scheduling and charging station locations, assuming on-route charging technology in a transit line (referred to as EVSP-LS). Our objective is to minimize the total operational costs of the fleet size and charging infrastructure while incorporating hard constraints for the maximum charging time for each trip, as charging actions occur on-route and impact passenger travel time. A comprehensive computational experiment using a general purpose solver has been conducted to assess two variants to analyze the impact of operational and strategic decisions: the operational version of our EVSP-LS considering only scheduling decisions of electric vehicles (referred to as EVSP) and the original EVSP-LS. We use randomly generated instances and a real scenario in our experimental stage, where the numerical results show that for the EVSP formulation,

optimal solutions are obtained in an average time of 40 seconds. In particular, we observed significant levels of charging station usage when the maximum recharging time per trip varies. Our experiments carried out for the EVSP-LS aim to analyze the usage of different charging stations at different locations, indicating that only some recharging stations should be used to significantly reduce infrastructure expenses.

This work suggests potential avenues for further research. Firstly, conducting an extensive analysis with various objective functions, such as minimizing total travel time for all buses, minimizing recharging time, or maximizing the use of charging stations for strategic decisions, could be insightful. We performed preliminary experimentation with several objectives; however, the problem's intractability increases when considering any of the latter objective functions. Additionally, analyzing the effects of a non-linear charging function, a heterogeneous fleet, or a multi-line system essentially generalizing the problem to adopt electromobility would be interesting. Based on the experiments conducted with the EVSP-TF model, we observed that providing flexibility in bus departure times can lead to a reduction in the size of the fleet, as expected. Moreover, we note that allowing greater flexibility in the timetable results in a higher number of instances where the objective function improves; however, this comes at the expense of increased computational time.

5.2 THE BI-OBJECTIVE CTOL-VSI PROBLEM

The second problem developed in this thesis introduces a novel bi-objective optimisation problem that integrates timetabling and vehicle scheduling problems, assuming interlining for bus transportation systems. We design a novel mathematical formulation in terms of scheduling variables, and we can find Pareto optimal solutions by implementing an ε -constraint and a Hierarchical Non-dominated Sorting Biased Random-Key Genetic Algorithm, which exhibits the trade-off between improving service regularity and reducing operational costs. Our findings indicate that the integrated approach enhances the efficiency of public bus services by minimising the total deviation from the ideal joint heads and the size of the fleet, significantly contributing to operational flexibility and cost-effectiveness. The experimental results on randomly generated instances showcase the algorithm's efficacy in managing the complexities of urban bus networks, offering transportation planners a robust tool for optimising overlapping lines.

In general, our study underscores the importance of integrated optimisation approaches in improving the efficiency and sustainability of public transportation systems. By offering insights into the trade-off between service quality and operational costs, this study contributes valuable knowledge to urban transit planning and management, paving

the way for more informed and effective decision making. Future research may explore the integration of passenger demand models to further refine service regularity measures or integrate driver scheduling decisions to enhance the definition of the problem. Another research area is the extension of a robust optimisation approach to account for travel time variability, similar to the one proposed by Chávez-Hernández et al. (2024) using fuzzy synchronisation functions. Moreover, our experiments consider only coordination at common route segments for overlapping lines. However, without loss of generality, we could define a unique set $L_1^s = l$ for each stop $s \in S$, where only the line l would be optimised to account for the operation of independent lines. Finally, another research area is the comparison of the optimisation of different waiting times (independent lines, overlapping lines, and transfer times) across an entire transit network or a generalised timetabling problem, as well as the inclusion of other objective functions to model user costs or additional operational characteristics.

As an additional contribution, this thesis introduces the Generalized Vehicle Scheduling Problem (G-VSP), presented in Appendix A. The formulation extends the traditional VSP by simultaneously incorporating several operational and technological characteristics within a unified optimization framework. In particular, it considers multiple transit lines operated from a single depot, a heterogeneous fleet in which conventional and electric buses may coexist, predefined interlining possibilities, charging infrastructure location, deadheading movements, and different charging technologies, including depot and on-route charging. In this sense, the G-VSP builds upon the characteristics analyzed separately in the main problems of this thesis and provides a broader framework for representing more heterogeneous public transportation operations. Overall, the results obtained throughout this thesis demonstrate that vehicle scheduling decisions are strongly influenced by planning decisions and operational characteristics that are traditionally considered separately. In the context of electromobility, battery limitations, charging opportunities, charging duration, and charging infrastructure directly affect the feasibility and efficiency of vehicle schedules. Similarly, when multiple transit lines are considered, timetabling and interlining provide additional flexibility for the assignment and sequencing of trips, creating opportunities to improve fleet utilization while introducing trade-offs with service-related objectives.

5.3 FUTURE RESEARCH

Several research directions arise from the developments presented in this thesis. For the electric vehicle scheduling problem, future work may consider alternative objective functions related to operational time, charging activities, or charging infrastructure uti-

lization. More realistic representations of battery charging behavior could also be incorporated, particularly through nonlinear charging functions, together with additional sources of variability in energy consumption and operating conditions. For the bi-objective vehicle scheduling problem, future research may incorporate passenger demand to provide a more detailed representation of service quality and passenger-related objectives. Driver scheduling decisions could also be integrated to extend the operational scope of the problem. Another relevant direction is the incorporation of uncertainty in travel times through robust or other uncertainty-aware optimization approaches. In addition, the coordination framework could be extended to consider different types of synchronization, including independent lines, overlapping lines, and passenger transfers, within a more general network-wide timetabling framework. Finally, the G-VSP provides a natural basis for several extensions of this research. A more extensive computational study is required to evaluate the behavior and scalability of the formulation for larger and more heterogeneous instances and to analyze in greater detail the interactions among vehicle technologies, charging strategies, charging infrastructure decisions, interlining possibilities, and deadheading movements. As the size and complexity of these instances increase, the development of specialized exact, heuristic, or matheuristic solution approaches represents an important direction for future research.

