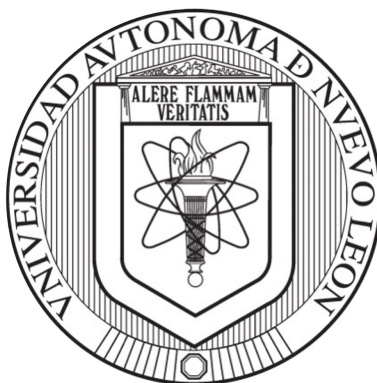


UNIVERSIDAD AUTÓNOMA DE NUEVO LEÓN
FACULTAD DE INGENIERÍA MECÁNICA Y ELÉCTRICA
DIVISIÓN DE ESTUDIOS DE POSGRADO



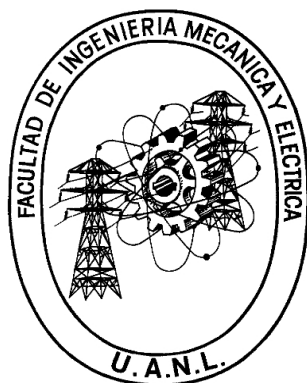
**IDENTIFICATION OF MODAL DYNAMIC PATTERNS IN
ELECTRICAL POWER SYSTEMS BASED ON
SYNCHROPHASOR DATA**

POR:
RODRIGO DAVID REYES DE LUNA

COMO REQUISITO PARA OBTENER EL GRADO DE:
DOCTOR EN INGENIERÍA ELÉCTRICA

SEPTIEMBRE 2026

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Posgrado

Los miembros del Comité de Evaluación de Tesis recomendamos que la tesis "Identificaction of modal dynamic patterns in electrical power systems based on synchrophasor data", realizada por el estudiante Rodrigo David Reyes de Luna, con número de matrícula 2082187, sea aceptada para su defensa como requisito parcial para obtener el grado de Doctorado en Ingeniería Eléctrica.

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*“If you want to go fast, go alone;
if you want to go far, go together.”*

— African proverb

Dedicatoria:

*A mi esposa **Jessi**, cuyo amor y apoyo incondicional han sido la fuerza que me sostuvo en cada paso de este camino. Gracias por tu paciencia y por estar siempre conmigo.*

*A mis hijas **Caro** y **Vale**, que con su amor y alegría me impulsaron a continuar esforzándome. Ustedes son mi inspiración y mi mayor orgullo.*

*Y a mis padres **Román** y **Margarita**, con gratitud infinita por todo lo que me dieron, por su amor, sus enseñanzas y su sacrificio. Este logro también es suyo, porque sin ustedes nada de esto habría sido posible.*

Abstract

Identification of Modal Dynamic Patterns in Electrical Power Systems based on Synchrophasor Data

Publication No. _____

Rodrigo David Reyes de Luna

Universidad Autónoma de Nuevo León

Facultad de Ingeniería Mecánica y Eléctrica

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Real-time monitoring of electromechanical oscillations in electrical grids is essential for maintaining optimal operational control and making informed decisions in adverse situations. The maintenance of system safety, reliability, stability, and continuity is of utmost importance in control centers. This requires system observability at as many points as possible and an appropriate algorithm that indicates the system's status. This can be achieved through synchrophasor measurement units (PMUs), which are connected to current and potential transformers in substations and measure electrical parameters such as voltage, current, angles, active power, reactive power, etc. These values are stored with their timestamp in a server, which allows the user to perform offline and real-time queries. Once these electrical parameters are available, it is possible to conduct studies on the system's status and behavior, enabling accurate decisions and keeping the system safe.

There are several methods and algorithms for system evaluation through signal reading or digital signal processing. A very important form of system evaluation is obtaining and analyzing signals after an event or disturbance, also called Ringdown event, this is because the system is excited, and mode extraction is more feasible through a mathematical algorithm. By extracting oscillation modes, it is possible to extract dynamic patterns and analyze the signal and, consequently, the system's behavior.

Mathematical algorithms include linear, nonlinear, and filter-based methods. Each has its differences, advantages, and disadvantages, which will be described in this work.

This research proposes two filter-based approaches with cosine and Bessel functions to identify modal patterns, such as damping and frequency, from multivariate and multi-modal oscillation signals from single and multiple PMUs and locations. The key idea of this proposal is to introduce filtering based on the first kind of Bessel functions (J_0) and cosine functions to perform a time-frequency analysis that allows for the modal identification of nonlinear signals that arise after a disturbance in the network.

Therefore, this research develops the theoretical foundations of both filtering approaches, forming a biorthonormal pair of bases that support their analysis and synthesis equations. It is also proposed to optimize the filters individually to obtain the mono-components in the optimal way possible and thus avoid undesirable frequencies in the mono-component signals. Mono-component signals are subsequently extracted using spectral analysis, and their modal patterns are extracted using the Hilbert transform. To optimally obtain the highest energy mono-component signals or the most excited signals, optimal sampling is proposed to ignore mono-components with undesirable frequencies.

Numerical experiments performed on synthetic oscillating signals with damped and poorly damped oscillations, simulated signals extracted from a known power system, and real data from synchrophasors captured by the Mexican WAMS demonstrates the applicability of the proposal. The results show high accuracy and relatively low computation time compared to other methods.

The goal of this work is to propose an alternative for extracting modal parameters from time-stamped data series from PMUs for power system analysis. This analysis

can be performed offline or in real time; therefore, the result delivery time must be very fast and reliable. It also demonstrates a substantial improvement in computing time, which allows it to be an online tool. This work demonstrates that the proposed algorithm meets these two requirements for implementation in control centers.

Resumen

El monitoreo en tiempo real de oscilaciones electromecánicas en los sistemas eléctricos es esencial para mantener un control operativo óptimo y tomar decisiones fundamentadas ante situaciones adversas. En los centros de control, la preservación de la seguridad, confiabilidad, estabilidad y continuidad del sistema es de suma importancia. Para lograrlo, es necesario disponer de la máxima observabilidad posible del sistema en diversos puntos, así como de un algoritmo adecuado que indique el estado del mismo. Esta observabilidad puede alcanzarse mediante el uso de unidades de medición sincrofasorial (PMUs, por sus siglas en inglés), las cuales se conectan a transformadores de corriente y de potencial en subestaciones para medir parámetros eléctricos como voltaje, corriente, ángulos, potencia activa, potencia reactiva, entre otros. Estos valores son almacenados junto con su estampa de tiempo en un concentrador, lo que permite al usuario realizar consultas tanto en tiempo real como de forma diferida. Una vez que se dispone de estos parámetros eléctricos, es posible realizar estudios sobre el estado y comportamiento del sistema, lo que permite tomar decisiones precisas y mantener la seguridad operativa del mismo.

Existen diversos métodos y algoritmos para la evaluación del sistema mediante la lectura de señales o el procesamiento digital de señales. Una forma particularmente relevante de evaluación consiste en obtener y analizar señales posteriores a un evento o perturbación importante, conocido como evento Ringdown. En este contexto, el sistema se encuentra excitado, lo cual facilita la extracción de los modos de oscilación mediante un algoritmo matemático. Al extraer estos modos, es posible identificar patrones dinámicos y analizar la señal, y, por ende, el comportamiento del sistema.

Los algoritmos matemáticos empleados incluyen métodos lineales, no lineales y basados en filtros. Cada uno de estos enfoques presenta diferencias, ventajas y desventajas que serán descritas a lo largo del presente trabajo.

Esta investigación propone dos enfoques basados en filtros utilizando funciones coseno y funciones de Bessel para la identificación de patrones modales, tales como la frecuencia y el amortiguamiento, a partir de señales oscilatorias multivariadas y multimodales provenientes de uno o varios PMUs ubicados en distintos puntos del mismo sistema. La idea central de esta propuesta consiste en introducir un proceso de filtrado basado en funciones de Bessel de primera especie (J_0) y funciones coseno escaladas, para llevar a cabo un análisis tiempo-frecuencia que permita la identificación modal de señales no lineales que emergen tras una perturbación en la red.

Este trabajo desarrolla los fundamentos teóricos de ambos enfoques de filtrado, formando un par de bases biortonormales que sustentan las ecuaciones de análisis y síntesis. Asimismo, se propone la optimización individual de los filtros con el fin de obtener señales mono-componente de manera óptima, evitando la presencia de frecuencias no deseadas. Las señales mono-componente se extraen posteriormente mediante análisis espectral, y sus patrones modales se determinan utilizando la transformada de Hilbert. Para obtener de forma óptima las señales mono-componente con mayor energía o las más excitadas, se plantea un muestreo óptimo que permite ignorar aquellas componentes con frecuencias no deseadas.

Los experimentos numéricos realizados sobre señales oscilatorias sintéticas con amortiguamiento fuerte y débil, señales simuladas extraídas de un sistema eléctrico conocido, así como datos reales provenientes de sincrofasores del sistema WAMS de México, demuestran la aplicabilidad de la propuesta. Los resultados muestran una alta precisión y un tiempo de cómputo relativamente bajo en comparación con otros métodos existentes.

El objetivo de este trabajo es proponer una alternativa para la extracción de parámetros modales a partir de series de datos con su respectiva estampa de tiempo proporcionadas por los PMUs, con fines de análisis del sistema eléctrico de potencia. Este análisis puede realizarse tanto en modo diferido como en tiempo real; por lo tanto, el tiempo de entrega de resultados debe ser rápido y confiable. Además, se demuestra una mejora sustancial en el tiempo de cómputo, lo cual permite su uso como herramienta en línea. Se demostrará que el algoritmo propuesto cumple con ambos requisitos para su implementación en centros de control.

Contents

Acknowledgments	iii
Abstract	vii
Resumen	ix
List of tables	xiii
List of figures	xvi
Acronyms	xvii
Nomenclature	xviii
1 Introduction	1
1.1 Literature Review	2
1.2 Problem statement	5
1.3 Hypothesis	5
1.4 Objectives	6
1.4.1 General objective	6
1.4.2 Specific objectives	6

1.5	Contributions	7
1.6	Outline of the dissertation	8
2	Mathematical Foundations	10
2.1	Cosine filter	10
2.2	Bessel filter	13
2.2.1	Bessel functions	13
2.2.2	First kind J_0 Bessel filter	15
2.3	Methods for multiple channels	18
2.4	Optimal calculation of parameter L	19
2.5	Modal pattern extraction	20
2.5.1	Amplitude and Phase	21
2.5.2	Frequency estimation	21
2.5.3	Damping and Damping Ratio estimation	22
2.6	Single Exponential Smoothing	22
2.7	Signal Decomposition	23
2.8	Conclusions	24
3	Application to synthetic signals	26
3.1	Single-channel multi-mode damped case	26
3.2	Single-channel multi-mode undamped Case	33
3.3	Multi-channel multi-mode damped case	35
3.4	Conclusions	38

4	Application to Power Systems Oscillations	40
4.1	The NETS-NYPS 68-bus system	40
4.2	Real event - Mexican system	43
4.3	Discussion	47
4.3.1	Noise Tolerance	47
4.3.2	Computational analysis	48
4.4	Conclusions	50
5	Conclusions, future work and products	51
5.1	Conclusions	51
5.2	Future work	53
5.3	Products derived from this work	54
5.3.1	Journals	54
5.3.2	Conference	55
5.3.3	Projects	55
5.3.4	Technical report	55
A	Eigensystem Realization Algorithm	56
A.1	Introduction	56
A.1.1	Eigensystem Realization Algorithm for single-channel	56
A.1.2	Eigensystem Realization Algorithm for multiple-channel	58
	Bibliography	58

List of Tables

3.1	Parameters for multiple synthetic mixed signals in s_a (multivariate analysis).	27
3.2	Analysis through cosine filters, Bessel filters, and ERA in single-channel multi-mode damped case. Frequency and damping values are represented by their average value.	32
3.3	Parameters for multiple synthetic mixed oscillating signals in s_b (Multivariate analysis).	34
3.4	Analysis through cosine filters, Bessel filters, and ERA in single-channel multi-mode undamped case. Frequency and damping values are represented by their average values.	36
3.5	Parameters of multiple synthetic mixed signals (Multichannel analysis).	36
3.6	Analysis through cosine filters, Bessel filters, and ERA for multichannel multi-mode damped case. Frequency and damping values are represented by their average value.	38
4.1	Multivariate analysis through Bessel filter and cosine filters, and ERA for multiple channels in the NETS-NYPS grid. Frequency and damping values are represented by their average values.	43
4.2	Multivariate analysis through cosine and Bessel filters, and ERA for multiple channels in the Mexican NIS. Frequency and damping values are represented by their average values.	46

List of Figures

2.1	FIR filters corresponding to the cosine filter for decomposing oscillating signals.	13
2.2	(a) Oscillating signal under study. (b) k mono-component signals obtained from the filter bank.	13
2.3	Bessel functions. (a) The first five functions for $v = 0, 1, 2, 3, 4$. (b) Their Fourier spectra.	16
2.4	Bessel filter for decomposing oscillating signals. (a) Scaled Bessel functions. (b) Their corresponding FIR filters.	17
2.5	Calculation of parameter L . (a) Energy per mono-component, and (b) sum of energy per mono-component in m modes.	20
2.6	Flowchart for the optimal calculation of the parameter L	21
2.7	Flowchart of the general procedure of the proposed strategy.	24
3.1	(a) Theoretical oscillating signal s_a . (b) The Fourier spectrum of s_a . .	28
3.2	Percentage of energy in 3 modes as a function of L	28
3.3	The first 15 compressed basis functions of: (a) Bessel filter; and (b) cosine filter.	29
3.4	The first 15 filter frequencies of: (a) Bessel biorthonormal basis; and (b) cosine biorthonormal basis.	29

3.5	Plots (a1) and (b1) respectively illustrate 15 and 3 mono-components of the signal s_a by processing with the Bessel filter bank. Plots (a2) and (b2) represent 15 and 3 mono-components of the signal s_a by decomposing with the cosine filter bank.	30
3.6	Percentage of energy per mode with $L_{Bessel} = 619$ and $L_{cosine} = 626$. .	31
3.7	Three modes by Bessel/cosine-Filter calculated by Hilbert.	31
3.8	Modal pattern extraction via Bessel filter bank. (a) Instantaneous frequency estimates $\hat{f}_k(t)$. (b) Instantaneous damping estimates $\hat{\sigma}_k(t)$. .	32
3.9	Modal pattern extraction via cosine filter bank. (a) Instantaneous frequency estimates $\hat{f}_k(t)$. (b) Instantaneous damping estimates $\hat{\sigma}_k(t)$. .	33
3.10	(a) Theoretical signal s_b . (b) Its frequency spectrum	34
3.11	Three mono-components obtained by: (a) Bessel-filter; and (b) cosine-filter.	35
3.12	(a) Multiple channel measurements rendered by synthetic signals. (b) Their frequency spectra.	37
3.13	Multi-channel analysis performed by the Bessel filter bank. (a) Mono-components. (b) Their frequency spectra.	37
4.1	Test system: 16-machine and 68-bus New-England power system. . .	41
4.2	Frequency deviations during ringdown at buses 2 to 5. (a) Waveforms, and (b) their Fourier spectra.	42
4.3	Frequency measurements captured by 5 FDRs in the Mexican National Interconnected System.	45
4.4	(a) Analysis window (Ringdown period selected), and (b) their Fourier spectra.	45
4.5	Sensitivity analysis for different Gaussian noise levels. (a) for single-channel multi-mode damped case and (b) for undamped case of Chapter 3.	48

4.6	Time cost analysis of the cosine and Bessel-based filters and the ERA method for different Gaussian noise levels applied to the same signal.	49
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Acronyms

ANF	Adaptive Notch Filter
CWT	Continuous Wavelet Transform
DFT	Discrete Fourier Transform
DTFT	Digital Taylor Fourier Transform
ERA	Eigensystem Realization Algorithm
EWT	Empirical Wavelet Transform
FBSE	Fourier Bessel Series Expansion
FFT	Fast Fourier Transform
FIR	Finite Impulse Response
FLL	Frequency Locked Loop
FOSL	Forced Oscillation Source Localization
HHT	Hilbert Huang Transform
HT	Hilbert Transformer
KF	Kalman Filter
LFEO	Low Frequency Electromechanical Oscillations
MP	Matrix Pencil
NIS	National Interconnected System
OMD	Optimal Mode Decomposition
OSG	Orthogonal Signal Generator
PMU	Phasor Measurement Unit
SNR	Signal Noise Ratio
SOGI	Second Order Generalized Integrator
SVD	Singular Value Decomposition
VMD	Variational Mode Decomposition
WAMS	Wide Area Monitoring System
WGN	Wight Gaussian Noise
WT	Wavelet Transform

Nomenclature

$\bar{z}$	Complex conjugate of z
$\hat{x}$	Estimation of parameter x
$!$	Factorial
J_0	First kind Bessel function
Γ	Gamma function
$\mathbf{I}$	Identity matrix
Im	Imaginary number
j	Imaginary unit
$\mathbf{M}^\dagger$	Pseudoinverse of matrix M
Re	Real number
$\mathbf{M}^T$	Transposed of M
σ_v^2	Variance of input noise

Chapter 1

Introduction

Electrical power systems are always subject to adversities, such as tripping of transmission lines or any primary equipment, sudden shutdown of generation plants, changes in network topology, or even poor tuning of the generator controls. Additionally, the massive integration of non-inertia-providing renewable energy sources to the system makes the electrical system increasingly vulnerable [1]. All these conditions must be identified to timely know the possible risks that expose the system to harmful conditions. Monitoring the status of the electrical system at all times allows operators and analysts to generate situational awareness to assess the security and stability of the system under any condition at any time. Thus, real-time system monitoring is imperative to maintain operational control in an optimal state in any adverse situation [2].

Currently, Wide Area Monitoring Systems (WAMS), based on synchronized Phasor Measurement Units (PMU) that are connected to voltage and current transformers in substations, allow the real-time reading of electrical parameters, providing important information for the system's observability. These synchrophasors feature the same time stamp, making it possible to compare data obtained from any PMU. WAMS platforms are also rich in historical data, since they concentrate and store all phasor information on servers and make it user-available to carry out offline studies [3].

In control centers, it is of utmost importance to be aware of the power system's dynamic state at all times, especially following a major disturbance. It is essential to take timely actions and avoid endangering the electrical system. The WAMS

tool provides real-time raw data about system conditions. Raw data are signals that initially offer limited information; digital signal processing transforms them into valuable insights for understanding the state of the system. This dissertation proposes a novel methodology for the identification of dynamic modal patterns from synchrophasor data, which provides critical information to control centers for the proper operation and management of the power system.

1.1 Literature Review

In power systems, two perspectives are used to analyze low-frequency oscillations, these are model-based and measurement-based approaches [4]. Although an electrical system has an inherently nonlinear behavior, both strategies linearize around an operating point to model the response of the electrical system [2].

In the model-based approach, the procedure involves constructing the state-space equations for the power system and then linearizing them at a specific operating point [5], [6]. From the linearized model, eigenvalues are obtained to assess the dynamic modal parameters of the power oscillations. This strategy is more focused on offline studies, being a more suitable option for medium and long-term studies or power system planning.