APPENDIX A

A GENERALIZED VEHICLE SCHEDULING PROBLEM

The Vehicle Scheduling Problem (VSP) represents a key operational decision in public transportation planning, as it determines the assignment and sequencing of scheduled trips to the available vehicles. A feasible schedule must ensure trip coverage while satisfying the temporal and operational requirements of the system and promoting an efficient use of the available fleet. However, the incorporation of electric buses introduces additional challenges to the traditional problem due to their limited driving range, the need for charging operations, and their dependence on specific charging infrastructure. The incorporation of these characteristics implies that electric bus scheduling does not depend solely on the temporal compatibility between consecutive trips. The feasibility of a trip sequence may also depend on the vehicle's battery level, the available charging opportunities, and the time required to access and use the charging infrastructure. These considerations become particularly relevant in networks where conventional and electric vehicles with different charging technologies coexist, as each technology may impose different operational requirements. The configuration of the network may also introduce additional conditions for trip sequencing. Transit lines may have different operational structures and terminal locations, and therefore, connections between consecutive trips may require deadheading movements. In addition, interlining allows a vehicle to operate consecutive trips belonging to different lines, providing greater flexibility in fleet utilization. Combining these characteristics with different vehicle technologies and charging alternatives expands the scheduling possibilities while also increasing the requirements that must be considered to ensure the feasibility of the resulting trip sequences.

The problems developed in the previous chapters of this thesis have analyzed some of these interactions under specific settings. In particular, electric bus scheduling has been studied jointly with charging infrastructure location and timetabling decisions, while timetabling and interlining have also been incorporated into a bi-objective vehicle scheduling framework. These problems allow specific interactions among different planning deci-

sions to be examined; however, they rely on particular assumptions regarding the network, fleet, and available technologies.

Building upon these developments, this chapter proposes the *Generalized Vehicle Scheduling Problem* (G-VSP), which extends the traditional VSP by jointly representing several operational and technological characteristics within a unified modeling framework. The G-VSP considers a network with multiple lines operated from a single depot and a heterogeneous fleet in which conventional and electric buses may coexist. Each line is preassigned to a vehicle technology and, for electric services, to a specific charging technology. The formulation also incorporates predefined interlining possibilities, dead-heading movements, charging infrastructure decisions, and different charging alternatives for electric buses, including both depot and on-route charging. In this way, the G-VSP enables the analysis of how the coexistence of these characteristics affects trip assignment and sequencing, fleet requirements, charging operations, and the charging infrastructure required for operation.

The remainder of this chapter is organized as follows. First, a detailed description of the problem and its main operational and technological characteristics is provided. The mathematical formulation is then presented, beginning with the definition of the sets, parameters, and decision variables, followed by the objective function and the corresponding constraints. Finally, computational experiments are presented to illustrate the behavior of the proposed formulation and to analyze the effects of the different configurations considered.

A.1 PROBLEM DESCRIPTION

The network considered in the G-VSP consists of a single depot and a set L of lines, over which a set I of scheduled trips is defined. Each trip $i \in I$ is associated with a line $l_i \in L$ and has a predetermined departure time, denoted by dt_i . Based on these trips and their corresponding schedules, the G-VSP aims to determine feasible sequences of trips to be operated by the available vehicles. The network considered in the G-VSP may include lines with different operational configurations. In particular, both circular and bidirectional lines are considered. On a circular line, trips start and end at the same terminal, whereas a bidirectional line operates between two different terminals, with trips performed in both directions. These line configurations may coexist within the same transit network. The location of the line terminals may also vary with respect to the depot. For some lines, one of the terminals may be located at the depot, whereas for others, both terminals may be located outside the depot. As a result, deadheading movements may be required to

connect different points of the network. These movements may occur from the depot to the initial terminal of the first trip, from the final terminal of the last trip back to the depot, or between the final terminal of one trip and the initial terminal of the following trip. Figure 25 illustrates an example of a network configuration that can be represented by the G-VSP. The network contains circular and bidirectional lines with different terminal locations, as well as charging stations located at different points of the network. The dotted connections represent possible deadheading movements between the depot and line terminals or between terminals belonging to different lines. This configuration illustrates how different line structures, charging opportunities, and deadheading movements may coexist within the same network.

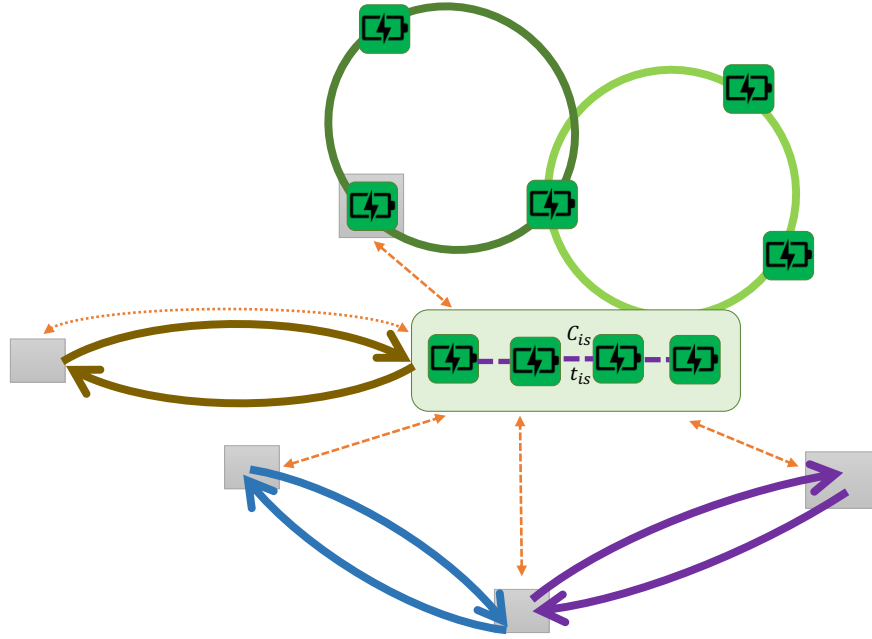


Figure 25: Example of a network configuration considered in the G-VSP.

Charging infrastructure is distributed between the depot and selected locations within the network. Charging stations are located at the depot and can be used whenever vehicles have access to these facilities. In addition, some lines have charging stations located along their routes. When permitted by the network configuration, the same on-route charging station may provide charging opportunities for more than one line. The G-VSP determines which candidate stations should be opened and coordinates charging operations over time, preventing conflicts when different vehicles require access to the same charging resource.

The network is operated by a heterogeneous fleet composed of different vehicle types, defined by the set V , with a maximum availability k_v for each type $v \in V$. Each line is

preassigned to a specific vehicle technology and, for lines operated by electric buses, to a specific charging technology. Consequently, diesel-operated lines and electric-bus lines with different charging alternatives may coexist within the same network. The technology assigned to each line determines the type of vehicle that can operate its trips and, for electric vehicles, the conditions under which charging operations can be performed. Two charging technologies are considered for electric buses: slow charging and fast charging. Lines associated with slow-charging technology are operated by vehicles that can recharge exclusively at the depot and, therefore, do not have charging opportunities while performing their trips. In contrast, some lines operate with fast-charging technology, or opportunity charging, which allows charging operations to be performed both at the depot and at stations located along the line. These technological configurations and their corresponding charging alternatives are illustrated in Figure ???. Thus, the charging opportunities available to an electric vehicle depend on the technology assigned to the line it operates.

The location of the terminals and the sequence of trips may generate deadheading movements, or deadheading. At the beginning of a vehicle block, a vehicle may need to travel from the depot to the initial terminal of its first trip and, at the end of the block, from the final terminal of its last trip back to the depot. During the operating day, a vehicle may also need to be repositioned when the final terminal of a trip i' does not coincide with the initial terminal of the subsequent trip i . In addition, electric buses may travel to the depot to perform a charging operation before continuing with the next trip. Therefore, deadheading movements may arise from initial or final vehicle positioning, repositioning between consecutive trips, or access to charging infrastructure located at the depot. In the latter case, traveling to the depot to perform a charging operation between two trips is feasible only when the available time interval is sufficient to complete the required movements and charging operation before the scheduled departure time of the subsequent trip.

When two consecutive trips i' and i are connected directly, the travel time required to move from the ending point of trip i' to the starting point of trip i is denoted by $\lambda_{i'i}$, while $\rho_{i'i}$ represents the corresponding battery consumption. For both trips to be consecutively included in the same vehicle block, sufficient time must be available to complete the preceding trip, perform the required movement, and begin the subsequent trip according to its scheduled departure time. For electric buses, the connection must also be compatible with the available battery level. When a charging operation is performed at the depot between two consecutive trips, the connection no longer corresponds to a direct movement between their respective terminals. To represent this situation, the parameters $\lambda_{i's}$ and λ_{si} denote, respectively, the travel time required to move from the ending point of trip i'

to charging station s , and from that station to the starting point of trip i . Similarly, $\rho_{i's}$ and ρ_{si} represent the battery consumption associated with these movements. The energy recovered during the charging operation depends on its duration and the corresponding charging rate $f_i(\cdot)$. Accordingly, the G-VSP can distinguish between a direct connection and a connection involving a visit to the depot for charging, accounting for the differences in travel time and battery consumption associated with each alternative.

To represent electric bus operations, the formulation explicitly tracks the evolution of the battery level throughout the operating day. A minimum safety battery level β is imposed and must be maintained during operation. Each trip i is associated with a set S_i of stations that may be visited during its execution. For each station $s \in S_i$, s_i^- and s_i^+ identify the preceding and subsequent stations along the trip, respectively, while s_i^{last} denotes the last station associated with the trip. The parameter t_{is} represents the travel time required to traverse the segment between a station $s \in S_i$ and the subsequent station s_i^+ , whereas C_{is} represents the corresponding battery consumption. This information allows the battery level to be updated as the vehicle progresses along the trip and ensures the energy feasibility of the resulting operation.

For each trip i , the parameter α_i establishes the maximum allowable duration of charging operations that may be performed during that trip. Its value is defined individually according to the characteristics of the trip and its associated technology. For trips that allow on-route charging, α_i limits the cumulative duration of such charging operations, thereby controlling their impact on the scheduled operation. For trips operated by diesel buses, this parameter is assigned a sufficiently large value so that the corresponding condition does not impose an effective restriction. The network also includes predefined interlining possibilities, whereby a vehicle completing a trip on one line may subsequently operate a trip on another line. These possibilities are not determined by the G-VSP but are provided as input data and represented by the binary parameter $A_{i'i}$, which indicates whether a connection between trips i' and i is allowed. The definition of these connections accounts for the compatibility of the technologies assigned to the corresponding lines. Consequently, the G-VSP may use an interlining connection to construct a vehicle block only when that connection has been predefined as feasible and the corresponding vehicle can operate the trips of both lines.