In the measurement-based approach, the procedure is through direct measurements from the PMUs, getting data of electrical parameters with their time-stamp instantly [4, 7, 8, 9, 10, 11, 12, 13]. Such measurements are evaluated in their ringdown time window (oscillatory response after a disturbance), which uses different digital signal processing algorithms to extract modal information (amplitude, frequency, damping ratio, and phase), and it is more suitable for online analysis [14, 15, 16, 17]. This perspective analyzes the dynamic state of the system through the ringdown response of the system and considers the system as a black box without access to its information. In this way, the power system can be monitored by analyzing its measurements without knowing the system model itself.

Within digital signal processing, there are different methods such as linear, non-linear, and filtering. Among linear methods, there are Prony, eigensystem realization algorithm, matrix pencil [18, 19, 20, 21], Total Least Square-Estimation of Signal Parameters via Rotational Invariance Techniques (TLS-ESPRIT) [22, 23], Optimal

Mode Decomposition (OMD) [24], which have been widely used and tested with good results. However, to obtain accurate results, the computational burden becomes large due to the large size of the matrices that they use. Another drawback is that the more precise the result is required, the more sensitive it becomes to signal noise. To prevent this, a low energy threshold is considered in the case of ERA and MP, and the incorporation of a Savitzky-Golay filter mitigates noise sensitivity in the TLS-ESPRIT performance [25].

Nonlinear methods such as Hilbert Huang transform, phase-locked loop, [26, 27, 28, 29, 30, 31, 32, 33, 34], Empirical Mode Decomposition (EMD) [35], and Damped Second Order Generalized Integrator (Damped-SOGI) [36], are known for their robustness, precision, and low computational burden. Nevertheless, they have disadvantages such as noise sensitivity, complex implementation, and unpredictable results when multiple electromechanical modes are present. The Damped-SOGI method directly estimates magnitude, frequency, and damping of electromechanical oscillations in a power system [37]. This is built by a block for orthogonal signal generator (OSG) which is used in a direct integration for adapting the center frequency of SOGI and this has resulted in the SOGI-Frequency-Locked Loop (FLL) and the Adaptive Notch Filter (ANF). However, its accuracy may be unpredictable with scenarios involving several modes.

Another widely used and studied methodology for signal processing is based on filters for analyzing power system low-frequency electromechanical oscillations (LFEOs). In [38], the authors focus on the study of LFEOs through a filter bank based on Zolotarev polynomials, in which, a filter bank designed for the decomposition of the signal coming from PMU is proposed based on Zolotarev polynomials to decompose the signal into mono-component signals.

In [39] and [40], the authors provide a different sort of filter based on the continuous wavelet transform (CWT). The wavelet transform (WT), especially, creates an alternative tool to face with non-stationary analysis of LFEOs since its multi-resolution property is very useful for recognizing parameters of a signal in both time and frequency domains. This algorithm consists of the use of a suitable formulation of the mother-wavelet function to improve the time-frequency resolution. Also in [41], authors apply a data-driven approach to power systems for Forced Oscillation Source Localization (FOSL), using the Synchronized Compression Wavelet Transform (SCWT), a time-frequency domain-based method, and a frequency domain.

Compared with the CWT method, this method improves the accuracy of forced oscillation frequency identification. Moreover, the method can provide multidimensional FOSL information on the power system.

Recently, in [42], the authors developed the digital Taylor-Fourier transform (DTFT), where the algorithm lies in the inclusion of a Taylor polynomial of finite degree in the signal model, creating a decomposition of the signal by spectral analysis using a filter bank, and providing information on its instantaneous damping and frequency. In [43] and [44], the method is implemented by incorporating a Taylor-Fourier filter bank designed with O-splines. The O-splines are derived from the discrete-time Taylor-Fourier transform (DTTFT), considerably reducing the computational load.

Another technique is the Fourier-Bessel method for digital signal processing. In [45], the author presents the advantages over the Fourier transform (FT) in the analysis of non-stationary signals. These advantages are both in the results' accuracy as well as in computational time. In [46], the decomposition of a signal is presented in terms of an expansion of the orthogonal Fourier-Bessel series. Recently, in [47], a new method for time-frequency (TF) representation of non-stationary signals has been presented, where the empirical wavelet transform (EWT) has been enhanced using Fourier-Bessel series expansion (FBSE) to obtain improved TF representation of non-stationary signals. The FBSE method has been used for the spectral representation of the multi-component signals, achieving a good frequency resolution. Then, the wavelet-based filter banks have been generated to decompose non-stationary multi-component signals into narrow-band components. Finally, the normalized Hilbert transform has been applied for the estimation of amplitude envelope and instantaneous frequency functions from the narrow-band components and the TF representation of the analyzed non-stationary signal.

Another recent advancement involves hybrid methods, such as variational mode decomposition (VMD) combined with machine learning [48]. This proposal combines machine learning online optimization to select the optimal parameters, requiring more computational resources than the traditional VMD method in [49].

1.2 Problem statement

Due to the connection and disconnection of transmission lines and transformers, sudden consumption in smelting plants, as well as various unscheduled conditions such as tripping of primary equipment (transmission lines, transformers, and generators), power systems are exposed to various threats that alter their operating condition. All of this can result in changes in the system's state. These changes undergo a transient period known as power oscillations until they reach an equilibrium or steady state. However, this is not always the case. The system may not reach a stable state and oscillate, damaging equipment and threatening the system's safety and reliability.

Low-frequency electromechanical oscillations are very often in power systems; however, when these are poorly damped or even negatively damped, they indicate a serious threat to the stability and safety of the grid, since they can lead to: damage electrical generators, reduce power exchange in tie-lines, form islands, or even worse, the total system collapse [50, 51].

Currently, there are different digital signal processing methods for capturing modal patterns from PMU data. All methods have advantages and disadvantages, where the most common and used approaches in research are: linear methods such as Prony, Eigensystem Realization Algorithm (ERA) and Matrix Pencil (MP) [52, 53, 54, 55]; nonlinear methods such as Hilbert Huang transform (HHT) and Variational Mode Decomposition (VMD) [49, 56, 57]; and filtering approaches such as Wavelets, Taylor-Fourier Transform, [39, 42, 58]. The problem with linear methods is that the large amount of processing required to work with large matrices results in a high computational cost to obtain accurate solutions. In the case of filtering methods, it is necessary to know the frequencies involved in each case, and to adjust the filter banks to extract the signals of interest.

1.3 Hypothesis

The proposed filter-based approaches, utilizing cosine and Bessel functions, enable the accurate and efficient decomposition of the signal into mono-components to extract modal parameters from PMU time-series data, outperforming traditional methods in terms of both accuracy and computational efficiency for both real-time and

offline power system analysis. These filter-based approaches maintain good accuracy but also significantly improve the computational efficiency for the modal parameter extraction from PMU data compared to existing methods.

1.4 Objectives

1.4.1 General objective

To develop a methodology for wide-area oscillation monitoring of large power systems through a ringdown analysis using digital filters, such as cosine-filter and Bessel-filter.

1.4.2 Specific objectives

- To propose a generalized strategy for recognizing the state of the power system using synchrophasor measurements.
- To detect electromechanical modes with poor or negative damping in the initial phase of an event by reading power oscillation signals. This strategy focuses on time-frequency analysis and is related to ringdown analysis using Bessel and cosine filters.
- To establish the mathematical formulation for obtaining single-component signals using Bessel and cosine filters.
- To formulate the Bessel and cosine filter algorithms to allow simultaneous analysis of multiple signals, generating a set of estimated modes.
- To validate and evaluate the performance of the proposed methods using simulated signals and measured data from a real power system for the detection and identification of electromechanical modes using single and multiple channels.

1.5 Contributions

This research aims to propose two filtering approaches to identify modal patterns (damping and frequency) from multivariate and multi-modal oscillation signals stemming from multiple PMUs and locations. The key idea behind this proposal lies in introducing the cosine and Bessel-based filtering to conduct a time-frequency analysis for dealing with the modal identification of nonlinear signals that arise after disturbance in the grid.

Basis functions of the Bessel and cosine types are adopted to build a set of digital finite impulse response (FIR) bandpass filters that are designed with equidistant frequencies to take advantage of the biorthonormal pair of bases to decompose oscillating power signals into individual components using the least squares method. These filters are stable because they do not have feedback loops.

The motivation behind selecting Bessel and cosine functions for the design of digital FIR filters emerged as an alternative filtering approach for ringdown analysis in power systems. The similarity between power oscillations (low-frequency and typically damped) and zero-order Bessel functions (J_0) was a key factor in this choice, as both exhibit comparable characteristics [45]. For the cosine-based functions, they were initially considered due to their filtering properties in commercial protection relays [59].

This research develops the theoretical foundations of both filtering perspectives, forming a biorthonormal pair of bases that support their analysis and synthesis equations. Then, mono-component signals are extracted by spectral analysis, and their modal patterns are extracted via Hilbert transform. Numerical experiments conducted on synthetic oscillating signals, simulated-based signals extracted from a well-known power grid, and synchrophasors captured by the Mexican WAMS, confirm the applicability of the proposal to actual power grids. The major contributions of this work are, as follows:

1. The adaptation of first-kind Bessel functions with defined frequencies and scaled cosine functions to create a filter bank for decomposing low-frequency electromechanical oscillations.
2. The implementation of the biorthonormal pair of bases based on Bessel/cosine

functions to segregate multivariate and multi-modal oscillating signals into mono-components for the extraction of modal patterns.

3. The proposed filtering is static and valid for the frequency range of concern, in which low-frequency electromechanical oscillations appear, and no frequency adjustments are required or previous knowledge of the modes' frequencies, preventing a pre-calculation step and saving computational time.
4. The optimization of the filter bank is achieved by correctly selecting the number of samples (L) of the biorthonormal pair of bases functions to separate the mono-components of interest from noise or undesirable components.
5. The proposed strategies are extensively assessed under noisy conditions through simulated signals and measured data from a real power system.
6. The proposal of these filters for obtaining power oscillation modes focuses on monitoring electrical systems from synchrophasor measurements, with different damping levels (low, medium and high) and a short analysis window from real PMU signals, which are quickly processed by the algorithm, as well as the emission of results.

1.6 Outline of the dissertation

This section presents the order and a brief description of each chapter of this thesis.

Chapter 1 presents the Literature Review, the problem statement, the Hypothesis, the objectives, and the contributions of this work.

Chapter 2 develops the mathematical formulation of cosine and Bessel filters, as well as the optimal way to obtain mono-component signals through these filters. The objective is to show the basis functions of these filters and how the matrices are generated for single and multi-channel, as well as the optimal sampling of the filters to excite more the frequencies of interest of the signals and obtain optimal results. The extraction of modal parameters is performed using Hilbert.

Chapter 3 presents the application of cosine and Bessel filters to decompose a synthetic signal and identify low-frequency oscillation modes with different damping characteristics. The option of multiple channels for ringdown analysis is presented.

The objective is to develop an oscillation monitoring system based on wide-area measurements from PMUs to detect electromechanical modes with poor or negative damping in power systems. In this chapter, synthetic signals with known modal parameters are presented, which are compared with the results of other well-known linear methods to compare the accuracy of results and the computation time, in which advantages are observed in the filtering methods, especially in the computation time.

Chapter 4 describes the application of the proposed methods for oscillation mode analysis using multiple PMU measurements through multidimensional analysis. The accuracy and efficiency of the proposed methods are tested using simulated and measured data from real power systems. They offer a significant advantage over other methods: static filters, as they do not require prior knowledge of the frequencies embedded in the signal. Ringdown signals obtained from power system simulations, as well as real signals from an event in the Mexican electrical system, are multi-signal, in which modal parameters are obtained using filters and compared with other methods, yielding good results. Signals under Gaussian noise conditions and their effect on the proposed method are also analyzed. In this chapter, the advantages of the filtering methods proposed in this thesis are demonstrated in depth.

Finally, Chapter 5 summarizes conclusions, future research, and the products derived from this work.

Chapter 2

Mathematical Foundations

This section is devoted to explaining the key mathematical findings related to cosine and Bessel filters that help us characterize the modal dynamic parameters of oscillating signals from synchrophasor data of electric power systems.

Both cosine and Bessel filters are given by a biorthonormal pair of bases; therefore, they are obtained via the inverse of the Gram matrix. They are used as resonators at equally spaced central frequencies. Then, they are designed to analyze multivariate oscillating signals through multi-channel analysis that obtains mono-components. Finally, the oscillation modal parameters are extracted from the cosine and Bessel's coefficients by applying the Hilbert transform [42], and they are also post-processed by exponential smoothing [60].

2.1 Cosine filter

A cosine filter bank is proposed to decompose a multi-mode oscillating signal into a set of cosine functions of the form:

$$\begin{aligned} s_1(t) &= A_1 \cos(2\pi f_1 t), \\ s_2(t) &= A_2 \cos(2\pi f_2 t), \\ &\vdots \\ s_k(t) &= A_k \cos(2\pi f_k t), \end{aligned} \tag{2.1}$$

sinusoidal signals in which the frequencies $f_1, f_2, \dots, f_k$ are equally spaced in the following frequency range $[f_1, f_k]$, where $f_1 = f_{min}$ and k is the number of modes initially specified, where k is chosen such that the highest frequency corresponds to f_{max} . This range of frequencies is defined by the range of frequencies known in low-frequency oscillations in power systems, which is $[f_{min} = 0.1, f_{max} = 2.0]$ Hz.

Considering the length of the filters equal to one cycle of the lower frequency $f_1 = 0.1\text{Hz}$, so the filter length must be at least $T_{min} = f_{min}^{-1} = 10\text{s}$, and a cycle is sampled with L_{min} samples, then we have the following cosine sequences from (2.1), if their central frequency is also separated by f_1 Hz:

$$\begin{aligned} s_1[n] &= A_1 \cos[2\pi f_1 n], \\ s_2[n] &= A_2 \cos[2\pi f_2 n], \\ &\vdots \\ s_k[n] &= A_k \cos[2\pi f_k n], \end{aligned} \tag{2.2}$$

where $n = \{n_1, n_2, \dots, n_L\}$ represents a discrete-time vector of L_{min} samples uniformly distributed over the interval $[-\frac{T_{min}}{2}, \frac{T_{min}}{2}]$.

For this investigation, the minimum signal length (L_{min}) must obey the following rule:

$$L_{min} = \frac{2f_s}{f_{min}}. \tag{2.3}$$

where f_s is the raw sampling frequency.

These scaled cosine functions are the vector basis of our oscillating signal model. They can be put in the following matrix $\mathbf{C}$.

$$\mathbf{C}_{k,n} = \begin{bmatrix} \cos[2\pi f_1 n_1] & \cos[2\pi f_1 n_2] & \dots & \cos[2\pi f_1 n_L] \\ \cos[2\pi f_2 n_1] & \cos[2\pi f_2 n_2] & \dots & \cos[2\pi f_2 n_L] \\ \vdots & \vdots & \ddots & \vdots \\ \cos[2\pi f_k n_1] & \cos[2\pi f_k n_2] & \dots & \cos[2\pi f_k n_L] \end{bmatrix}. \tag{2.4}$$

The filter bank for an oscillating signal model can be obtained in the rows of the pseudoinverse matrix of $\mathbf{C}$, since the elements of the vector basis are not orthonormal.

Then, we have:

$$\mathbf{C}^\dagger = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T, \quad (2.5)$$

where rows in $\mathbf{C}^\dagger$ are the elements of the biorthonormal basis. Then, the cosine-filter coefficients ($\hat{\xi}$) are obtained using the least-squares solution, as follows:

$$\hat{\xi} = \mathbf{C}^\dagger \mathbf{s}, \quad (2.6)$$

where $\mathbf{s}$ consists of discrete signal samples. Then, the analysis and synthesis equations become (2.8) and (2.7), respectively:

$$\hat{\xi} = \mathbf{C}^\dagger \mathbf{s}, \quad (2.7)$$

$$\hat{\mathbf{s}} = \mathbf{C} \hat{\xi}, \quad (2.8)$$

where matrix $\mathbf{C}$ is the cosine matrix and $\mathbf{C}^\dagger$ is its pseudo-inverse, whose rows contain the impulse response of the filter bank, $\hat{\xi}$ are the estimated coefficients. Each coefficient enables to reconstruct a mono-component signal by means of the synthesis equation (2.8). For each complex mode $\hat{s}_k$, it is possible to calculate their real and imaginary parts, such as ($\mathbb{R}\{\hat{s}\}$ and $\mathbb{I}\{\hat{s}\}$). The signal reconstruction ($\hat{\mathbf{s}}$) for each mode is made by using (2.8), such as:

$$\begin{aligned} \hat{\mathbf{s}}_0 &= \mathbf{C}_0 \hat{\xi}_0, \\ \hat{\mathbf{s}}_1 &= \mathbf{C}_1 \hat{\xi}_1, \\ &\vdots \\ \hat{\mathbf{s}}_k &= \mathbf{C}_k \hat{\xi}_k, \end{aligned} \quad (2.9)$$

where k mono-components are obtained from the maximum spectral components of the oscillations. To achieve this, cosine functions in (2.2) are assembled into the scaling matrix in (2.4) that is orthonormalized by (2.5) to form the biorthonormal base of the FIR filters, whose frequency responses are displayed in Fig. 2.1.

Figure 2.2 depicts the signal's decomposition $s(t)$ into k mono-components, where each signal $\hat{s}_k(t)$ contains a unique frequency and damping to obtain modal parameters subsequently, as describes in Section 2.5. Essentially, the cosine filter decomposes the oscillating signal into mono-components to study each signal as its oscillatory modes.

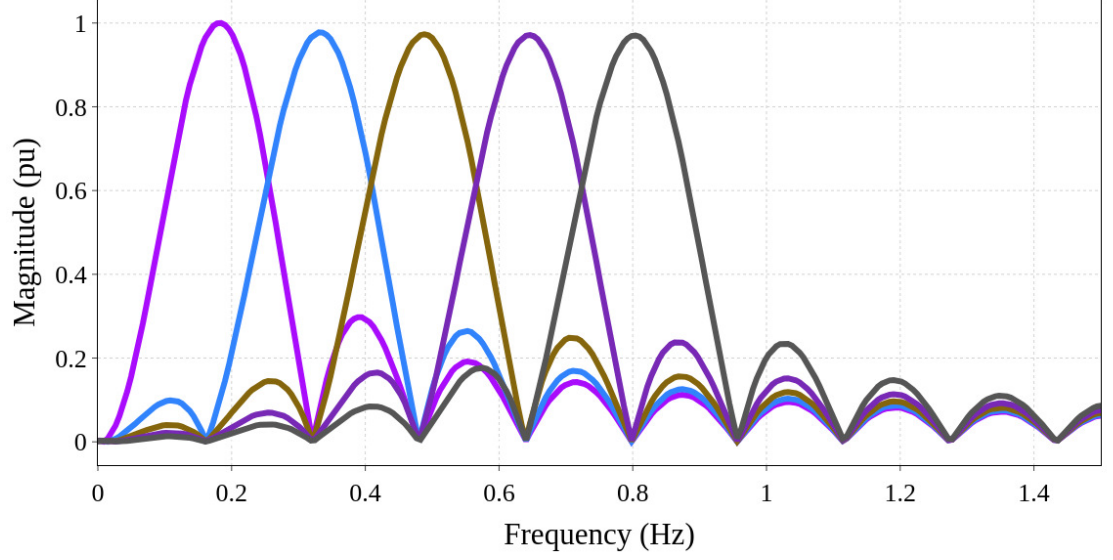


Figure 2.1: FIR filters corresponding to the cosine filter for decomposing oscillating signals.

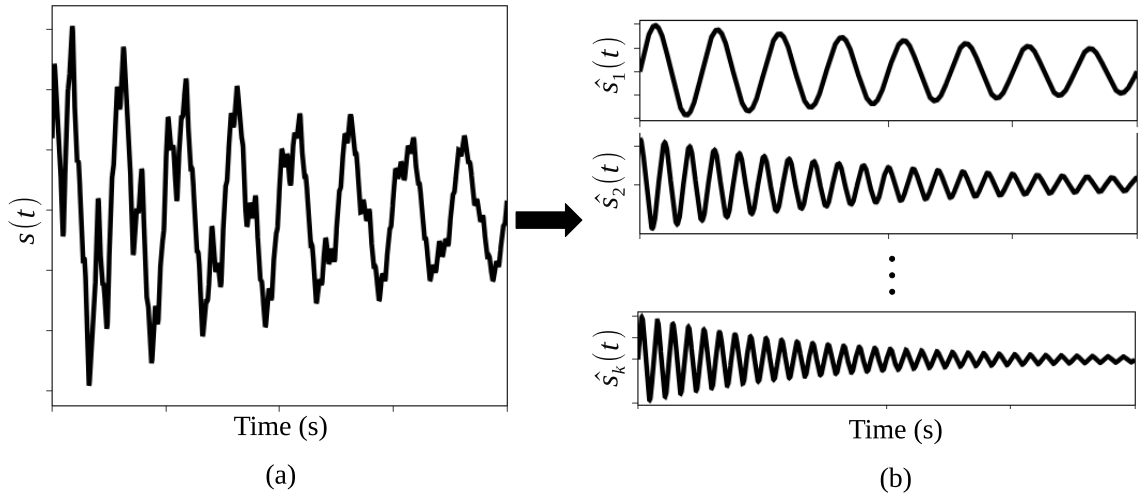


Figure 2.2: (a) Oscillating signal under study. (b) k mono-component signals obtained from the filter bank.

2.2 Bessel filter

2.2.1 Bessel functions

Given that the first and second kind Bessel functions represent an oscillating signal, these signals will be used to develop a Bessel-based biorthonormal pair of bases to

filter mono-components from multi-modal oscillating signals.

Bessel functions of the first kind, denoted as $J_v(x)$, is the solution of Bessel's equation that remains finite at $x = 0$ for $v > -1$. Bessel functions arise from the solution to the second-order homogeneous differential equation, called Bessel's differential equation [61]:

$$x^2 y'' + xy' + (x^2 - v^2)y = 0, \quad (2.10)$$

where v is a fixed parameter, which can be a real or complex number, which is the order of the Bessel function, x is any real number, and $y = f(x)$. In the context of the Bessel differential equation, the parameter v refers to a constant value that remains fixed throughout the solution process. In differential equations, this parameter can be a coefficient of the equation and can affect either the general or specific solution of the differential equation. In this case, the parameter v is the order of the Bessel equation $J_v(x)$.

In (2.10), $x = 0$ is a singular point, and has no more singular points in the complex plane, then it is possible to solve by means Frobenius method and will converge for $0 < x < \infty$, such as:

$$y = \sum_{n=0}^{\infty} c_n x^{n+r}, \quad (2.11)$$

where c_n are the coefficients, with $n = 0, 1, 2, \dots, \infty$, r are the roots of the exponents. Substituting (2.11) in (2.10) and reducing terms, the following is obtained:

$$y = c_0(r^2 - v^2)x^r + x^r \sum_{n=0}^{\infty} c_n x^{n+2} + x^r \sum_{n=1}^{\infty} c_n [(n+r)^2 - v^2] x^{n+r}. \quad (2.12)$$

Based on the first term in (2.12), it is seen that the indicial equation is $r^2 - v^2 = 0$ so that the indicial roots are $r = v$. By applying this criterion, all odd coefficients are zero $c_{2n-1} = 0$, and even coefficients are:

$$c_{2n} = \frac{(-1)^n}{2^{2n+v} n! \Gamma(1+v+n)}, \quad (2.13)$$

the same applies to the exponent where $r = -v$:

$$c_{2n} = \frac{(-1)^n}{2^{2n-v} n! \Gamma(1 - v + n)}, \quad (2.14)$$

where $\Gamma(z)$ is the gamma function, a shifted generalization of the factorial function to non-integer values [62]. Finally, substituting (2.13) and (2.14) in (2.11) the following is obtained:

$$\begin{aligned} J_v(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(1 + v + n)} \left(\frac{x}{2}\right)^{2n+v}, \\ J_{-v}(x) &= \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(1 - v + n)} \left(\frac{x}{2}\right)^{2n-v}. \end{aligned} \quad (2.15)$$

Functions $J_v(x)$ and $J_{-v}(x)$ are called the first kind Bessel functions. Depending on the value of v , (2.15) may contain negative powers of x and thus be convergent at range $[0, \infty]$.

$$y = c_1 J_v(x) + c_2 J_{-v}(x). \quad (2.16)$$

The general solution of the differential equation (2.10) is denoted by (2.16). These Bessel functions are non-stationary in nature [47, 45]. Any Bessel wave is a function that describes a repetitive and smooth oscillation, which in its basic form is damped, where its frequency tends to be π , which can be manipulated by generating functions at different established frequencies. Besides the natural form of the Bessel functions fits very well with low-frequency power oscillations that are mostly damped, allowing the filter with the Bessel functions to be a good resonator for decomposing oscillating signals. This makes the Bessel filter suitable for the analysis of signals with time-varying parameters.

2.2.2 First kind J_0 Bessel filter

This investigation uses the first kind of Bessel functions J_0 to create the biorthonormal pair of bases and generate the Bessel filter bank.

Considering the first kind Bessel function J_0 where $v = 0$, in discrete time and a

frequency dependent on factor p_i such as:

$$J_0(p_i x_j) = \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(1+n)} \left(\frac{p_i x_1}{2} \right)^{2n} + \cdots + \sum_{n=0}^{\infty} \frac{(-1)^n}{n! \Gamma(1+n)} \left(\frac{p_i x_{L_{min}}}{2} \right)^{2n}, \quad (2.17)$$

$$p_i \in [p_1, p_k] \forall i = 1, \dots, k.$$

where k is the number of Bessel functions with the i -th frequency, and L is the number of samples, whose minimum value of L_{min} indicated in (2.3). For instance, the first five Bessel functions and their Fourier spectra are respectively depicted in Fig. 2.3(a)-(b). Note that the frequency of the functions is almost equal. These

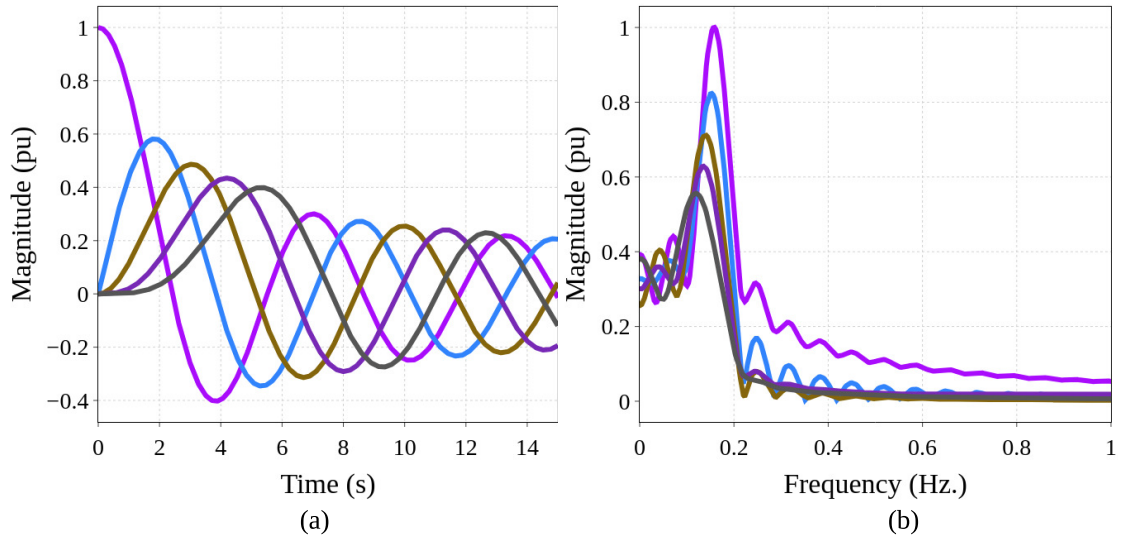


Figure 2.3: Bessel functions. (a) The first five functions for $v = 0, 1, 2, 3, 4$. (b) Their Fourier spectra.

unity Bessel functions are scaled by the factor p_i in a range from the minimum up to the maximum frequency contained in the signals under study.

To build the Bessel filter bank, the k scaled Bessel functions in (2.17) are assembled in the columns of matrix $\mathbf{J}_{k,n}$, the rows are containing the functions in the time range $[0, 2\pi]$ with L_{min} samples, such that:

$$\mathbf{J}_{k,n} = \begin{bmatrix} J_0(p_1 n_1) & J_0(p_1 n_2) & \dots & J_0(p_1 n_{L_{min}}) \\ J_0(p_2 n_1) & J_0(p_2 n_2) & \dots & J_0(p_2 n_{L_{min}}) \\ \vdots & \vdots & \ddots & \vdots \\ J_0(p_k n_1) & J_0(p_k n_2) & \dots & J_0(p_k n_{L_{min}}) \end{bmatrix}, \quad (2.18)$$

where matrices $\mathbf{C}$ and $\mathbf{J}$ in (2.4) and (2.18) have dimensions (L_{min}, k) , respectively. Then, the biorthonormal basis ($\mathbf{C}^\dagger$) is built using the Bessel functions' matrix ($\mathbf{J}$) and applying the Gram matrix, thus the same procedure from (2.5) to (2.9) is followed.

Figure 2.4(a) depicts the first five scaled Bessel functions J_0 built by (2.17), which are assembled into the scaling matrix in (2.18) and orthonormalized by (2.5) to form the biorthonormal basis of the FIR filters, whose frequency response are displayed in Fig. 2.4(b).

To obtain the mono-components, the procedure is similar to the cosine filter, the coefficients $\hat{\xi}$ are obtained using (2.6), the synthesis and analysis equations are derived with (2.8) and (2.7), respectively, and the mono-components are extracted as in (2.9).

The Bessel filter, as well as the cosine filter, is a tool for decomposing the oscillating signal into mono-components, the difference lies in its naturally damped Bessel functions.

Algorithm 1 shows the pseudo-code for creating the cosine and Bessel resonator filters, as well as, the formulation for extracting the highest energy mono-components. Where initial parameters needed, such as number of modes m , number of the filters' functions k , initial filter sampling L_{min} , scaling vector α , and Bessel ($J_0(\cdot)$) and cosine ($\cos(\cdot)$) functions [63].

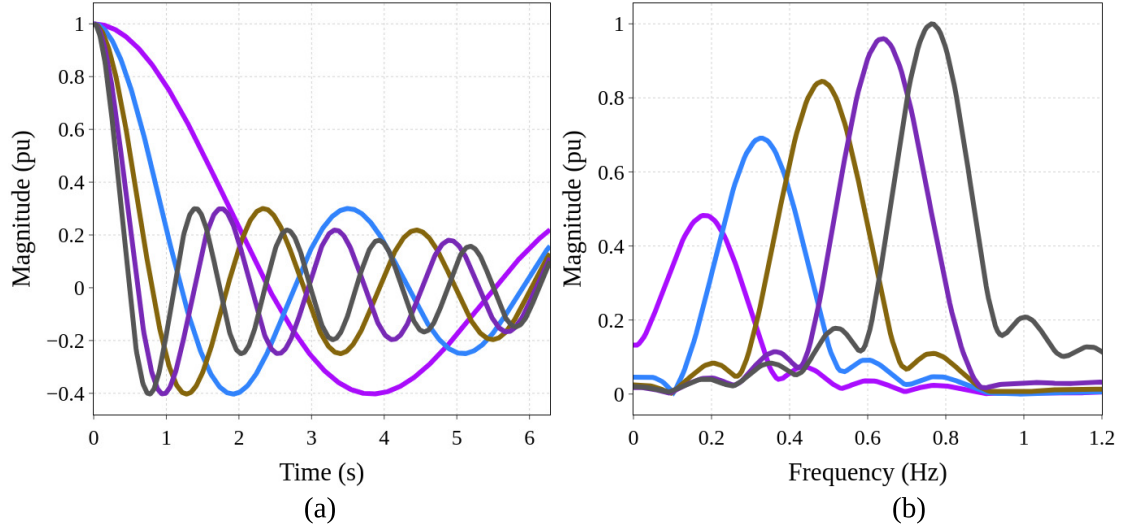


Figure 2.4: Bessel filter for decomposing oscillating signals. (a) Scaled Bessel functions. (b) Their corresponding FIR filters.

Algorithm 1 Design of cosine and Bessel resonator filters.

```

1: Input:  $m, k, L_{min}, \alpha, J_0(n), \cos(n)$ 
2: Output:  $\hat{s}_0, \hat{s}_1, \dots, \hat{s}_m$ 
3: Consider:  $L_{min} = \frac{2f_s}{f_{min}}$ , and  $L_{max} = 2L_{min}$ 
4: Create Matrices:  $\mathbf{J}_{k,L_{min}}(\alpha n)$  or  $\mathbf{C}_{k,L_{min}}(\alpha n)$ 
5: Get Basis:  $\mathbf{J}^\dagger = (\mathbf{J}^T \mathbf{J})^{-1} \mathbf{J}^T$  or  $\mathbf{C}^\dagger = (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T$ ;
6: for  $i = 0$  to  $N - L_{min}$  do
7:    $\hat{\xi}_k \leftarrow (\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \mathbf{s}_i$ ;
8:    $\hat{s}_k \leftarrow \mathbf{C} \hat{\xi}_k$ ;
9:    $(\hat{s}_1, \hat{s}_2, \dots, \hat{s}_k) \leftarrow \hat{s}_k$ ;
10: end for
11: Get  $m$  modes with higher energy  $\hat{s}_1, \hat{s}_2, \dots, \hat{s}_m \gg \hat{s}_{k-1}, \hat{s}_k$ 
12: return  $\hat{s}_0, \hat{s}_1, \dots, \hat{s}_m$ 

```

2.3 Methods for multiple channels

This section presents the ability of the proposed filters to process multiple channels and produce a set of mono-components simultaneously. All channels under analysis can come from different electrical/mechanical variables, however, they must come from the same event, in which the same oscillation modes are propagated. To analyze multiple signals using the cosine and Bessel filters, the synthesis and analysis equations (2.5) and (2.6), respectively, are extended for a set of m channels or signals, as follows:

$$\begin{aligned}
 [\hat{\xi}_1 \ \hat{\xi}_2 \ \dots \ \hat{\xi}_m] &= \mathbf{C}^\dagger [\mathbf{s}_1 \ \mathbf{s}_2 \ \dots \ \mathbf{s}_m], \\
 [\hat{s}_1 \ \hat{s}_2 \ \dots \ \hat{s}_m] &= \mathbf{C} [\hat{\xi}_1 \ \hat{\xi}_2 \ \dots \ \hat{\xi}_m],
 \end{aligned} \tag{2.19}$$

and for the Bessel matrix $\mathbf{J}$, it is as follows:

$$\begin{aligned}
 [\hat{\xi}_1 \ \hat{\xi}_2 \ \dots \ \hat{\xi}_m] &= \mathbf{J}^\dagger [\mathbf{s}_1 \ \mathbf{s}_2 \ \dots \ \mathbf{s}_m], \\
 [\hat{s}_1 \ \hat{s}_2 \ \dots \ \hat{s}_m] &= \mathbf{J} [\hat{\xi}_1 \ \hat{\xi}_2 \ \dots \ \hat{\xi}_m],
 \end{aligned} \tag{2.20}$$

where $\mathbf{s}_i$ and $\hat{\mathbf{s}}_i$ are the discrete signal samples and its estimated respectively. $\mathbf{C}$ is the cosine matrix and $\mathbf{C}^\dagger$ is its pseudo-inverse, whose rows contain the impulse response of the filter bank, this also $\hat{\xi}_i$ are the estimated coefficients.

2.4 Optimal calculation of parameter L

The parameter L is the number of samples of the filter bank functions. In this research, the functions have a length in the range $[0, 2\pi]$, however, the sampling is defined by L .

To determine the optimal value of L , it is necessary to establish the number of m modes in the signal by taking a Fourier analysis that allows a prior evaluation of the magnitude and how many modes are involved. Then, modes are selected within the k number of mono-components, considering that the modes with the higher energy are the modes with the most influence on the signal. Thus, the sampling L makes the proposed filters resonate more strongly across the frequencies integrated into the signal. It is straightforward to select the mono-components with higher participation in the signal and prevent frequencies that are not of concern. Such participation is computed in the sense of energy contributions, assuming an L given as L_j . Then, the energy of each mono-component, $(\%E_i)$, is computed in terms of the total energy in descending order, as follows:

$$\%E_k = \frac{\sum_{n=1}^L \hat{s}_k^2(n)}{\sum_{k=1}^m \sum_{n=1}^L \hat{s}_k^2(n)} 100. \quad (2.21)$$

Graphically, it is observed in Fig. 2.5(a) that the first modes concentrate most of the signal energy. Then, the energy's sum of the m selected modes is obtained through (2.22) and displayed in Fig. 2.5(b), where an energy of 99.37% is reached when $L = 625$.

$$\%E_{k,j}(L) = \sum_{k=1}^m \%E_k. \quad (2.22)$$

The complete procedure is carried out by sweeping L_j within a range $[L_{min}, L_{max}]$ and obtaining the total energy at each point up to the L_j achieves its optimal value (a local maximum). In this research, the sweep of L is performed in a range within $[600, 1200]$.