At the end of the operating day, electric vehicles return to the depot and have an overnight period PN available for charging operations. The charging requirements during this period depend on the battery level at which the vehicles complete their respective blocks. This condition accounts for the battery replenishment requirements following daily operations and the use of depot charging infrastructure to prepare the vehicles for the subsequent operating day.

Within this context, the G-VSP consists of determining the sequence of trips assigned to the vehicles, the movements required to connect them, and the charging operations required by the electric fleet. The G-VSP may use the predefined interlining possibilities and simultaneously determines the charging infrastructure to be opened and the use of the available charging stations. The formulation must ensure that all scheduled trips are covered and that the availability k_v of each vehicle type is respected, while maintaining the temporal feasibility of trip connections and the technological compatibility of the resulting vehicle blocks. For electric buses, additional conditions account for battery evolution and the minimum battery level, energy consumption associated with both revenue trips and deadheading movements, available charging opportunities and charging limits, charging infrastructure availability and utilization, and end-of-day charging requirements. Therefore, the resulting vehicle blocks must be feasible from operational, temporal, technological, and energy perspectives. The objective function of the G-VSP minimizes three cost components. The first corresponds to the cost of the vehicles used to cover the scheduled trips, promoting an efficient use of the available fleet. The second represents the cost of opening the charging infrastructure required to support electric bus operations. Finally, the third component represents the cost associated with charging operations that generate additional non-revenue movements. This component penalizes charging operations that require deadheading movements to the depot and, consequently, indirectly reduces such movements when more convenient operational alternatives are available.

Based on the operational and technological characteristics described above, the G-VSP is formulated as a mathematical programming model that jointly represents vehicle scheduling, charging, and infrastructure decisions while accounting for the operational, temporal, technological, and energy requirements of the system. The mathematical formulation is presented next, beginning with the definition of the sets, parameters, and decision variables, followed by the objective function and the corresponding constraints.

A.2 MATHEMATICAL MODEL

This section presents the mathematical formulation of the G-VSP. The proposed model simultaneously considers vehicle scheduling, interlining, charging operations, and charging infrastructure decisions. Different vehicle types may coexist within the same transit network, and each trip is associated with a specific transit line and a sequence of stations that may be visited during its operation. For electric vehicles, the formulation explicitly accounts for battery consumption, charging opportunities, and minimum state-of-charge requirements.

The sets and parameters used throughout the formulation are summarized in Table 11. The artificial trip 0 is introduced to represent the beginning and end of each vehicle block.

The decision variables used in the mathematical formulation are defined as follows:

$$x_{i'i} = \begin{cases} 1, & \text{if trip } i' \text{ is operated immediately before trip } i, \\ 0, & \text{otherwise,} \end{cases} \quad i', i \in I_0.$$

$$y_{is} = \begin{cases} 1, & \text{if trip } i \text{ charges at station } s, \\ 0, & \text{otherwise,} \end{cases} \quad i \in I, s \in S_i.$$

$$z_s = \begin{cases} 1, & \text{if charging station } s \text{ is opened,} \\ 0, & \text{otherwise,} \end{cases} \quad s \in S.$$

$$g_{i'is} = \begin{cases} 1, & \text{if trip } i' \text{ arrives at station } s \text{ before trip } i, \\ 0, & \text{otherwise,} \end{cases} \quad i', i \in I_0, s \in S.$$

$$tc_{is} = \text{charging time at station } s \text{ during trip } i, \quad i \in I, s \in S_i.$$

The following auxiliary variables are defined through the corresponding operational relationships:

$$dh_{i'i} = \left(1 - \sum_{s \in S_{i'}} y_{i's} \right) \lambda_{i'i} + \sum_{s \in S_{i'}} y_{i's} (\lambda_{si} + \lambda_{i's}), \quad i', i \in I. \quad (\text{A.1})$$

$$cd_{i'i} = \left(1 - \sum_{s \in S_{i'}} y_{i's} \right) \rho_{i'i} + \sum_{s \in S_{i'}} y_{i's} (\rho_{si} + \rho_{i's}), \quad i', i \in I. \quad (\text{A.2})$$

$$et_i = dt_i + \sum_{s \in S_i} (t_{is} + tc_{is} + \lambda_{is} y_{is}), \quad i \in I. \quad (\text{A.3})$$

Table 11: Sets and parameters used in the mathematical formulation.

Notation	Description
I_v	Set of trips associated with vehicle type $v \in V$.
I_0	Set of trips including the artificial trip 0.
I	Set of scheduled trips excluding the artificial trip 0.
S	Set of stations.
S_i	Set of stations that can be visited during trip $i \in I$.
$S_i^-(s)$	Set of stations preceding station s during trip $i \in I$.
V	Set of vehicle types.
L	Set of transit lines.
s_i^+	Station following station $s \in S_i$ during trip i .
s_i^-	Station preceding station $s \in S_i$ during trip i .
s_i^{last}	Last station associated with trip i .
α_i	Maximum charging time allowed during trip i .
k_v	Number of available vehicles of type v .
t_{is}	Travel time from station $s \in S_i$ to station s_i^+ during trip i .
C_{is}	Battery consumption from station $s \in S_i$ to station s_i^+ during trip i .
β	Minimum battery safety buffer.
λ_{is}	Travel time from the ending point of trip i to station s .
$\lambda_{i'i}$	Travel time from the ending point of trip i' to the starting point of trip i .
λ_{si}	Travel time from station s to the starting point of trip i .
$f_i(\cdot)$	Charging rate associated with trip i .
l_i	Transit line associated with trip i .
$A_{i'i}$	Binary parameter equal to 1 if interlining between lines $l_{i'}$ and l_i is allowed, and 0 otherwise.
dt_i	Scheduled departure time of trip i .
$\rho_{i'i}$	Battery consumption associated with the deadheading movement from the ending point of trip i' to the starting point of trip i .
$\rho_{i's}$	Battery consumption associated with traveling from the ending point of trip i' to station s .
ρ_{si}	Battery consumption associated with traveling from station s to the starting point of trip i .
$\lambda_{i's}$	Travel time from the ending point of trip i' to station s .
λ_{si}	Travel time from station s to the starting point of trip i .
$BusCost$	Cost associated with using one vehicle.
$StationCost_s$	Cost of opening charging station s .
$ChCost_i$	Cost associated with the corresponding deadheading movement related to trip i .
M	Sufficiently large positive constant used in the logical constraints.
PN	Time period available for nighttime charging.