The general procedure to obtain the parameter L is summarized in the Flowchart 2.6 and Algorithm 2, which indicates an initial number of modes m must be established, and subsequently the signals must be decomposed $L_{max} - L_{min}$ times. Then, each signal is evaluated as a function of the energy of the m highest energy modes

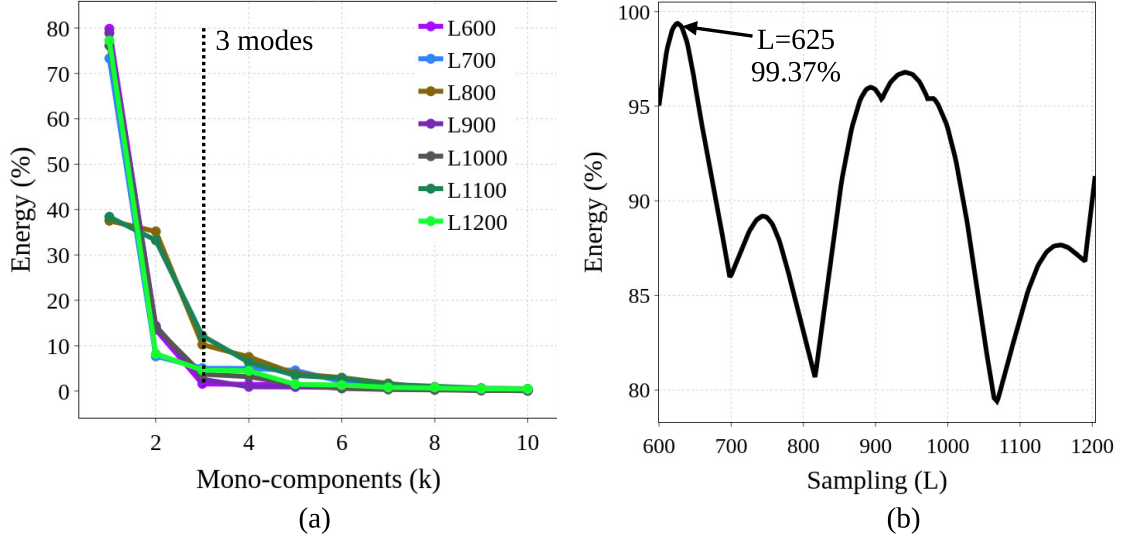


Figure 2.5: Calculation of parameter L . (a) Energy per mono-component, and (b) sum of energy per mono-component in m modes.

to determine the optimal L .

2.5 Modal pattern extraction

To derive the modal parameters such as amplitude (a_k), frequency (f_k), damping (σ_k), and phase (φ_k), for the k -th mono-component, the methodology is explained in this section.

Algorithm 2 Optimization of the parameter L .

```

1: Input:  $m, k$ 
2: Output:  $\%E_{k,j}(L)$ 
3: Consider:  $L_{min} = \frac{2f_s}{f_{min}}$ , and  $L_{max} = 2L_{min}$ 
4: for  $L_j = L_{min}$  to  $L_{max}$  do
5:    $(\mathbf{C}_{k,j}(L), \mathbf{B}_{k,j}(L)) \leftarrow (\cos(\omega_k n_j), J_0(\omega_k n_j))$ ;
6:    $\hat{\mathbf{s}}_k \leftarrow \mathbf{C}(\mathbf{C}^T \mathbf{C})^{-1} \mathbf{C}^T \mathbf{s}$ ; or  $\hat{\mathbf{s}}_k \leftarrow \mathbf{B}(\mathbf{B}^T \mathbf{B})^{-1} \mathbf{B}^T \mathbf{s}$ ;
7:    $(\hat{s}_1, \hat{s}_2, \dots, \hat{s}_k) \leftarrow \hat{\mathbf{s}}_k$ ;
8:    $\%E_{i,j}(L) \leftarrow \frac{\sum_{n=1}^N \hat{s}_i^2(n)}{\sum_{k=1}^m \sum_{n=1}^N \hat{s}_k^2(n)} 100$ 
9: end for
10: return  $\%E_{k,j}(L)$ 

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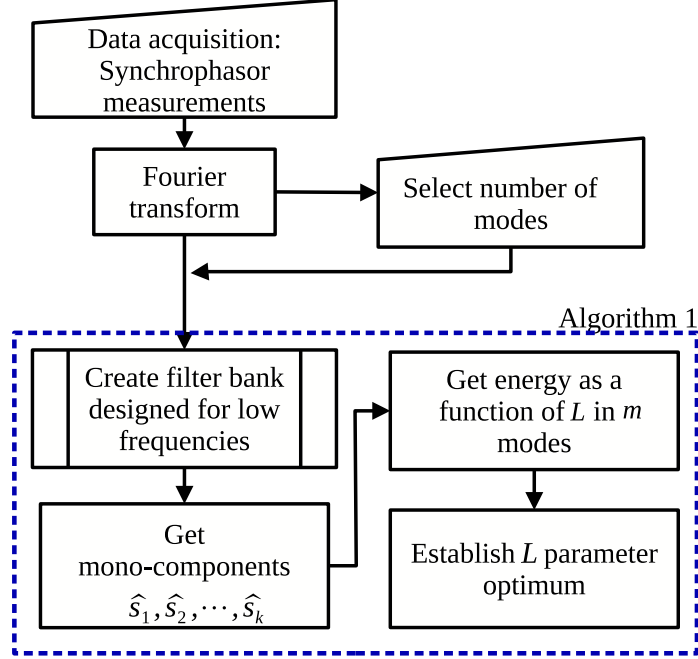


Figure 2.6: Flowchart for the optimal calculation of the parameter L .

2.5.1 Amplitude and Phase

With the information of the analysis equation in (2.7), the amplitude and phase estimates are:

$$\hat{a}_k(t) = 2|\hat{\xi}_k|, \quad (2.23)$$

$$\hat{\varphi}_k(t) = \angle \hat{\xi}_k. \quad (2.24)$$

2.5.2 Frequency estimation

From the synthesis equation (2.8), each component $\hat{s}_k$ may be written as:

$$\hat{s}_k = \mathbb{R}\{\hat{s}_k\} + j\mathbb{I}\{\hat{s}_k\}, \quad (2.25)$$

where $\mathbb{R}\{\hat{s}_k\}$ is the real part of $\hat{s}_k$, denoted by $\hat{s}_{\mathbb{R}}$, and $\mathbb{I}\{\hat{s}_k\}$ is the imaginary part, denoted by $\hat{s}_{\mathbb{I}}$. Therefore, the instantaneous frequency may be computed through the Hilbert transform as:

$$f_k(t) = \frac{\hat{s}_{\mathbb{R}}(t)\dot{\hat{s}}_{\mathbb{I}}(t) - \dot{\hat{s}}_{\mathbb{R}}(t)\hat{s}_{\mathbb{I}}(t)}{2\pi(\hat{s}_{\mathbb{R}}^2(t) + \hat{s}_{\mathbb{I}}^2(t))}. \quad (2.26)$$

The instantaneous frequency is estimated as in (2.27).

$$\hat{f}_k(t) = f_k + \frac{\dot{\hat{\varphi}}_k}{2\pi}, \quad (2.27)$$

where the first phase derivative is given by:

$$\dot{\hat{\varphi}}_k = \frac{\mathbb{I}(\dot{\hat{\xi}}_k e^{-i\hat{\varphi}_k(t)})}{\hat{a}_k(t)}. \quad (2.28)$$

2.5.3 Damping and Damping Ratio estimation

The instantaneous damping for the k -th mode may be estimated as:

$$\hat{\sigma}_k(t) = -\frac{\dot{\hat{a}}_k(t)}{\hat{a}_k(t)}, \quad (2.29)$$

where the first derivative of amplitude $\dot{\hat{a}}_k(t)$ is estimated in through the analysis equation, then:

$$\dot{\hat{a}}_k(t) = 2\mathbb{R}\{\dot{\hat{\xi}}_k e^{-i\hat{\varphi}_k(t)}\}, \quad (2.30)$$

and $\hat{\varphi}_k(t)$ is the estimated phase.

2.6 Single Exponential Smoothing

Once instantaneous frequency $\hat{f}_k(t)$ and damping $\hat{\sigma}_k(t)$ are estimated, a single exponential smoothing (SES) [60] is applied to smooth the parameters given by (2.31), in which α value is a constant given by a range $[0 - 1]$. This method takes the estimation for the previous period and adjusts it using the estimation error.

$$\hat{y}_t = \hat{y}_{(t-1)} + \alpha(y_{(t-1)} - \hat{y}_{(t-1)}). \quad (2.31)$$

The estimation $\hat{y}_t$ is based on weighting the most recent observation $y_{(t-1)}$ with a weight value α , and weighting the most recent estimation $\hat{y}_{(t-1)}$ with a weight of $1 - \alpha$. Thus, it can be interpreted as a weighted average of the most recent estimation and the most recent observation. This way, by using (2.31), the new estimate is simply

the previous estimate plus a percentage of the error of the previous estimate. When α is close to 1, the new estimate will include an adjustment more influenced by the error of the previous estimate. Conversely, as α approaches 0, the new estimate will have minimal influence.

Another way of writing (2.31) is:

$$\hat{y}_t = \alpha y_{(t-1)} + (1 - \alpha) \hat{y}_{(t-1)}. \quad (2.32)$$

Therefore, it can be understood as a weighted average of the most recent estimate and the most recent observation, where the estimate and observation have respective weights based on the α parameter. In this work, the weight value α was considered as $\alpha = 0.005$. A small smoothing factor ($\alpha=0.005$) was selected to produce a smoother series by reducing the influence of short-term fluctuations.

2.7 Signal Decomposition

The general procedure of the proposed strategy is described in the flowchart shown in Fig. 2.7. Initially, the signals are received from the PMUs with their timestamp, which contains the ringdown event window, then an analysis is performed by the Fourier transform to determine the number of modes m needed to extract via the filter bank.

Then, a cosine/Bessel filter bank is created without additional adjustments, since the filter bank is static and designed for low frequencies. The optimal computation of L follows this to improve results. The proper selection of the parameter L allows obtaining the most representative mono-components of the signal through the filter bank.

The filter will obtain k mono-components, however, with the m modes, it is established that the m highest energy modes will be selected for the analysis. Each mono-component contains its respective frequency and damping parameters. Subsequently, the identification of modal patterns is carried out using the Hilbert transform.

Finally, the result of the damping and instantaneous frequency is subjected to Single Exponential Smoothing to attenuate variations in the instantaneous results.

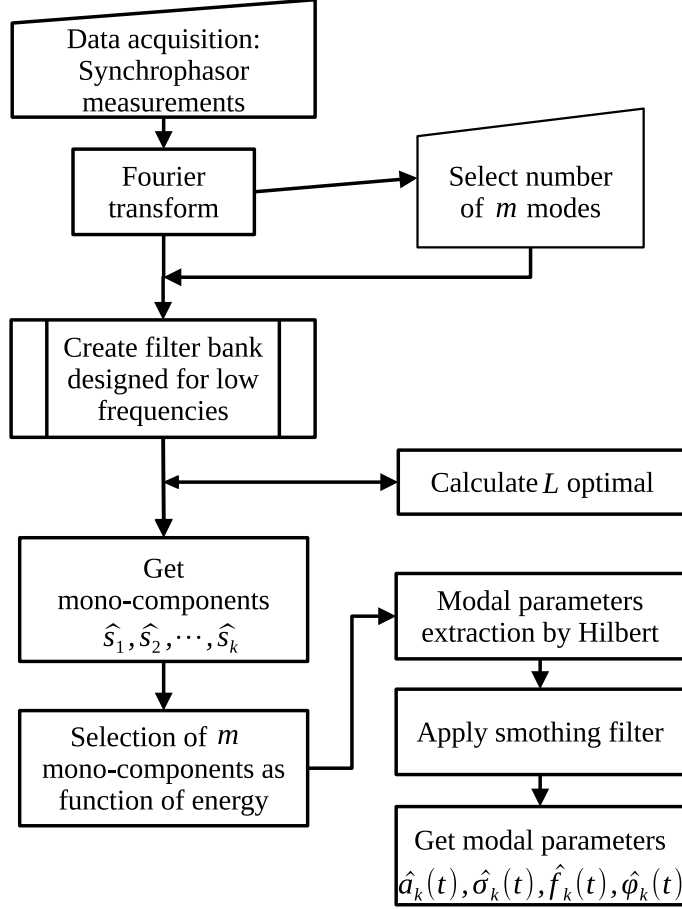


Figure 2.7: Flowchart of the general procedure of the proposed strategy.

2.8 Conclusions

This chapter establishes the mathematical foundations for creating cosine and Bessel filters and their use to identify modal dynamic patterns in power systems.

Initially, the filter functions are described from the scaled cosine and Bessel functions of the first kind. These are finite-scaled functions. Using the Gram process, a biorthonormal basis is obtained for creating the filters. These resonator filters are static; that is, they do not have to adapt to the signal being analyzed. These filters are designed for low frequencies found in power systems' inter-area oscillation. This is a major advantage over other filters, as they do not have to adapt to the frequencies to be obtained. Filters contain k functions that excite different modes of the signal, thus obtaining the highest energy modes (m), preventing spurious frequen-

cies. It is important to note that the number of filter functions depends on the user. This can improve the analysis and results, but will generate a greater computational load and therefore response time. The mathematical formulation for single-signal and multi-signal analysis, which is necessary for systems with several synchrophasor measurements at different locations in the system, is detailed.

To improve results, the optimal parameter L is proposed, which allows obtaining more cleanly the higher energy modes immersed in the signal under study and neglecting the lower energy modes and noise.

The mathematical formulation of the Hilbert transform is also described to obtain the modal parameters of amplitude and instantaneous frequency from the mono-components obtained from the filters. These modal dynamic parameters are necessary to describe the state of a power system through the reading of synchrophasor data.

In the following chapters, results and computation time will be seen, and it will be concluded that the mathematical procedure is robust, precise, and requires little computational effort to obtain the dynamic modal parameters of a system.

Chapter 3

Application to synthetic signals

In this chapter, cosine and Bessel filters are applied to identify low-frequency electromechanical modes in synthetic oscillating signals that are assumed as synchrophasor data. To this end, synchrophasor signals with stable and unstable modes are proposed to demonstrate the applicability of the proposed methods.

In the following sections, cosine and Bessel filters are described, as well as the modal pattern extraction in nonlinear signals. To evaluate the performance of the methods, two cases are initially analyzed, each with single-channel signals with stable and unstable modes, respectively. Finally, a multi-channel case is analyzed using different modes obtained from the same system. The results obtained with the proposed methods are compared with conventional time-domain techniques, demonstrating their performance. The calculation to obtain the parameter L and its impact on obtaining the mono-component signals are also investigated.

Finally, modal patterns such as damping, damping ratio, and frequency are estimated simultaneously using ringdown analysis via the Hilbert transform. These parameters provide important information to power system operators for assessing system conditions.

3.1 Single-channel multi-mode damped case

To demonstrate the multivariate performance of the proposed strategy, synthetic signals are assembled considering different damping levels and frequencies, where

both are exactly known. Let's form multiple oscillating signals composed of the sum of the combined amplitude-modulated (AM) primitive signals, as follows:

$$s_a(t) = s_1(t) + s_2(t) + s_3(t), \quad (3.1)$$

where:

$$\begin{aligned} s_1(t) &= a_1 e^{\sigma_1 t} \cos(2\pi f_1 t + \theta_1), \\ s_2(t) &= a_2 e^{\sigma_2 t} \cos(2\pi f_2 t + \theta_2), \\ s_3(t) &= a_3 e^{\sigma_3 t} \cos(2\pi f_3 t + \theta_3). \end{aligned} \quad (3.2)$$

All parameters for signals s_1 , s_2 , and s_3 are summarized in Table 3.1, whose sampling rate is assumed of $f_s = 30$ Hz, as synchrophasor data.

Table 3.1: Parameters for multiple synthetic mixed signals in s_a (multivariate analysis).

Index	Magnitude	Damping	Frequency	Angle
i	a_i	σ_i	f_i	θ_i
1	20	-0.02	0.20	$\pi/6$
2	15	-0.05	0.50	0
3	10	-0.09	0.90	0

The multi-composed damped signal in (3.1) and its Fourier spectrum are depicted in Fig. 3.1. Note that the frequency spectrum contains 3 main frequencies at 0.2, 0.5, and 0.9 Hz.

The selection of the parameter L must obey (2.3). Figure 3.2 exhibits the L behavior for cosine and Bessel filters, where L is also computed assuming that most of the signal's energy is concentrated in 3 modes for each filter. With $f_s = 30$ Hz and considering the minimum frequency in power swing oscillations is $f_{min} = 0.1$ Hz [64], the minimum value of L is $L_{min} = 600$. A local maximum for the Bessel filter is found at $L_{bessel} = 619$. For the cosine filter, the local maximum is located at $L_{cosine} = 626$. It is noteworthy to remark that the larger $E(L)$, the more energy is concentrated in the k -th modes.

Afterward, cosine and Bessel functions are assembled into cosine and Bessel matrices in (2.4) and (2.18), respectively, and the Gram matrix in (2.5) is applied to these primitive functions to obtain the biorthonormal bases which will be the bases to design the FIR filters' impulse responses. In this case, 15 basis functions are used

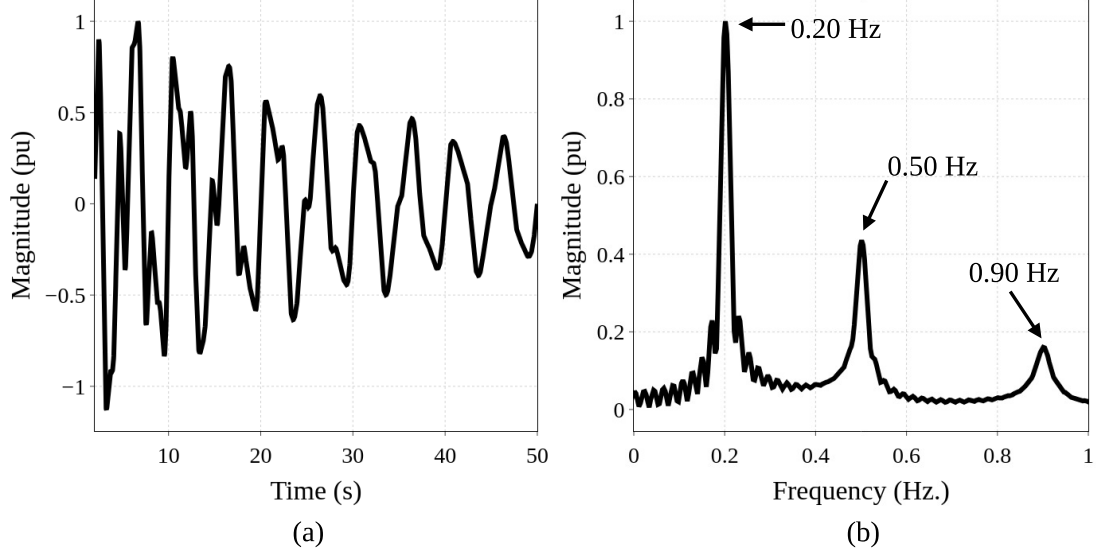


Figure 3.1: (a) Theoretical oscillating signal s_a . (b) The Fourier spectrum of s_a .

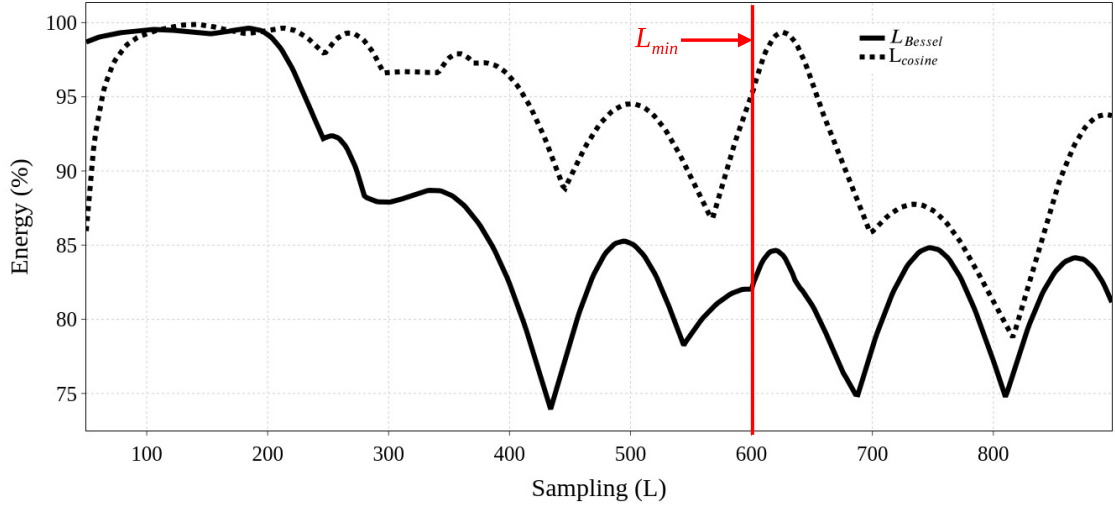


Figure 3.2: Percentage of energy in 3 modes as a function of L .

for each filter. The functions described in Fig. 3.3(a) are the first-class Bessel functions compressed in (2.18) to generate the filter bank. In the same way, Fig. 3.3(b) showcases cosine functions scaled according to (2.4) to form the filter bank. The frequency responses of the cosine and Bessel filters are displayed in Fig. 3.4, where the frequencies are uniformly distributed in a spectrum from 0.1 up to 2.3 Hz. Notice that the Bessel basis in Fig. 3.4(a) has the advantage that the different frequencies have less participation among others, even adjacent frequencies have less influence among them, not so in the cosine basis in Fig. 3.4(b) where the first frequency has an important participation in the second and gradually less in the subsequent ones,

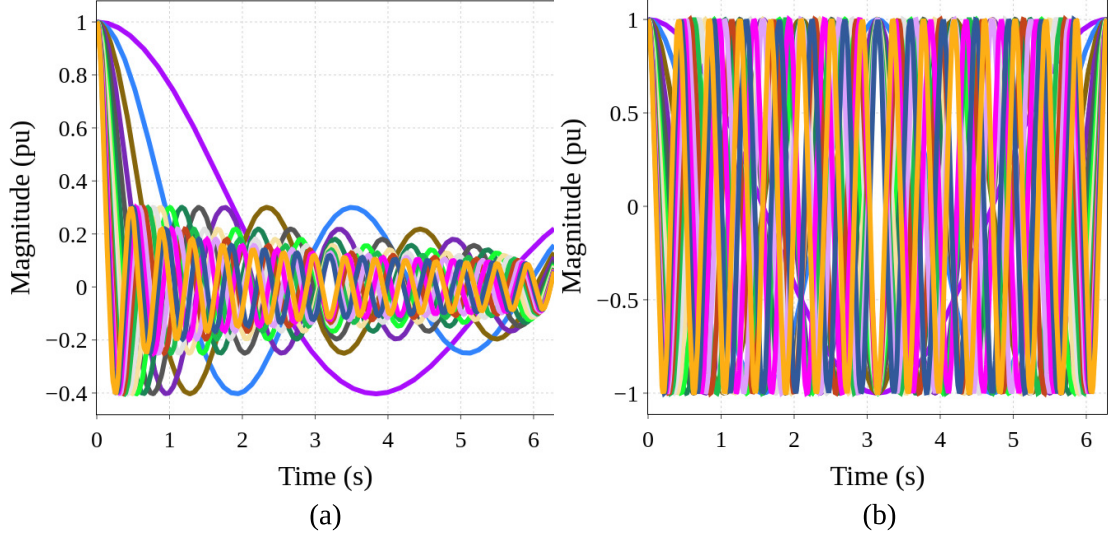


Figure 3.3: The first 15 compressed basis functions of: (a) Bessel filter; and (b) cosine filter.

but the participation among all of them is evident.

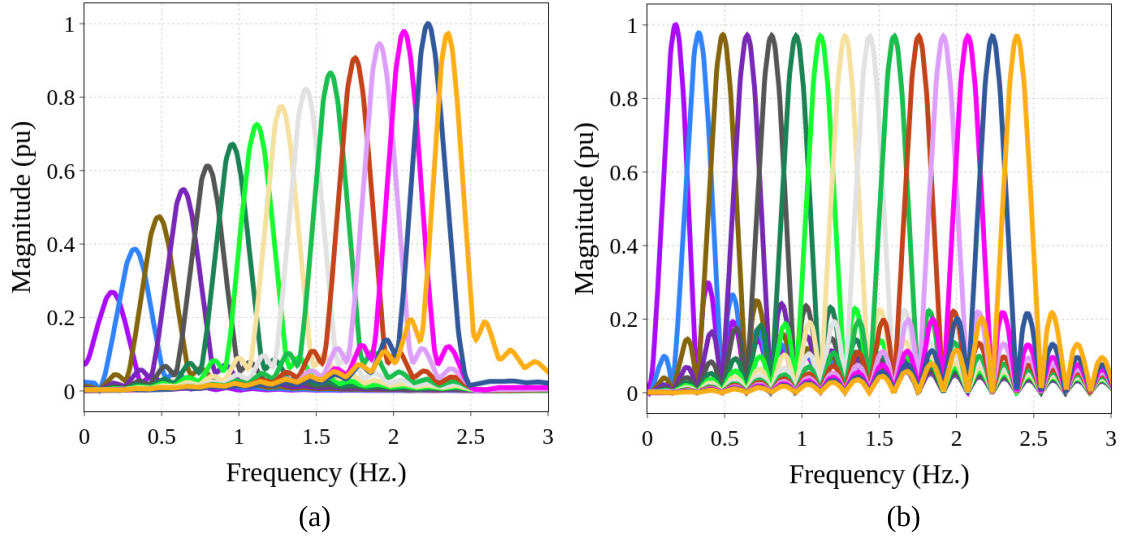


Figure 3.4: The first 15 filter frequencies of: (a) Bessel biorthonormal basis; and (b) cosine biorthonormal basis.

After computing the pseudoinverse in (2.5), the filter coefficients are estimated in the sense of the least-squares solution, as in (2.6). Then, the mono-component signals are estimated through the synthesis equation in (2.8), resulting in the decomposition illustrated in Fig. 3.5, where 15 sub-band signals are obtained by each method. Notice that the modes f_3 , f_6 , and f_7 have greater energy than others, which are exhibited in Figs. 3.5(b1) and 3.5(b2), and in turn, they capture the higher energy

of the oscillating signal s_a , and allow an accurate reconstruction.

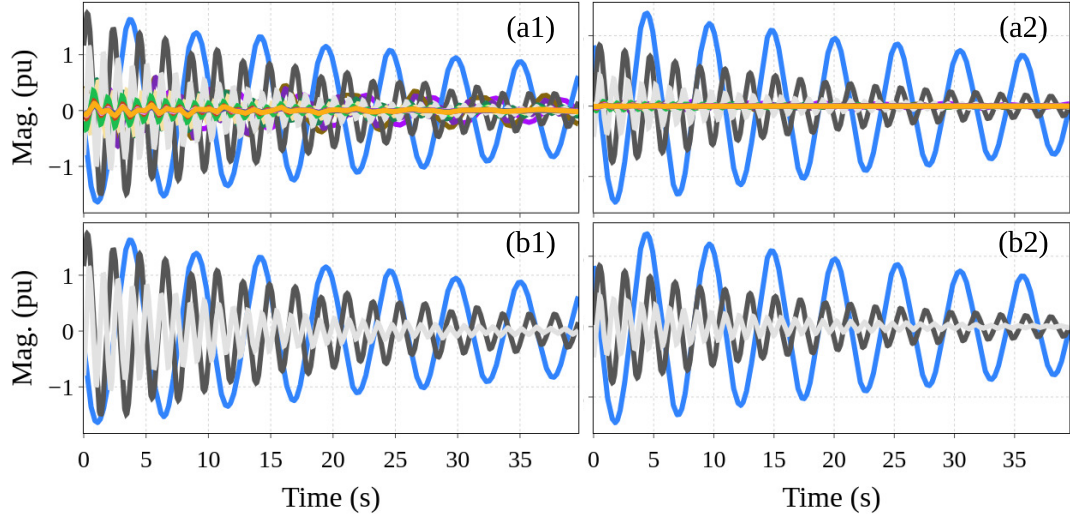


Figure 3.5: Plots (a1) and (b1) respectively illustrate 15 and 3 mono-components of the signal s_a by processing with the Bessel filter bank. Plots (a2) and (b2) represent 15 and 3 mono-components of the signal s_a by decomposing with the cosine filter bank.

Figure 3.6 shows the modes sorted from the higher to the lower energy. Note that the 15th modes calculated by cosine and Bessel filters indicate that more than 99% of the signal energy is concentrated in the first 3 modes. In the Bessel filter, the energy of the first 3 modes showcases a certain decreasing linearity; in contrast, in the cosine filter, the first mode is almost all the signal energy, and the other 2 are less representative. In both cases, the other modes are negligible.

After obtaining signal's mono-components by each method, modal patterns such as amplitude, damping, and frequency are extracted according to Section 2.5. Plots (a1), (b1), and (c1) in Fig. 3.7 display the attained results using the Bessel filter bank. Meanwhile, plots (a2), (b2), and (c2) in Fig. 3.7 showcase the results obtained using the cosine filter bank. Notice that three mono-components are efficiently identified by both methods, even when their magnitudes are different. The oscillation frequency and damping for each mono-component are very similar, which can be corroborated in Figs. 3.8 and 3.9, where the instantaneous frequency and instantaneous damping are illustrated, respectively, for both methods. Thus, the instantaneous frequency and damping are estimated using (2.27) and (2.29), respectively, and the continuous lines in Figs represent them. 3.8 and 3.9, for the cosine and Bessel filters' bank, respectively.