$$u_{is} = dt_i + \sum_{s' \in S_i^-(s)} (t_{is'} + tc_{is'} + \lambda_{is} y_{is'}), \quad i \in I, s \in S_i. \quad (\text{A.4})$$

$$ca_{is} = ca_{i,s_i^-} + f_i tc_{i,s_i^-} - C_{i,s_i^-}, \quad i \in I, s \in S_i. \quad (\text{A.5})$$

$$ic_i = (100 - \rho_{si})x_{0i} + \sum_{i' \in I} x_{i'i} \left(ca_{i's_{i'}^{last}} + f_i tc_{i's_{i'}^{last}} - C_{i's_{i'}^{last}} - cd_{ii'} \right), \quad i \in I. \quad (\text{A.6})$$

Using the sets, parameters, and decision variables previously defined, the proposed mathematical formulation is presented below. The model determines the sequence of trips assigned to each vehicle while simultaneously accounting for interlining possibilities, charging operations, charging infrastructure availability, and battery requirements. The objective function minimizes the total cost of the system, subject to constraints that ensure feasible vehicle schedules and charging operations.

$$\min \sum_{i \in I} BusCost x_{0i} + \sum_{s \in S_i} StationCost_s z_s + \sum_{(i,i') \in I} ChCost_i cd_{ii'} \quad (\text{A.7})$$

subject to:

$$\sum_{i' \in I_0 - \{i\}} x_{i'i} = 1 \quad \forall i \in I \quad (\text{A.8})$$

$$\sum_{i \in I_v} x_{0i} \leq k_v \quad \forall v \in V \quad (\text{A.9})$$

$$g_{ii's} + g_{i'is} \leq 1 \quad \forall i, i' \in I, s \in S \quad (\text{A.10})$$

$$u_{i's} + tc_{i's} \leq u_{is} + M(3 - y_{is} - y_{i's} - g_{i'is}) \quad \forall i, i' \in I, s \in S \quad (\text{A.11})$$

$$et_{i'} + dh_{i'i} \leq dt_i + M(1 - x_{i'i}) \quad \forall i', i \in I \quad (\text{A.12})$$

$$ca_{is} - C_{is} \geq \beta \quad \forall s \in S, i \in I \quad (\text{A.13})$$

$$x_{ii'} + x_{i'i} \leq 1 \quad \forall i', i \in I \quad (\text{A.14})$$

$$\sum_{i' \in I_0 - \{i\}} x_{i'i} - \sum_{i' \in I_0 - \{i\}} x_{ii'} = 0 \quad \forall i \in I \quad (\text{A.15})$$

$$x_{i'i} \leq A_{i'i} \quad \forall i, i' \in I \quad (\text{A.16})$$

$$\sum_{s \in S_i} tc_{is} \leq \alpha_i \quad \forall i \in I \quad (\text{A.17})$$

$$tc_{is} \leq y_{is} \quad \forall i \in I, s \in S \quad (\text{A.18})$$

$$y_{is} \leq z_s \quad \forall i \in I \ s \in S \quad (\text{A.19})$$

$$\sum_{i \in I} x_{i0} \left(100 - \left(ca_{is_i^{last}} + f_i tc_{is_i^{last}} - C_{is_i^{last}} \right) \right) \geq \sum_{s \in S} z_s * PN \quad (\text{A.20})$$

$$x_{i'i}, y_{is}, z_s, g_{i'is} \in \{0, 1\}, \quad (\text{A.21})$$

$$tc_{is}, u_{is}, dh_{i'i}, cd_{i'i}, et_i, ca_{is}, ic_i \in \mathbb{R}_+. \quad (\text{A.22})$$

The objective function (A.7) minimizes the costs associated with system operation and charging infrastructure. The first term represents the cost of the vehicles used to cover the scheduled trips, while the second accounts for the cost of opening charging stations. The third term represents the cost associated with deadheading movements, including both repositioning movements required to connect consecutive trips and those required to access charging infrastructure and perform charging operations. Constraint (A.8) ensures that each scheduled trip has exactly one predecessor, which may be another trip in set I or the artificial trip 0, representing the beginning of the operation from the depot. Constraint (A.9) ensures that the number of vehicles used of each type v does not exceed the corresponding fleet availability k_v . Constraint (A.10) establishes the order between two trips that pass through the same station s , determining which trip uses the station first, while Constraint (A.11) uses this ordering to prevent overlapping charging operations at the same station. Constraint (A.12) ensures the temporal feasibility of consecutive trips assigned to the same vehicle. Constraint (A.13) ensures that the battery level of electric vehicles remains above the minimum required level during operation. Constraint (A.14) prevents two trips from immediately preceding each other simultaneously, while Constraint (A.15) imposes flow conservation and maintains the continuity of the trip sequences. Constraint (A.16) restricts connections between trips according to the predefined interlining possibilities. Constraint (A.17) limits the total charging time during each trip i according to parameter α_i . Constraint (A.18) links the charging time tc_{is} to the corresponding charging decision y_{is} , while Constraint (A.19) links charging operations to the availability of charging infrastructure. Finally, Constraint (A.20) accounts for the charging requirements of electric vehicles at the end of the operating day, considering the energy required to restore their battery level and the period PN available for overnight charging at the depot. Expressions (A.21) and (A.22) define the domains of the variables used in the formulation.