Once the instantaneous frequency $\hat{f}_i(t)$ and damping $\hat{\sigma}_i(t)$ are estimated, a single

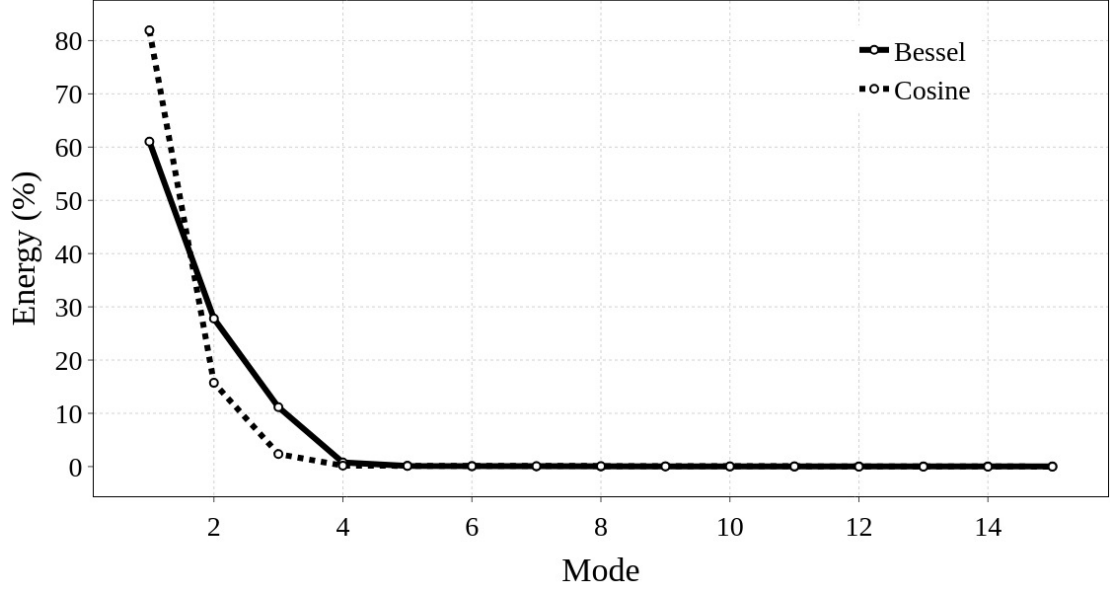


Figure 3.6: Percentage of energy per mode with $L_{Bessel} = 619$ and $L_{cosine} = 626$.

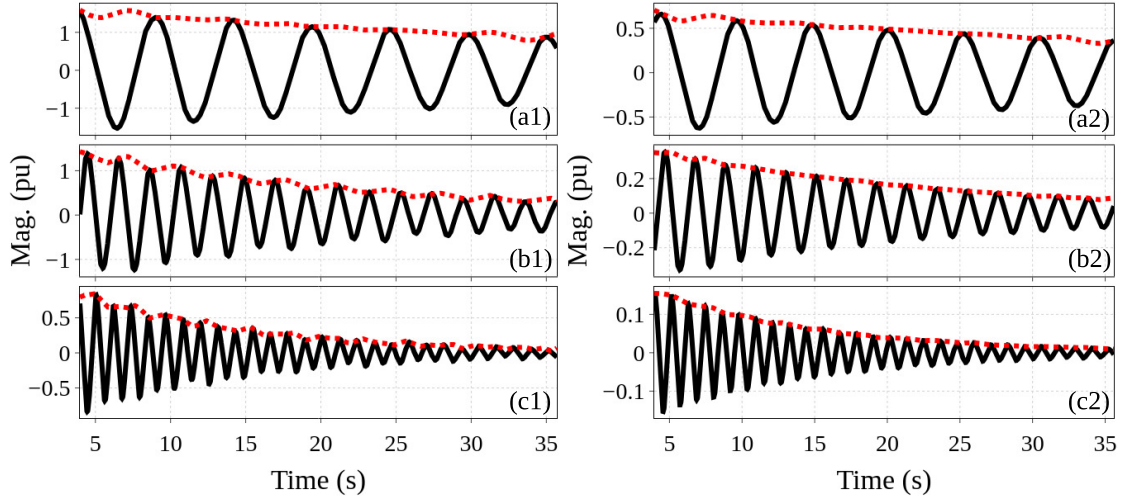


Figure 3.7: Three modes by Bessel/cosine-Filter calculated by Hilbert.

exponential smoothing is applied to smooth the estimated parameters using (2.31) with $\alpha = 0.005$. So, the smoothing parameters are represented by the dashed lines in Figs. 3.8 and 3.9, for the cosine and Bessel methods, respectively. Notice that the estimated modal frequencies are around 0.19, 0.48, and 0.87 Hz for the Bessel filter, whereas the estimated modal frequencies are around 0.19, 0.48, and 0.86 Hz for the cosine filter, which are both very close to the actual values shown in Table 3.1. Similarly, the estimated modal damping for both methods is 0.017, 0.048, and 0.083 for the Bessel filter and 0.021, 0.045, and 0.087 for the cosine filter, respectively.

Again, both methods give similar values to the actual values depicted in Table 3.1.

In summary, the values obtained for this case are summarized in Table 3.2, which includes the quantitative results and the advantage in computation time. The latter demonstrates an advantage over the ERA method, yielding highly accurate results.

The instantaneous estimates of the cosine and Bessel filters' bank are depicted in Figs. 3.8 and 3.9.

Table 3.2: Analysis through cosine filters, Bessel filters, and ERA in single-channel multi-mode damped case. Frequency and damping values are represented by their average value.

Signals' Mode	Synthetic		ERA	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.20	0.02	0.21	0.019
s_2	0.50	0.05	0.49	0.051
s_3	0.90	0.09	0.91	0.091
Window size (L)	-		619	
Computation time (s)	-		0.135	

Signals' Mode	cosine-filter		Bessel-filter	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.19	0.017	0.19	0.017
s_2	0.48	0.048	0.48	0.048
s_3	0.87	0.083	0.87	0.083
Window size (L)	626		619	
Computation time (s)	0.037		0.037	

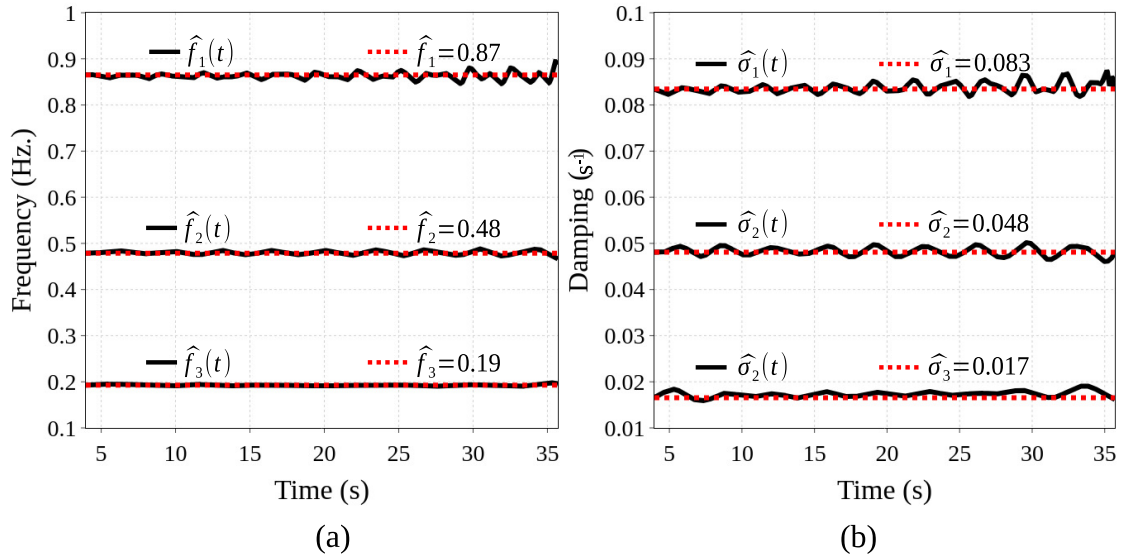


Figure 3.8: Modal pattern extraction via Bessel filter bank. (a) Instantaneous frequency estimates $\hat{f}_k(t)$. (b) Instantaneous damping estimates $\hat{\sigma}_k(t)$.

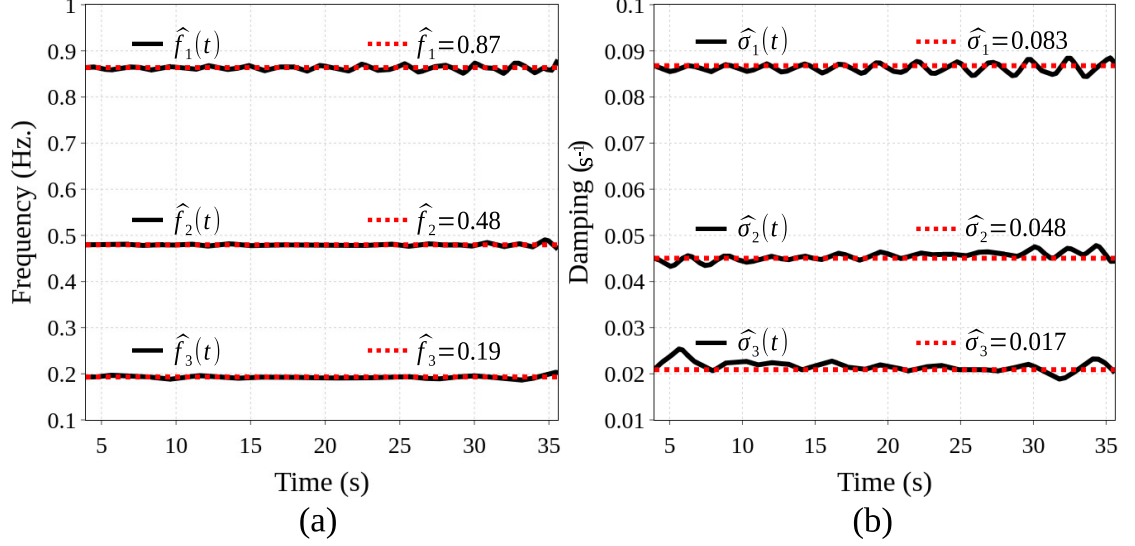


Figure 3.9: Modal pattern extraction via cosine filter bank. (a) Instantaneous frequency estimates $\hat{f}_k(t)$. (b) Instantaneous damping estimates $\hat{\sigma}_k(t)$.

Table 3.2 shows the results obtained with the cosine filter and the Bessel filter, where each value approximates the synthetic values quite well. The results from the ERA method are also included to compare results and computation time, where the proposed filter-based method demonstrates an advantage.

3.2 Single-channel multi-mode undamped Case

This case consists of a signal with 3 mono-components, but one with negative damping (unstable mode). This is to investigate how adaptable both methods are for these cases. Initially, the stable mono-components with positive damping predominate, but as the stable modes are damped. Then, the unstable mode gradually grows, and its amplitude is greater than the other modes, and all modes remain mixed.

For the signal s_b , the same values of s_a are considered, but the 3rd mode is modified by a negative damping, as shown in Table 3.3.

To demonstrate the multivariate performance of the proposed methods, considering conditions for different damping ratio levels, values of damping ratio and frequency are exactly known, as well as in the signal s_a . The multiple oscillating

signals are composed of the sum of the combined AM signals, as follows:

$$s_b(t) = s_1(t) + s_2(t) + s_3(t), \quad (3.3)$$

where

$$\begin{aligned} s_1(t) &= a_1 e^{\sigma_1 t} \cos(2\pi f_1 t + \theta_1), \\ s_2(t) &= a_2 e^{\sigma_2 t} \cos(2\pi f_2 t + \theta_2), \\ s_3(t) &= a_3 e^{\sigma_3 t} \cos(2\pi f_3 t + \theta_3). \end{aligned} \quad (3.4)$$

Parameters for oscillating signals s_1 , s_2 and s_3 are summarized in Table 3.3.

Table 3.3: Parameters for multiple synthetic mixed oscillating signals in s_b (Multivariate analysis).

Index	Magnitude	Damping	Frequency	Angle
i	a_i	σ_i	f_i	θ_i
1	20	-0.02	0.20	$\pi/6$
2	15	-0.05	0.50	0
3	10	0.005	0.90	0

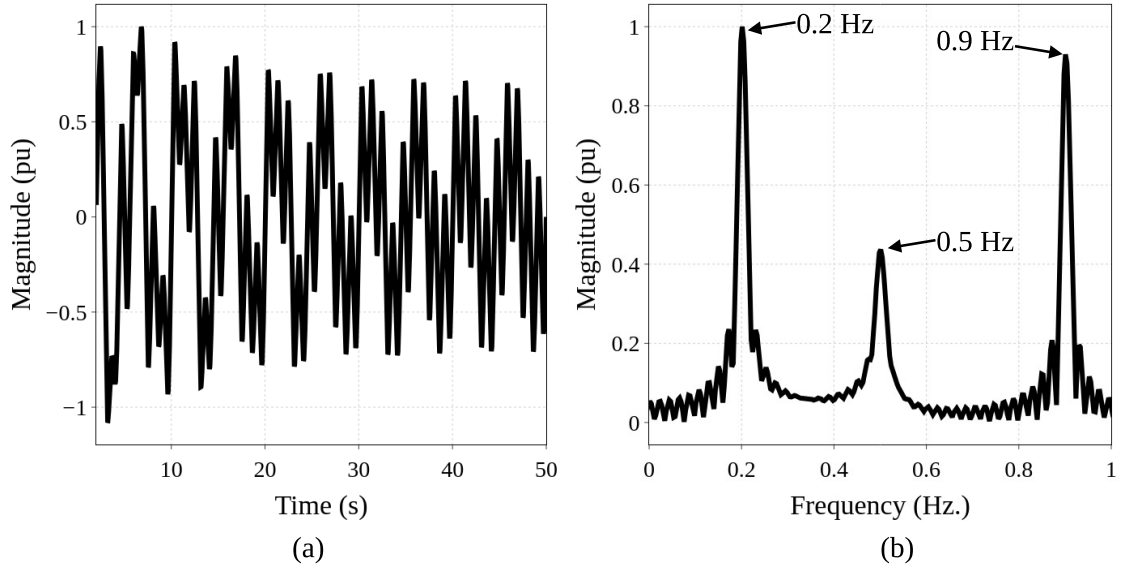


Figure 3.10: (a) Theoretical signal s_b . (b) Its frequency spectrum

Signal s_b is depicted in Fig. 3.10(a), exhibiting an unstable mode with the highest frequency. Figure 3.10(b) shows the Fourier frequency spectrum, where frequencies and their amplitude relationship among them are correlated. The 3rd mode is initially the one with the lowest magnitude; however, because it is unstable or has

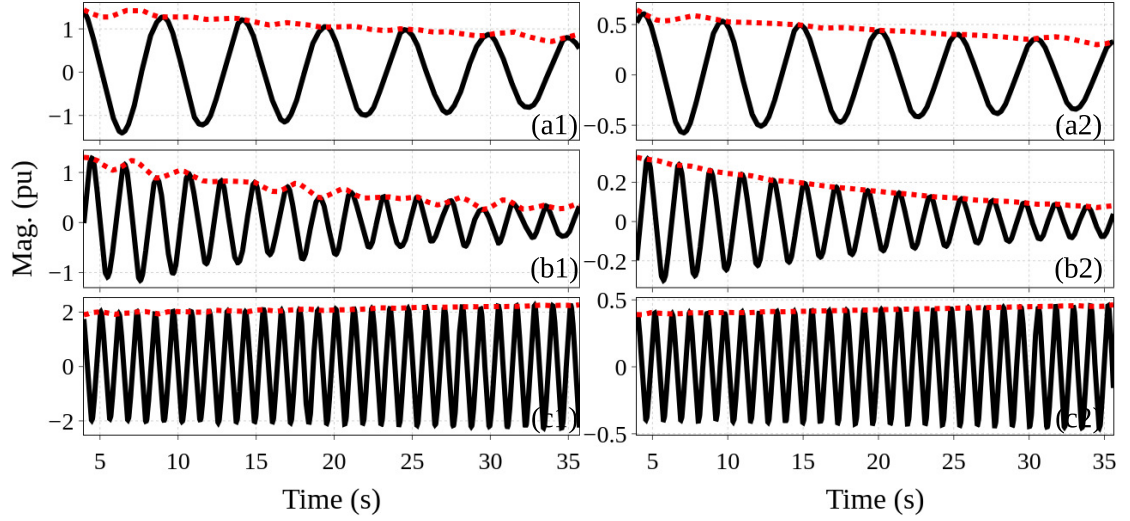


Figure 3.11: Three mono-components obtained by: (a) Bessel-filter; and (b) cosine-filter.

negative damping, it gradually grows, and its magnitude in the frequency spectrum is high.

In Fig. 3.11(b), it can be observed a mode with negative damping, which is well filtered by both methods.

Table 3.4 displays the results obtained by the Bessel filter, as well as the cosine filter, where it can be seen that there is an error margin of less than 8%. Each value obtained approximates the synthetic values quite well. Besides, the results obtained from the ERA and MP methods are added to Table 3.4. In the results for signal s_b , an unstable mode begins with a lower magnitude with respect to the others, and it grows due to its instability. Both analysis methods manage to obtain the modal dynamic patterns, gradually decreasing their amplitudes.

3.3 Multi-channel multi-mode damped case

In this case, the multivariate performance of the proposed strategy is demonstrated by considering different damping ratio levels. Commonly, in this case, measurements are assumed to be captured at different locations after perturbations take place; they generally exhibit the same modes excited and are observed in measurements such as frequency, active power, voltage phase angles, etc. This case consists of a set of 3 synthetic signals whose values are previously known. These signals contain 3

Table 3.4: Analysis through cosine filters, Bessel filters, and ERA in single-channel multi-mode undamped case. Frequency and damping values are represented by their average values.

Signal s_b Mode	Synthetic		ERA	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
1	0.2	0.02	0.19	0.019
2	0.5	0.05	0.48	0.0501
3	0.9	-0.005	0.87	-0.0049
Window size (L)	-		619	
Computation time (s)	-		0.135	
Signal s_b Mode	cosine-filter		Bessel-filter	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
1	0.19	0.016	0.19	0.016
2	0.48	0.040	0.48	0.040
3	0.83	-0.005	0.87	-0.005
Window size (L)	626		619	
Computation time (s)	0.037		0.037	

mono-components according to Table 3.5, where all three modes are poorly damped (less than 5%). Figure 3.12(a) depicts 3 multi-component signals, whose Fourier spectra are shown in Fig. 3.12(b). As this is a multivariate strategy, all 3 signals are supposed to come from the same system in the same event, thereby they have similar swing modes.

Table 3.5: Parameters of multiple synthetic mixed signals (Multichannel analysis).

a_1	σ_1	f_1	a_2	σ_2	f_2	a_3	σ_3	f_3
17.9	0.062	0.260	19.0	0.021	0.630	20.0	0.043	0.940
a_4	σ_4	f_4	a_5	σ_5	f_5	a_6	σ_6	f_6
18.1	0.061	0.259	19.1	0.021	0.631	20.1	0.041	0.939
a_7	σ_7	f_7	a_8	σ_8	f_8	a_9	σ_9	f_9
18.2	0.060	0.261	19.1	0.021	0.629	20.0	0.045	0.943

It is important to mention that thanks to the static filter shown in Fig. 3.4(a), all multiple synthetic signals can be decomposed into three main mono-components without making any additional adjustments to the filter bank. Thus, Fig. 3.13(a) displays the 3 mono-components obtained by the Bessel filter bank, and Fig. 3.13(b) showcases frequency spectra for each mono-component. For the cosine filter, similar mono-component signals are obtained using the static filter bank depicted in Fig. 3.4(b).

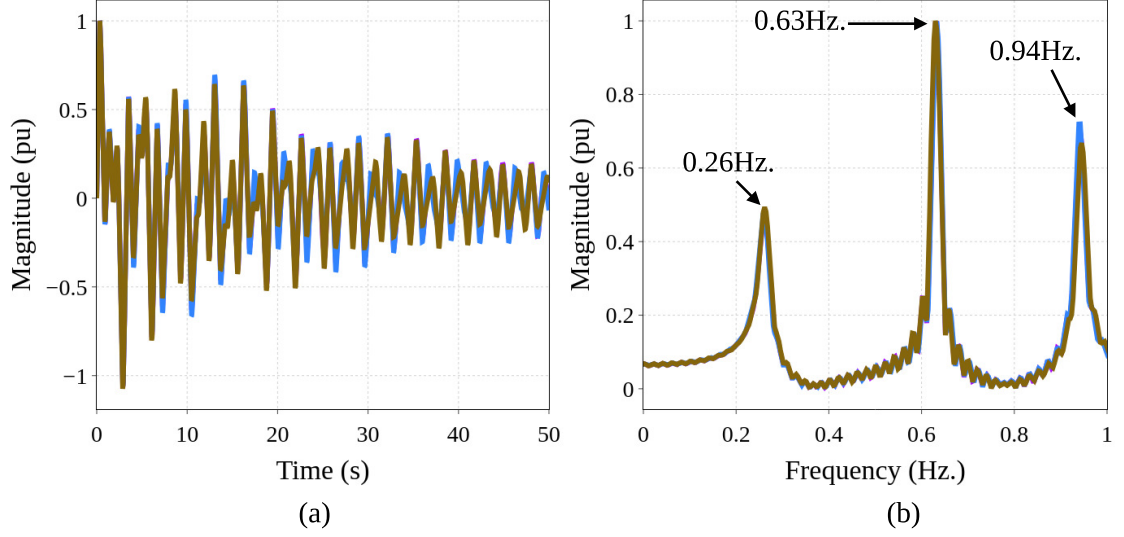


Figure 3.12: (a) Multiple channel measurements rendered by synthetic signals. (b) Their frequency spectra.

Table 3.6 summarizes the results of each filter, as well as the multi-ERA method for comparison purposes. It is noticeable that the multi-channel analysis obtained by the cosine and Bessel strategies behaves similarly, where the average values are taken. Nevertheless, the computation time for the cosine and Bessel filters shows an advantage over the ERA method, which is considerably slower, even though both filters and the ERA method have the same size of the analysis window to ensure a fair comparison.

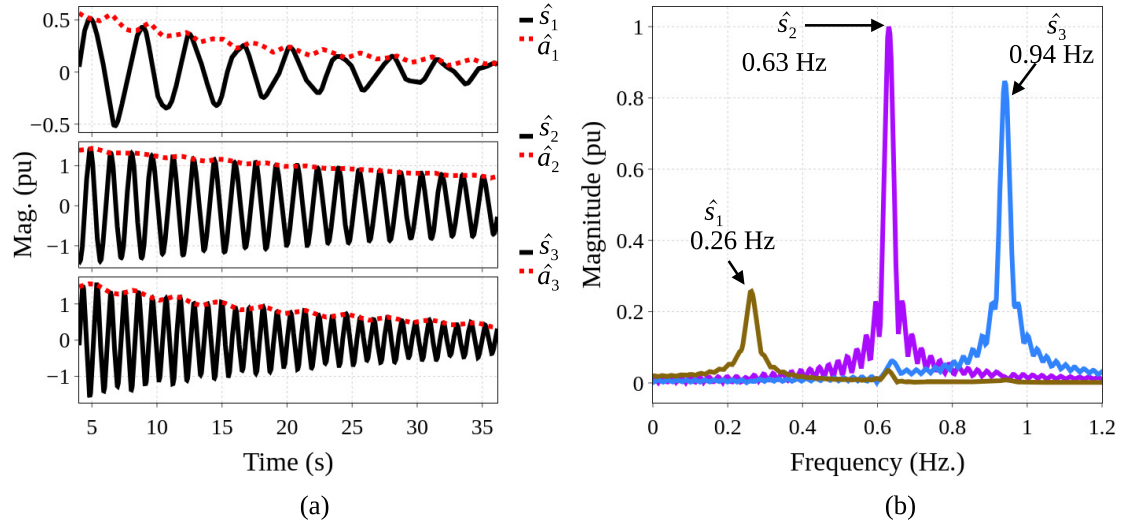


Figure 3.13: Multi-channel analysis performed by the Bessel filter bank. (a) Mono-components. (b) Their frequency spectra.

Table 3.6: Analysis through cosine filters, Bessel filters, and ERA for multi-channel multi-mode damped case. Frequency and damping values are represented by their average value.

Signals' Mode	Synthetic		Multi ERA	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.260	0.061	0.261	0.059
s_2	0.630	0.021	0.629	0.021
s_3	0.940	0.043	0.939	0.041
Window size (L)	-		560	
Computation time (s)	-		0.870	
Signals' Mode	Multi cosine-filter		Multi Bessel-filter	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.261	0.064	0.259	0.058
s_2	0.630	0.019	0.629	0.020
s_3	0.938	0.044	0.937	0.043
Window size (L)	588		560	
Computation time (s)	0.052		0.053	

3.4 Conclusions

In this section, synthetic cases were analyzed for damped oscillating signals, signals with negative damping, and multi-signal scenarios, where modal dynamic patterns are identified. The proposed filters demonstrated good performance in all cases. Due to the nature of these synthetic signals—noise-free and precisely defined—it is possible to perform an exact comparison between the estimated frequency and damping results and the true signal parameters. This allows for a reliable assessment of the filters' performance.

The results show that both filters are effective in estimating the frequency and damping of modal components. However, the Bessel filter tends to produce more accurate damping estimations. This can be attributed to the intrinsic properties of Bessel functions, which are naturally oscillatory and damped, making them particularly well-suited for identifying the damping characteristics of ringdown signals. On the other hand, the cosine-based filter yields better results in frequency estimation. This is due to the constant magnitude of the cosine function across its domain, which facilitates accurate frequency identification.

It was also demonstrated that the methods are capable of extracting negative damping in poorly damped or negatively damped signals. In multi-signal scenarios,

the filters successfully identify the dominant frequencies present in the ringdown signal set. This demonstrates the method's ability to extract modal dynamic patterns in a system with a set of synchrophasor measurements following a significant disturbance. In this case, the signals under analysis contain the same oscillation modes since they are in the same system.

Additionally, due to their significantly lower computational time compared to the ERA, these filters represent a viable and highly effective alternative for real-time studies. Their fast computation enables the immediate presentation of results in control centers, making them particularly suitable for online monitoring applications.

Chapter 4

Application to Power Systems Oscillations

In this chapter, the application of the cosine and Bessel filters is described to identify modal dynamic patterns in electrical power systems based on synchrophasor data, conventionally associated with low-frequency electromechanical modes or inter-area power oscillations. With such a purpose, data from transient simulations, as well as real system data, are used to demonstrate the applicability of the proposed filtering approaches.

In the following sections, the proposed filters and time-frequency analysis on the oscillating signals are described. To test the ability of the proposed algorithms for performing ringdown analysis, a well-known power system test case is adopted. A real power system event in Mexico is analyzed by using real synchrophasor data. Results obtained from the proposed method are compared with conventional time-domain techniques, demonstrating its performance. Both the damping factor and the frequency are simultaneously estimated from ringdown analysis.

4.1 The NETS-NYPS 68-bus system

In this section, both filters are applied to simulated signals obtained from the 16-machine test power system illustrated in Fig. 4.1 [65]. The system is completely modeled and simulated via Power System Toolbox (PST) [66]. All synchronous

generators are represented by a fourth-order model, equipped with a static excitation system, so that the system exhibits 80 oscillating modes. In this research, the emphasis is placed on electromechanical modes, conventionally associated with inter-area oscillations, where their observability depends on the measured signal and their locations.

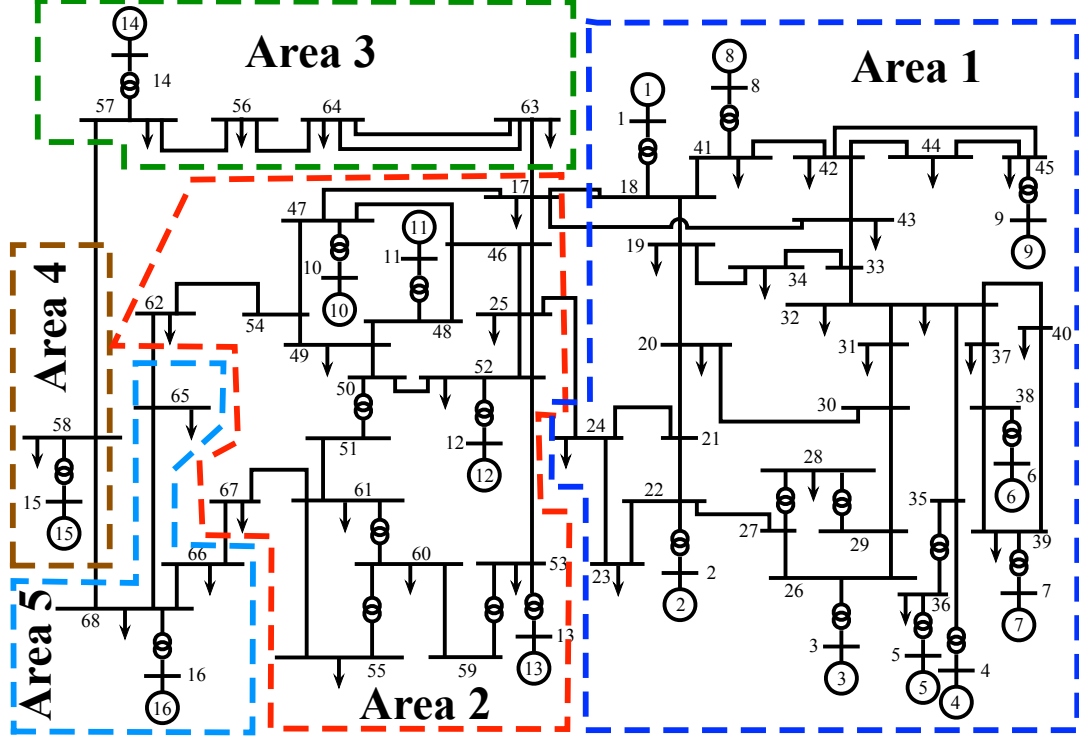


Figure 4.1: Test system: 16-machine and 68-bus New-England power system.

After running transient simulations in PST, including a three-phase fault at bus 13 and lasting 3 cycles, the ringdown period of the signals is captured and depicted in Fig. 4.2(a). Furthermore, it is important to mention that these signals are polluted with white Gaussian noise (WGN), with uniform power. In this simulation, a signal-to-noise ratio (SNR) of 66 dB is used, where the SNR is defined as follows:

$$SNR = 20 \log \left(\frac{\|s\|}{\|n\|} \right). \quad (4.1)$$

On the other hand, the Fourier spectra are displayed in Fig. 4.2(b), where the following inter-area modes are identified with frequencies at 0.37, 0.58, and 0.81 Hz.

The static filters' bank illustrated in Fig. 3.4 for the cosine and Bessel filters is used to identify modal dynamic patterns for the synchronized frequency measure-

ments. To quantitatively contrast the proposal against the multi-ERA method, Table 4.1 displays the modal parameters identified for the proposed methods, where it is evident that all techniques behave in accordance with the frequencies in the Fourier spectra. On the other hand, the damping identification is very similar, where the multi-ERA method identified damping values at 0.24, 0.3, and 0.12, whereas the cosine and Bessel filters identified damping values around 0.22, 0.27, and 0.24. These damping values correspond to well, medium, and poor damped levels, that is, damping ratios around 10%, 7%, and 3%, respectively, for each mode.