The proposed formulation integrates the main operational and technological characteristics considered in the G-VSP within a unified optimization framework. Once the mathematical formulation has been established, the next step is to validate that the model adequately represents the decisions and interactions for which it was designed. To this end, the following section presents a computational validation based on illustrative

instances, with the purpose of verifying the consistency and feasibility of the solutions generated by the model under different operational and technological configurations.

A.3 COMPUTATIONAL VALIDATION

This section presents the computational validation of the proposed G-VSP formulation. The main objective is to verify that the model adequately represents the operational and technological characteristics for which it was designed and that the resulting solutions are feasible and consistent with the conditions established in the formulation. To this end, a set of illustrative instances is used to analyze the main decisions generated by the model and to examine the interactions among the different components incorporated into the G-VSP.

The validation first considers the main aspects associated with vehicle scheduling. In particular, it verifies that all scheduled trips are properly assigned and sequenced, that the availability of each vehicle type is respected, and that the selected connections between trips are temporally feasible. The analysis also examines the use of the predefined interlining possibilities and their compatibility with the technologies assigned to the corresponding lines.

To evaluate the computational behavior of the proposed formulation, four different network configurations were considered. These configurations differ in terms of the number of lines, their operational structure, and the vehicle and charging technologies assigned to them. The purpose of considering these configurations is to progressively evaluate the formulation under different levels of network and technological complexity. The Type 1 configuration consists of two bidirectional lines operated by electric buses using slow-charging technology. The Type 2 configuration also considers two lines; however, both lines have a circular structure and are operated by electric buses using fast-charging technology. The Type 3 configuration represents a more heterogeneous network composed of four circular lines with different vehicle and charging technologies. One line is operated by diesel buses, two lines are operated by electric buses using fast-charging technology, and the remaining line is operated by electric buses using slow-charging technology. This last configuration allows the formulation to be evaluated in a setting where different vehicle and charging technologies coexist within the same network. Table 12 reports the computational results obtained for these network configurations. For each instance, the total number of scheduled trips, computational time, and final optimality gap are reported.

Table 12: Computational results for the G-VSP network configurations.

Network configuration	Number of trips	Computational time (s)	Gap (%)
Type 1	60	676.137	0%
	203	3600	55%
Type 2	56	1.58297	0%
	201	3600	33%
Type 3	115	2.99235	0%
	419	3600	86%

The computational results indicate that the difficulty of the G-VSP is influenced by both the size of the network and the combination of technological characteristics considered. As the number of lines and scheduled trips increases, a larger number of alternatives must be evaluated for the assignment and sequencing of trips. Moreover, the coexistence of different vehicle and charging technologies introduces additional operational requirements associated with energy consumption, charging opportunities, charging infrastructure, interlining possibilities, and non-revenue movements. Consequently, these decisions must be considered simultaneously when constructing feasible vehicle schedules. This behavior becomes particularly evident when the number of lines is increased. For configurations involving four lines, the computational performance of the formulation deteriorates considerably, and the exact optimization approach is no longer able to adequately solve the instances within the established computational time. These results identify an important computational limitation of the current solution approach as the size and heterogeneity of the network increase.

It is important to emphasize that the purpose of these experiments is not to provide an exhaustive scalability analysis of the G-VSP, but rather to perform an initial computational validation of the proposed formulation. The results demonstrate the ability of the G-VSP to represent networks in which lines operated with different vehicle and charging technologies coexist and to generate feasible schedules under the operational and technological conditions considered. At the same time, the experiments reveal the computational challenges that arise when these characteristics are simultaneously incorporated into the vehicle scheduling problem. Therefore, these results constitute a starting point for the continued development of the G-VSP. Future research will focus on extending the computational experiments to a broader set of instances and analyzing in greater detail the effect of the different operational and technological characteristics on the complexity of the problem. In particular, the development of alternative solution methodologies represents an important direction for addressing larger and more heterogeneous instances. Further research will therefore focus on developing solution approaches capable of improving the computational performance of the proposed formulation and obtaining high-quality so-

lutions for more complex instances. These developments would allow the G-VSP to be evaluated in larger networks with dimensions and technological configurations closer to those encountered in real-world public transportation systems.

APPENDIX A

LINEARIZATION TECHNIQUES

The mathematical program defined for the EVSP-LS is a non-linear model because multiple products of variables appear while defining auxiliary variables and constraints. Classical linearization techniques are implemented to formulate a mixed-integer linear program for the proposed EVSP, as detailed in Chapter 4 of MirHassani and Hooshmand (2019). In particular, the following new decision variables are introduced to represent the value of the products in the non-linear model.

- o_{is} to represent $y_{is}ca_{is}$
- p_{is} to define $y_{is}tc_{is}$
- $q_{i'is}$ standing for $y_{i's}ca_{i's}x_{i'i}$
- $r_{i'is}$ to linearize the product $y_{i's}x_{i'i}$
- $t_{i'is}$ to compute $x_{i'i}ca_{i's}$

In addition, the following inequalities are used to define the values of the linearization variables. Note that all variables are defined in terms of lower and upper bounds of the continuous/integer variables to express the products of variables. For example, the variable o_{is} with inequalities (A.1)–(A.3) represents the product of $y_{is}o_{is}$. Note that we include the following two scenarios: the variable o_{is} is 0 if the variable $y_{is} = 0$ is due to constraint (A.1); on the other hand, a variable $y_{is} = 1$ leads to inequalities $o_{is} \leq ca_{is}$ and $o_{is} \geq ca_{is}$, that is, $o_{is} = ca_{is}$, while constraint $o_{is} \leq 100$ is redundant because the variable o_{is} is a measure of battery charge (bounded by 100% of the battery level). The rest of the inequalities to define the linearization variables are analogous.

$$o_{is} \leq 100y_{is} \qquad \forall i \in I_0, s \in S \qquad (\text{A.1})$$

$$o_{is} \leq ca_{is} \qquad \forall i \in I_0, s \in S \qquad (\text{A.2})$$

$$o_{is} \geq ca_{is} - 100(1 - y_{is}) \quad \forall i \in I_0, s \in S \quad (\text{A.3})$$

$$p_{is} \leq My_{is} \quad \forall i \in I_0, s \in S \quad (\text{A.4})$$

$$p_{is} \leq tc_{is} \quad \forall i \in I_0, s \in S \quad (\text{A.5})$$

$$p_{is} \geq tc_{is} - M(1 - y_{is}) \quad \forall i \in I_0, s \in S \quad (\text{A.6})$$

$$q_{i'is} \leq 100y_{is} \quad \forall i', i \in I_0, s \in S \quad (\text{A.7})$$

$$q_{i'is} \leq 100x_{i'i} \quad \forall i', i \in I_0, s \in S \quad (\text{A.8})$$

$$q_{i'is} \leq ca_{is} \quad \forall i', i \in I_0, s \in S \quad (\text{A.9})$$

$$q_{i'is} \geq ca_{is} - 100(2 - y_{is} - x_{i'i}) \quad \forall i', i \in I_0, s \in S \quad (\text{A.10})$$

$$r_{i'is} \leq y_{is} \quad \forall i', i \in I_0, s \in S \quad (\text{A.11})$$

$$r_{i'is} \leq x_{i'i} \quad \forall i', i \in I_0, s \in S \quad (\text{A.12})$$

$$r_{i'is} \geq y_{is} + x_{i'i} - 1 \quad \forall i', i \in I_0, s \in S \quad (\text{A.13})$$

$$t_{i'is} \leq 100x_{i'i} \quad \forall i', i \in I_0, s \in S \quad (\text{A.14})$$

$$t_{i'is} \leq ca_{is} \quad \forall i', i \in I_0, s \in S \quad (\text{A.15})$$

$$t_{i'is} \geq ca_{is} - 100(1 - x_{i'i}) \quad \forall i', i \in I_0, s \in S \quad (\text{A.16})$$

Next, we replace the non-linear terms with the linearization variables to redefine the auxiliary variables et_i , u_{is} , ca_{is} and ic_i as follows:

- $et_i = dt_i + \sum_{s \in S} t_s + \sum_{s \in S} p_{is} + y_{is}\lambda$
- $u_{is} = dt_i + \sum_{s'=0}^{s-1} (t_{s'} + p_{is'} + y_{is'}\lambda)$
- $ca_{is} = ca_{i,s-1} - o_{i,s-1} + 100y_{i,s-1} - C_{s-1}$
- $ic_i = 100x_{0i} + \sum_{i' \in I} t_{i'i|S|} - q_{i'i|S|} + 100r_{i'i|S|} - C_{|S|}x_{i'i}$

Then, the mixed-integer linear program for the EVSP-LS is defined as follows:

$$\min \zeta_1 \left(\frac{\sum_{i \in I} x_{0i}}{|V|} \right) + \zeta_2 \left(\frac{\sum_{s \in S} z_s}{|S|} \right)$$

subject to:

$$(\text{A.1}) - (\text{A.16})$$

$$\sum_{i \in I} y_{is} \leq |I|z_s \quad \forall s \in S \quad (\text{A.1})$$

$$\sum_{i' \in I_0 - \{i\}} x_{i'i} = 1 \quad \forall i \in I \quad (\text{A.17})$$

$$\sum_{i \in I} x_{0i} \leq |V| \quad (\text{A.18})$$

$$g_{ii's} + g_{i'is} \leq 1 \quad \forall i, i' \in I, s \in S \quad (\text{A.19})$$

$$u_{i's} + p_{i's} + y_{i's}\lambda \leq u_{is} + M(3 - y_{is} - y_{i's} - g_{i'is}) \quad \forall i, i' \in I, s \in S \quad (\text{A.20})$$

$$et_{i'} \leq dt_i + M(1 - x_{i'i}) \quad \forall i', i \in I \quad (\text{A.21})$$

$$x_{ii'} + x_{i'i} \leq 1 \quad \forall i', i \in I \quad (\text{A.22})$$

$$ca_{is} - o_{is} + 100y_{is} - C_s \geq \beta \quad \forall s \in S, i \in I \quad (\text{A.23})$$

$$\sum_{i' \in I_0 - \{i\}} x_{i'i} - \sum_{i' \in I_0 - \{i\}} x_{ii'} = 0 \quad \forall i \in I \quad (\text{A.24})$$

$$\sum_{s \in S} p_{is} + \lambda y_{is} \leq \alpha \quad \forall i \in I \quad (\text{A.25})$$

$$\sum_{s \in S} y_{is} \leq n \quad \forall i \in I \quad (\text{A.26})$$

$$x_{i'i}, y_{is}, g_{ii's}, r_{i'is}, z_s \in \{0, 1\} \quad \forall i' \in I_0, i \in I, s \in S \quad (\text{A.2})$$

$$tc_{is}, et_i, u_{is}, ic_i, ca_{is}, o_{is}, p_{is}, q_{i'is}, r_{i'is} \in \mathbb{R}^+ \cup \{0\} \quad \forall i \in I, s \in S \quad (\text{A.3})$$