In this simulation case, multiple signals from the same event and system are considered. There are no initial values or previously established reference values for the signals. For frequency estimation, a comparison is made using the Fourier transform, which serves as a reference criterion. In contrast, damping ratio estimation is more complex to evaluate, as there are estimates from several methods, but no established ground truth value. Therefore, a baseline value must be defined for comparison. Damping values are compared with those obtained from the ERA method. Although not identical, the estimates are reasonably close.

Table 4.1 presents the results obtained using the multi-Bessel and multi-cosine filtering approaches, as well as the multi-ERA analysis. It also includes the analysis window size (L) used for each method, which is kept similar to ensure a fair comparison. Additionally, computation time is reported, revealing a substantial difference between the multi-ERA and the proposed filters.

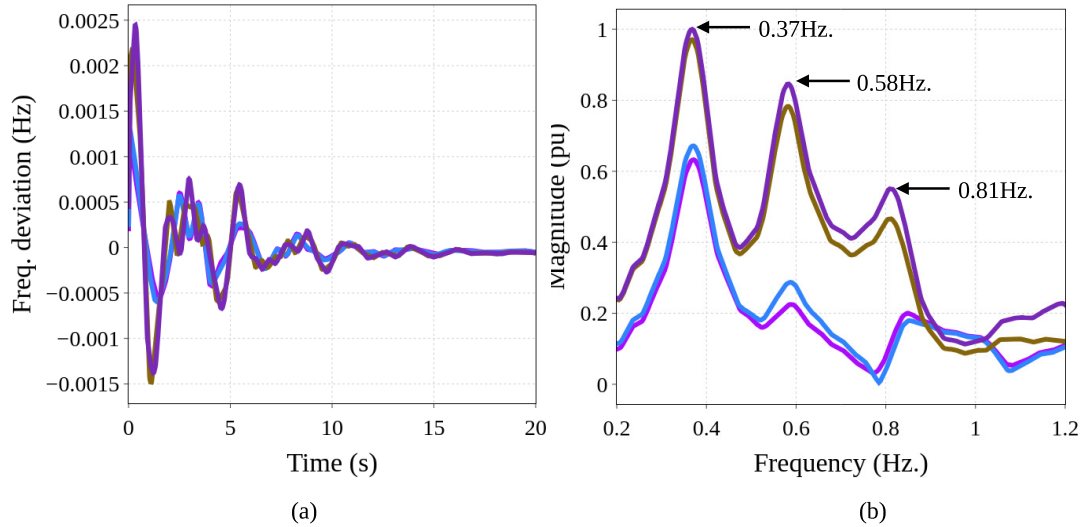


Figure 4.2: Frequency deviations during ringdown at buses 2 to 5. (a) Waveforms, and (b) their Fourier spectra.

Table 4.1: Multivariate analysis through Bessel filter and cosine filters, and ERA for multiple channels in the NETS-NYPS grid. Frequency and damping values are represented by their average values.

Signals'	Fourier		Multi ERA	
Mode	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.37	-	0.370	0.24
s_2	0.58	-	0.575	0.30
s_3	0.81	-	0.830	0.12
Window size (L)	-		330	
Computation time (s)	-		3.395	
Signals'	Multi cosine-filter		Multi Bessel-filter	
Mode	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.36	0.25	0.37	0.22
s_2	0.56	0.20	0.57	0.27
s_3	0.82	0.21	0.83	0.24
Window size (L)	479		330	
Computation time (s)	0.240		0.270	

4.2 Real event - Mexican system

In Mexico, four electrical systems operate in isolation: Muleje, Baja California (BCA), Baja California Sur (BCS), and the National Interconnected System (NIS), which is the largest one. The total installed capacity is approximately 83,120 MW with a maximum demand of about 50GW in 2019. The BCA system is interconnected to the California ISO (CAISO) through two synchronous 230kV transmission lines (TL).

The NIS is a mesh system with radial connections towards the northwest and southeast. Its transmission level mainly comprises TLs with operating voltages of 400kV, 230kV, and 130-69kV. This system covers from Quintana Roo to Sonora state, making up the main electrical power grid in Mexico, and it is split into 7 regions: Central, Eastern, Western, Northwestern, North, Northeastern, and Peninsular. It also has six trade international interconnections: (*i*) there are four asynchronous connections in the Northeastern region with the Electric Reliability Council of Texas (ERCOT) in USA by four tie-lines with 436MW capacity; (*ii*) there is one synchronous connection in the Eastern region with Guatemala via one tie-line with 240MW capacity; and (*iii*) there is also one synchronous connection in the Peninsular region with Belize by one tie-line with 55MW capacity. For the sake of brevity,

public official documents are available at [67].

On Dec 28th 2020, two of the main TLs that interconnect the Northwestern, North and Northeastern regions with the Central, Eastern, Western, and Peninsular regions were tripped, provoking a cascade event that tripped other TLs, such as the interconnection with Guatemala. Due to the loading capacity of TLs and the presence of power oscillations, the Northwestern, North, and Northeastern regions remained connected with the Central, Eastern, Western, and Peninsular regions by one TL. Thus, this sequence of events yielded that the power generation surpassed the load demand in the Northwestern, North, and Northeastern, causing a frequency excursion up to 61.14Hz that activated the stages of remedial action schemes to trip generation derived from high frequency. Meanwhile the load demand surpassed the power generation in the Central, Eastern, Western, and Peninsular regions, dropping down the frequency up to 58.76Hz and activating the load shedding stages as a consequence of the low frequency. As a result, the total affectation of power generation and load reached 8,696MW, representing 26% of the demand [67].

This case comprises the recorded measurements in the mentioned event, which are collected by five time-synchronized frequency disturbance recorders (FDRs) and a PMU installed in the following locations: 1389-Monterrey (Northeastern), 1422-Mazatlan (Northwestern), 1378-Guadalajara and 1408-Morelia (both Western), and 1424-Chetumal (Peninsular), and PMU-Chiapas (Eastern). Numbers denote the ID code for FDRs in the FNET/GridEye project ¹. The collected measurements from five regions are illustrated in Figure 4.3, where the measurements 1389 and 1422 exhibited a frequency increment, whereas the measurements 1378, 1408, and 1424 presented a frequency drop. For this real event, the window analysis is adopted of 20s, as shown in Figure 4.4(a). Likewise, their Fourier spectra are displayed in Figure 4.4(b), where two oscillating modes are indicated at 0.315 and 0.595 Hz.

Figure 4.3 shows the frequency signals of 5 FDR measurements. It is observed that there are mainly 2 groups, in the first group the frequency rises suddenly, while the second group drops. Both groups get back to a new operating point and begin to oscillate with each other. In Figure 4.4(a), the analysis window displays the oscillating signals, where 2 interarea modes at 0.595Hz and 0.315Hz are present, as can be seen in their Fourier spectra depicted in Figure 4.4(b). Therefore, it is expected that the multivariate analysis will show these two modes.

¹<http://fnetpublic.utk.edu/index.html>

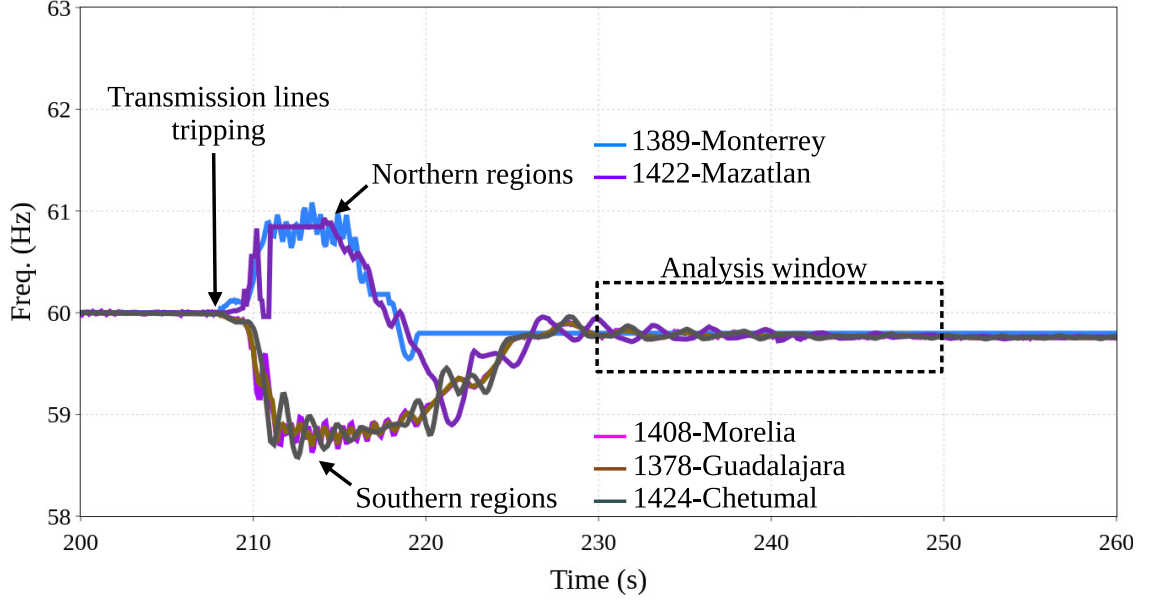


Figure 4.3: Frequency measurements captured by 5 FDRs in the Mexican National Inter-connected System.

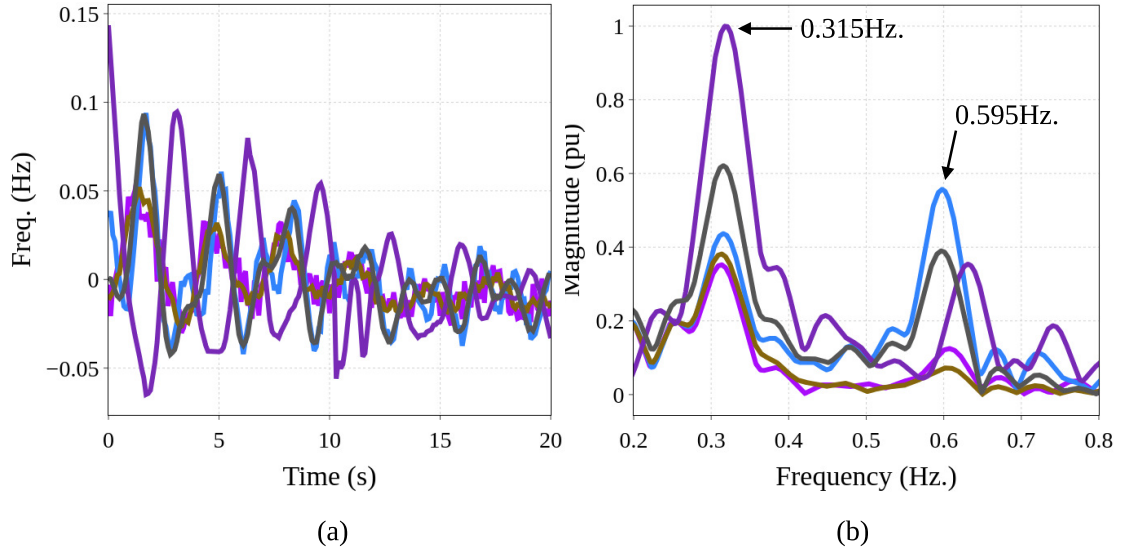


Figure 4.4: (a) Analysis window (Ringdown period selected), and (b) their Fourier spectra.

Once the measurements are available, the cosine and Bessel filters-based multi-channel analysis is accomplished. The identification of instantaneous modal patterns is achieved according to Section 2.5, resulting in two dominant modes, as described in Table 4.2. Notice that both cosine and Bessel filters precisely estimate the modal parameters contained in the ringdown analysis, where the modal frequencies match very well with those given by the Fourier and multi-ERA methods. For the estimated damping, both cosine and Bessel filters behave similarly, where the damping of the

Table 4.2: Multivariate analysis through cosine and Bessel filters, and ERA for multiple channels in the Mexican NIS. Frequency and damping values are represented by their average values.

Signals' Mode	Fourier		Multi ERA	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.315	-	0.312	0.112
s_2	0.595	-	0.601	0.062
Window size (L)		-		120
Computation time (s)		-		0.028
Signals' Mode	Multi cosine-filter		Multi Bessel-filter	
	Freq. (Hz)	Damp. (s^{-1})	Freq. (Hz)	Damp. (s^{-1})
s_1	0.303	0.113	0.311	0.108
s_2	0.580	0.043	0.591	0.049
Window size (L)		120		129
Computation time (s)		0.005		0.004

0.315 Hz mode is very similar compared with the multi-ERA estimated damping, because this mode has the most energy, as can be seen in the Fourier spectra of Fig. 4.4(b), corresponding to a medium-damped level mode, around 5%.

It can be observed from the results in Table 4.2 that the estimated frequencies exhibit good accuracy when they are compared with the Fourier analysis. However, for the estimated damping, there is no comparison criterion as precise as that provided by the Fourier in frequency analysis. For the damping of mode s_1 , the estimates obtained from both filters and ERA are very similar. For mode s_2 , the results differ more significantly among methods. This discrepancy is attributed to the fact that, being a lower-energy mode (see Table 4.2(b)), the methods may yield greater variability in the results. Nevertheless, the estimated values still demonstrate suitable performance. Another advantage is the computation time which is low for filter-based methods. In this case, with real values, the computation time is low due to the filter window size (L), which allows the algorithms to obtain results quickly.

4.3 Discussion

4.3.1 Noise Tolerance

The strategy based on the cosine and Bessel filters' bank must be able to identify modal patterns in the presence of noise. This capability in the methods has been examined using the simulated signals from single-channel multi-mode damped and undamped cases from Chapter 3. These signals are contaminated with additive WGN (AWGN) with constant power in an SNR range of 10 to 120 dB. Comparisons among filtering approaches against the ERA method are exhibited in Figs. 4.5(a)-(b).

The noise tolerance analysis is carried out by quantifying the impact of SNR variations on the output variance. These results are shown in Fig. 4.5(a) for all positive damping, and 4.5(b) for one negative damping, where cosine and Bessel filters present low sensitivity to noise. It is important to notice that the variance behavior depends on the signal modal patterns, as shown in Figs. 4.5(a) and (b).

Single-channel multi-mode damped case reaches the minimum output variance in the cosine filter at 8.061×10^{-3} and the Bessel filter at 9.217×10^{-3} , starting at 40 dB, when the variance stabilizes. For the ERA method, the output variance becomes slightly larger in comparison with cosine and Bessel filters, reaching a variance of 10.677×10^{-3} . Single-channel multi-mode undamped case reaches the minimum output variance in the Bessel filter at 31.25×10^{-3} and the cosine filter at 30.74×10^{-3} , starting at the same 40 dB, when the variance stabilizes. In the ERA method, the output variance is 33.1×10^{-3} .

The sensitivity analysis demonstrates that the proposed filters, like the ERA method, are capable of neglecting modes that are not of concern. The proposed filters have a low sensitivity to noise, due to the ease of choosing the number of modes according to their energy in the signal; low-energy modes can be discriminated in the presence of noise.

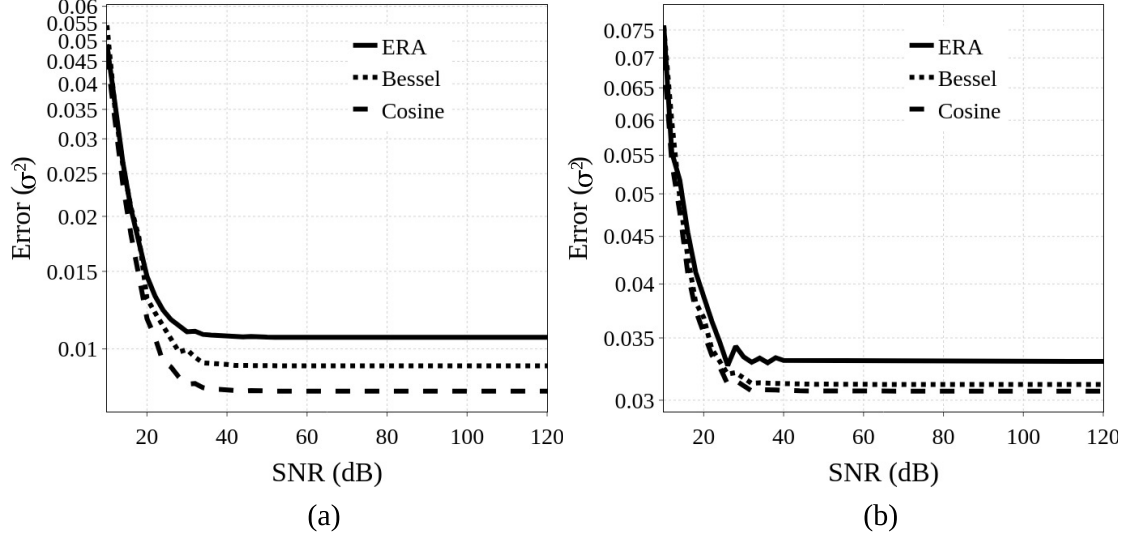


Figure 4.5: Sensitivity analysis for different Gaussian noise levels. (a) for single-channel multi-mode damped case and (b) for undamped case of Chapter 3.

4.3.2 Computational analysis

The procedure for determining modal dynamic patterns of a power system based on cosine and Bessel filters is described in Chapter 2. Both approaches follow the same strategy to generate the filter bank from a biorthonormal base. The difference lies in the formation of these filters, which are based on the first-order cosine and Bessel primitive functions with adequate scaling to obtain modes of interest in a signal.

The computing time is mainly based on the size of the biorthonormal filter, the number of Bessel/cosine primitive functions, the size of the signals to be analyzed, and the number of modes. The design and construction of the filters are the same. The extraction of mono-components and the computation of modal patterns are performed using the same approach. The only difference between the filters lies in the sampling parameter (L) used for the filter functions, which consequently indicates the size of the filter matrices. The parameter L takes on different values depending on the method used.

In the case of noise tolerance, 120 calculations were carried out with cosine and Bessel filters and by the ERA method to obtain the result of a signal contaminated with different Gaussian noise levels. Considering the same signal for all cases. In the case of ERA, 3 modes are identified with an energy threshold of 84%. In the case of cosine and Bessel filters, this was done by using 15 primitive functions in each one.

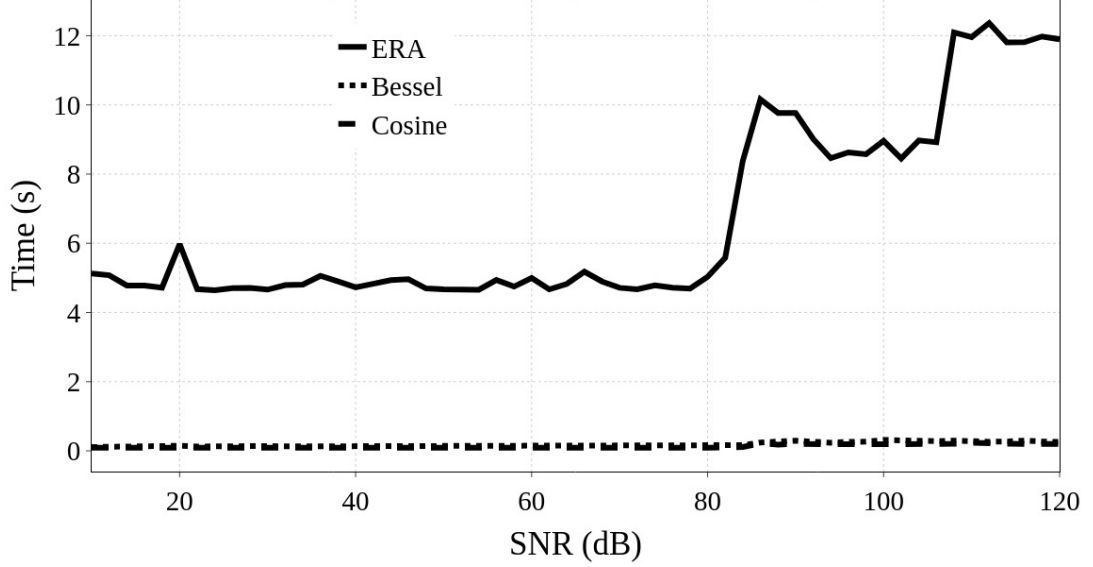


Figure 4.6: Time cost analysis of the cosine and Bessel-based filters and the ERA method for different Gaussian noise levels applied to the same signal.

The computation of the modal patterns is considered within the execution time. The calculation time of each method on each noisy signal is shown in Figure 4.6, where it is observed that the execution time of the cosine and Bessel-based filters is similar, low, and also more stable than the ERA method. Besides, it can be observed that the time behavior by filtering with cosine and Bessel is almost not affected by the noise signal.

By averaging all execution times for the ERA method, the average execution time is 6.64s; however, the execution time becomes 5.12s when 10 dB of WGN is added, and 11.89s with 120 dB. The tendency is to increase the execution time. Meanwhile, the execution times become 0.177s and 0.126s, respectively, when the proposed filtering strategies are used, demonstrating another advantage of the proposed filters. An important observation highlighted in Tables 3.2, 3.6, 4.1, and 4.2 is the comparison of computation time between the filter-based methods and the ERA method. These results demonstrate the substantial advantage in computation time offered by the cosine and Bessel filters over the ERA approach.

All these calculations are based on Python programming language version 3.10.12. Linux Ubuntu 22.04 environment, with an x86_64 architecture, 8 cores, model Intel(R) Core(TM) i7 CPU Q 720 @ 1.60GHz. The code of the algorithms is available online at [68].

4.4 Conclusions

In this chapter, a simulation of a disturbance in a well-known power system was analyzed, resulting in low-frequency electromechanical oscillations. A set of measurements was processed using both the cosine and Bessel filters applied to the ringdown window. Initially, the Fourier method was used to estimate the dominant modal frequencies; however, no accurate reference for damping values was available, making precise comparison of modal patterns uncertain. To address this, damping estimates obtained from the filters were compared against results from the ERA, showing good performance.

An additional analysis was conducted using real data from the Mexican power system, corresponding to a disturbance that occurred on December 28, 2020, which caused the system to oscillate. Fourier analysis identified two fundamental modes. Both the cosine and Bessel filters produced reliable results with significantly reduced computational time compared to ERA, supporting their suitability for real-time analysis applications.

When real-world signals were used, which inherently include noise, the filters' performance continued well. To further assess robustness, an additional experiment was conducted in which Gaussian noise was artificially added in a range from 0 to 120 dB. The performance of the two filters and the ERA method was evaluated under these conditions. All methods showed reliable results in terms of mode identification and damping estimation. However, the filters demonstrated a significantly lower computational cost, highlighting their advantage for use in time-sensitive applications such as online system monitoring.

In this chapter, data from power system simulations and real-time synchrophasor data from the Mexican system are analyzed. This is done to test the hypothesis that it is possible to determine the system state using cosine and Bessel filters. The results show the accuracy of the methods and the reduced computation time. These results present the idea of identifying modal dynamic patterns obtained from real-time synchrophasor data to display them in control centers with accurate and reliable information.

Chapter 5

Conclusions, future work and products

This final chapter presents a summary of the main results obtained throughout this thesis, as well as an overview of all the cases and their results. The proposed filters are analyzed under noisy conditions, as well as compared with known methods. The conclusions are presented, which summarize the most relevant findings concerning the stated objectives and the impact of the work developed.

Subsequently, lines of future work that could expand, deepen, or complement the results presented here are identified and discussed.

Finally, the work derived from this thesis is described, that is, the publications, projects, and scientific papers that have emerged as a direct consequence of this research.

5.1 Conclusions

This thesis has proposed, developed, and implemented novel filtering strategies based on scaled cosine and Bessel functions for precisely monitoring electromechanical oscillations in power systems. These strategies aim to extract single-component signals from a non-stationary multi-component signal. Bessel functions are special functions widely used in differential equations, applied mathematics, and mathematical physics. In this research, the first-order Bessel functions were utilized with a scaling

factor that allows the generation of an optimal filter for extracting low-frequency single-component signals, which are crucial to infer the power system stability.

Additionally, the cosine filter, designed optimally for extracting low-frequency mono-components, demonstrated high stability in results. One advantage of these filter banks is that they do not require special adaptations for specific frequencies in each signal; instead, they are general for frequencies ranging from 0.1 Hz up to 2 Hz, which are of interest in this research. Both filter banks were designed to minimize frequencies not present in the signal as much as possible and to excite functions with similar frequencies. This ensures the extraction of single-component signals as accurately as possible without needing a specific filter bank for each signal. This work demonstrates that a biorthonormal pair of bases with n functions is capable of extracting the signal's monocomponents based on a static basis; that is, it works for a given frequency range and does not require adjustment to each frequency within the signal under analysis.

Another advantage is their capability to provide precise results for each signal, regardless of noise contamination and undesired frequencies. It is noteworthy to remark that the computational cost of obtaining results is much lower compared to linear methods like ERA. One of the most important advantages demonstrated in this article is the significantly shorter computation time compared to traditional methods. Based on the results of the case studies, the average computational advantage is approximately 10 times faster, which represents the possibility of using this method online to obtain dynamic modal parameters and immediately reflect them in control centers.

Based on the results obtained from synthetic, simulated, and real examples, it was observed that damping estimation is more accurate when using the Bessel filter. This is attributed to the damped oscillatory nature of Bessel functions. On the other hand, the cosine filter exhibited better performance in frequency estimation, which can be explained by its periodic functions with constant frequency and damping characteristics. Both filtering approaches allow flexible extractions of k modes, either by energy threshold or directly by the desired modes.

This investigation reports examples of synthetic signals exhibiting different damping levels, as well as applications in power systems and real event data. Both methods performed well in obtaining anharmonic modes from a ringdown signal. The attained results were highly accurate, demonstrating the method's ability to work effectively

with signals contaminated under noisy conditions.

The importance of this doctoral research lies in presenting a new method for extracting modal dynamic patterns in electrical power systems based on synchrophasor data. This method is of great value for understanding the system status, predicting potential risks, formulating control strategies, and ensuring the safe and reliable operation of the electrical system. This is achieved by extracting modal parameters through synchrophasor measurement readings. Knowing this information in real time is of utmost importance in control centers.

The application of the proposed methods facilitates the identification of the system state based on the magnitude of the disturbance and the operating state of the system, and provides operators with reliable information in a short time to detect a potential threat and take corresponding actions to ensure system stability. To this end, this thesis investigates and establishes the advantages of cosine and Bessel filters to extract relevant modal dynamic information from synchrophasor measurements of the power system response following a major disturbance. The objective of these two methods is to detect the existence of poorly or negatively damped electromechanical modes in the system through a single-component filtering analysis using a biorthonormal pair of bases of cosine and Bessel functions.

In summary, the proposed Bessel- and cosine-based filtering approaches provide a computationally efficient and accurate alternative for modal identification in real-time power system monitoring. The results obtained from synthetic, simulated, and real synchrophasor data confirm their robustness and precision compared to traditional methods. Furthermore, the relatively low computational burden makes these approaches suitable for online implementation in control centers, strengthening situational awareness and supporting timely decision-making. This contribution represents a practical and scalable tool for enhancing the reliability and stability of modern electrical power systems.

5.2 Future work

According to our research, the following points may be explored in more depth:

- The investigation remains open to designing and testing these filters for a

different frequency range and assessing their performance in these scenarios.

- Other adaptable mathematical functions can be incorporated to design filters and extract single-component signals.
- Direct calculation of the sampling of the filter functions (L), performed by prioritizing the excitation of higher-energy modes while neglecting the lower-energy modes.
- Real-time implementation of the proposed methods to identify modal parameters and coherent groups using the extracted modal characteristics of power systems.

5.3 Products derived from this work

The following works have resulted from this research.

5.3.1 Journals

1. “Electromechanical modes identification based on an iterative Eigenvalue decomposition of the Hankel matrix”. IEEE Transactions on Power Systems. Published – 2022 [50].
2. “Web-based Wide-Area Monitoring Platform for Ringdown and Clustering Analytics in Power Systems”. International Journal of Electrical Power & Energy Systems. Published – 2025 [69].
3. “Bessel and Cosine Filtering Approaches for Electromechanical Modes Identification”. International Journal of Electrical Power & Energy Systems. Published – 2025 [70].
4. “Stability Analysis of Power Systems with High Renewable Penetration Using Electromechanical Modes Extracted via Fourier-Sine Filters and Lyapunov Exponents”. International Journal of Electrical Power & Energy Systems. Published – 2025 [71].
5. “Power Oscillation Estimation using Tuned Biorthonormal Taylor-Fourier Filters”. IEEE Transactions on Power Systems. Under review.

5.3.2 Conference

1. “A Python-based Ringdown Analysis Toolbox for Electromechanical Modes Identification”. International Conference on Smart Grid Synchronized Measurements and Analytics (SGSMA). Published – 2022 [72].
2. “On-line System Identification of Power System Linear Models”. IEEE Power & Energy Society General Meeting (PESGM). Published – 2023 [73].
3. “Data-Driven Transient Stability Assessment by Modal Patterns-based Severity Indices”. North American Power Symposium (NAPS). Published – 2024 [74].

5.3.3 Projects

1. Identification of Electromechanical Modes Driven by Ringdown Analysis. *Graphical User Interface for modal analysis of electrical power systems 2021*. Available in [75].
2. Wide Area Monitoring System Mexico. *Open-source research platform for monitoring in real-time the Mexican power system 2022*. Available in <http://wams.fie.umich.mx/cict/app/Home> [76].
3. Ringdown oscillations’ monitoring and analytics (ROMA). *WAMS - Ringdown analysis web page, 2022*. Available in <http://wams.fie.umich.mx/cict/roma> [77].

5.3.4 Technical report

1. “Application of spatio-temporal data-driven algorithms for security assessment”. Institute of Electrical and Electronics Engineers (IEEE). Published – 2022 [78].
2. “Application of Data-Driven and Machine Learning Algorithms for the Secure Operation of Transmission Systems”. Institute of Electrical and Electronics Engineers (IEEE) Task Force Published - 2024 [79].

Appendix A

Eigensystem Realization Algorithm

A.1 Introduction

The eigensystem realization algorithm (ERA) is a system identification technique that is widely used for the modal identification and reducing linear systems. The ERA is based on the singular value decomposition (SVD) of the Hankel matrix $\mathbf{H}_0$ associated with the ringdown behavior of a linear system [52, 53, 80]. The Hankel matrices are typically assembled using all of the available data such that the top left-most elements of $\mathbf{H}_0$ is y_0 value, and the bottom right-most element of the shifted $\mathbf{H}_1$ is y_N value.

A.1.1 Eigensystem Realization Algorithm for single-channel

Given the time-series data of recorded signal $y(k)$ with N samples, establish r which is the size of the Hankel matrix $\mathbf{H}$ and ensure that r meets:

$$r = \frac{N}{2} - 1, \tag{A.1}$$

this choice of r assumes that the number of data points is sufficient such that $r > n$, and holds $\mathbf{H}_0$ and $\mathbf{H}_1$.

To construct the Hankel matrix $\mathbf{H}_0$ and its shifted Hankel matrix $\mathbf{H}_1$ using the

elements of the recorded data signal $y(k)$, such as:

$$\mathbf{H}_0 = \begin{bmatrix} y_0 & y_1 & \cdots & y_r \\ y_1 & y_2 & \cdots & y_{r+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_r & y_{r+1} & \cdots & y_{N-1} \end{bmatrix}, \quad (\text{A.2})$$

$$\mathbf{H}_1 = \begin{bmatrix} y_1 & y_2 & \cdots & y_{r+1} \\ y_2 & y_3 & \cdots & y_{r+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{r+1} & y_{r+2} & \cdots & y_N \end{bmatrix}, \quad (\text{A.3})$$

then apply SVD to $\mathbf{H}_0$:

$$\mathbf{H}_0 = \mathbf{U} \mathbf{\Sigma} \mathbf{V}^T. \quad (\text{A.4})$$

To separate $\mathbf{H}_0$ into two components, a n large (nonzero in the case of noiseless measurements) and s small (zero in the case of noiseless measurements) singular values. This is to extract singular values with the higher energy.

$$\mathbf{H}_0 = \begin{bmatrix} \mathbf{U}_n & \mathbf{U}_s \end{bmatrix} \begin{bmatrix} \mathbf{\Sigma}_n & 0 \\ 0 & \mathbf{\Sigma}_s \end{bmatrix} \begin{bmatrix} \mathbf{V}_n^T \\ \mathbf{V}_s^T \end{bmatrix}, \quad (\text{A.5})$$

this way, it can be considered the approximate high-rank Hankel matrix $\mathbf{H}_0$ by a reduced-rank n matrix:

$$\mathbf{H}_0 \approx \mathbf{U}_n \mathbf{\Sigma}_n \mathbf{V}_n^T. \quad (\text{A.6})$$

Then compute the discrete system matrix $\mathbf{A}$

$$\mathbf{A} = \mathbf{\Sigma}_n^{-\frac{1}{2}} \mathbf{U}_n \mathbf{H}_1 \mathbf{V}_n^T \mathbf{\Sigma}_n^{-\frac{1}{2}}, \quad (\text{A.7})$$

then, z_i roots are obtained from:

$$z_i = \text{eig}(\mathbf{A}). \quad (\text{A.8})$$

From the roots, the parameters frequency $\hat{f}_i$ and damping $\hat{\sigma}_i$ can be calculated as $\hat{f}_i = \text{Im}(\lambda_i)$ and $\hat{\sigma}_i = \text{Re}(\lambda_i)$ where λ_i is:

$$\lambda_i = \frac{\ln(z_i)}{\Delta t}, \quad (\text{A.9})$$

where Δt is time (t) sampling.

A.1.2 Eigensystem Realization Algorithm for multiple-channel

For multiple outputs channels, a set of M signals is shaped in matrix form such that $\mathbf{Y}_m \in \mathbb{R}^{N \times m}$, that is m -column arrays corresponding to single channels, as follows:

$$\mathbf{Y}_m = [\mathbf{y}^{\{1\}} \ \mathbf{y}^{\{2\}} \ \dots \ \mathbf{y}^{\{q\}} \ \dots \ \mathbf{y}^{\{m\}}], \quad (\text{A.10})$$

with $\mathbf{y}^{\{q\}} = [y(0) \ y(1) \ \dots \ y(N-1)]^T$.

The Hankel matrix $\mathbf{H}_0$ can be also derived for multiple output channels by using sub-matrices that are constructed by each signal. This is called a block Hankel matrix and is given by

$$\tilde{\mathbf{H}}_0 = [\mathbf{H}^1 \ \mathbf{H}^2 \ \dots \ \mathbf{H}^m]^T. \quad (\text{A.11})$$

Thus, this block Hankel matrix and its shifted Hankel matrix $\tilde{\mathbf{H}}_1$ are replaced such as in (A.4), enabling to extend for multiple channels. Hence, the next steps are the same to estimate the roots z_i using multiple channels.

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