APPENDIX B

SENSITIVITY ANALYSIS FOR PARAMETER

$$\alpha$$

We perform a sensitivity analysis to evaluate the model’s response to variations in the parameter α , and we present the impact on key performance indicators, such as charging time, average recharging actions and percentage of station usage, previously discussed in detail in section 3.2.2.

A preliminary analysis of the EVSP model, presented in Section 3.2.2, was conducted using a random sample of 28 instances selected from a total of 70 available instances. In these instances, we obtain the same value of the objective function, but we evaluate the previously defined key performance indicators for different values of α , within a range of $\alpha = 0.01\text{MTT}$ to $\alpha = 0.10\text{MTT}$, with increments of 0.01. The selection of this range is justified within the context of scenarios that prioritize rapid recharging during vehicle routes. Values of α greater than 10% of the Maximum Travel Time (MTT) are deemed impractical, as the total time allocated for recharge would exceed the reasonable waiting time for users at the charging points, thus compromising the operational efficiency of the routes. Furthermore, it was observed that for values of α below 0.06MTT , some instances were infeasible. Consequently, the analysis focuses on the effective range of α , evaluating key performance indicators for the selected values. First, the total recharge time during trips is analyzed, as shown in Figure 26. The results indicate that these times remain relatively consistent in most values of α .

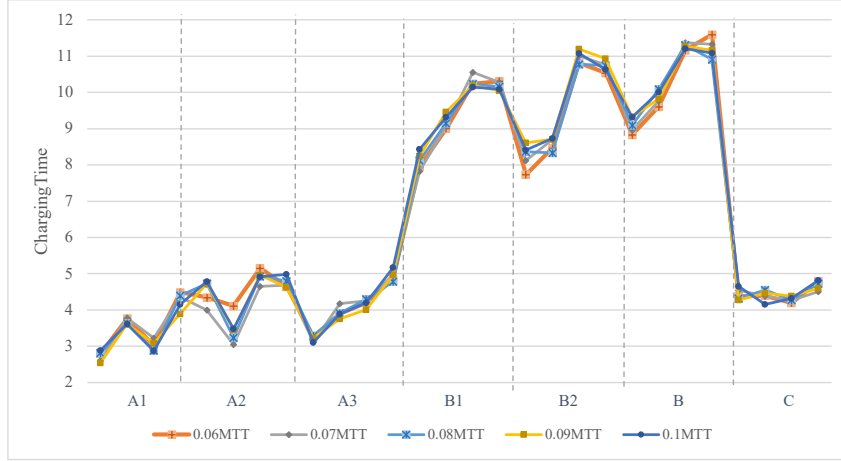


Figure 26: Charging times for different values of the parameter α

On the other hand, Figure 27 illustrates the average number of charging actions for the selected sample of instances. Unlike the previous indicator, this one exhibits greater variability with respect to changes in α . It should be noted that, in type B instances, characterized by longer travel distances, the indicator's behavior remains relatively consistent across all values of α . However, this is not the case when evaluating type A instances, since the parameter α has a more significant effect on the indicator of recharging actions. Specifically, using the smaller value of α ($\alpha = 0.06$ MTT), leads to a greater number of recharging actions.



Figure 27: Average recharging actions indicator for different values of α .

Finally, we analyze the indicator related to the use of charging stations. Figure 28 shows the average percentage of usage for each charging station at different values of α . It can be seen that the behavior of the charging action indicator along the route is very similar regardless of the chosen value α . It is important to note that the usage of station $S4$ increases significantly when α is set to 0.10MTT.

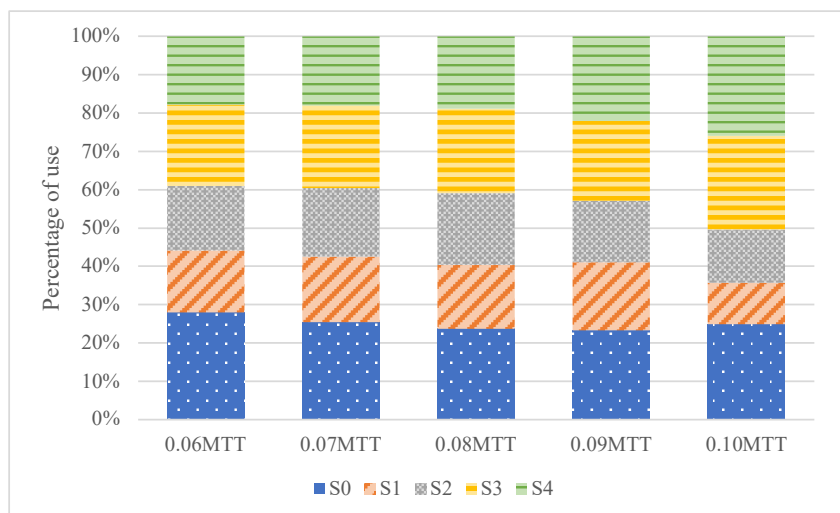


Figure 28: Average percentage of charging station usage for different values of α .

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