Application of the Cournot and Stackelberg Mixed Duopoly Models for Mexican Real Estate Market

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Abstract. This paper investigates the Mexican real estate market, especially the economy class homes. This sector plays an important role for Mexican social stability. Authors investigate the Cournot and Stackelberg Mixed Duopoly models where there is a competition between a state-owned public firm maximizing domestic social surplus, and a private firm, which maximizes its own profit. Such models are used for modeling markets with low number of participants and high impact of government.

1 Introduction

The crediting system has a crucial role for the society in Mexico due to many reasons. Some of them are: the deficit of apartments in the renting segment. Next, the problem of poverty is still very important to Mexico, as around 50% of population have monthly incomes below 3000-4000 pesos. This part of population depends on the state-owned projects of the construction and financing the real estate.

One of the most important projects is INFONAVIT system. Every officially working citizen has a saving account, to where the part of paid tax is being transferred. On house/apartment purchase, one can use saved money and the rest of real estate's volume is financed with credit. This credit has governmental guaranties in case of a default.

The computational game theoretic modeling tool offered in this paper is composed as a mixed complementarity problem (MCP). Numerical experiments are programmed in computing language GAMS.

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This paper is structured as follows: the introduction is followed by the section with mathematical description. The analysis of obtained average credit rates and conclusions will finish the paper.

At the present time, the mortgage system (both private and public sectors) is offering changing and fixed interest rates. Fixed rates are around 11-16% and special rates are about 7-8%, but being calculated on the base of minimal salaries. If inflation rate is higher than salary increments, the credit giving bank counts a loss in real money value. If minimum salary grows faster than the inflation, the bank has an increase in actual credit value. During last years, as the figure 1 illustrates, in average there is an increase of the minimal salary and inflation state quite constant (4.5%) (Villar (2007)).

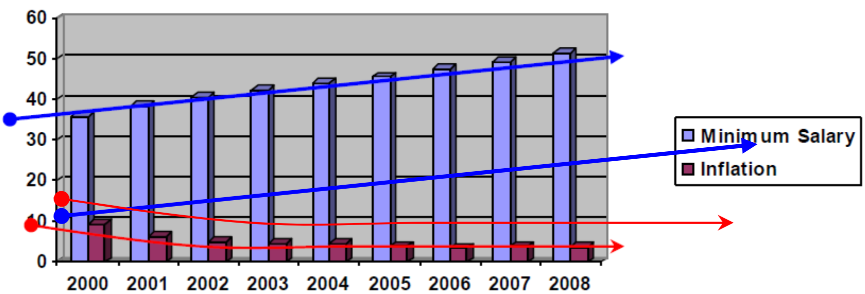

Fig. 1 The relationships between minimal salaries and inflation rate in Mexico.

The reason for using Cournot-Nash and Stackelberg Mixed models is mainly discussed in Section 2.1. These models are very popular for modeling markets with oligopolistic structures.

2 Formulation of Models

2.1 Formulation of the Model and Cournot-Nash Equilibrium

An examination of mixed oligopolies, in which social surplus-maximizing public firms compete against profit-maximizing private firms, have become increasingly popular in recent years. For pioneering works on mixed oligopolies, see Merrill and Schneider (1966), Harris and Wiens (1980), and Bös (1986, 1991). Excellent surveys can be found in Vickers and Yarrow (1988), De Fraja and Delbono (1990), Nett (1993).

The interest in mixed oligopolies is high because of their importance in economies worldwide (see Matsushima and Matsumura, 2003) for analysis of "herd behavior" by private firms in many branches of the economy in Japan.

In the paper by Matsumura (2003), the author investigates mixed duopoly and analyzes a desirable role (either leader or follower) of the public firm. This paper examines the desirable roles of both the foreign private agent and the domestic public firm. The following figure gives a graphical interpretation for different types of models (STRA: Cournot Nash model, STACK : Stackelberg, COMP : perfect competition).

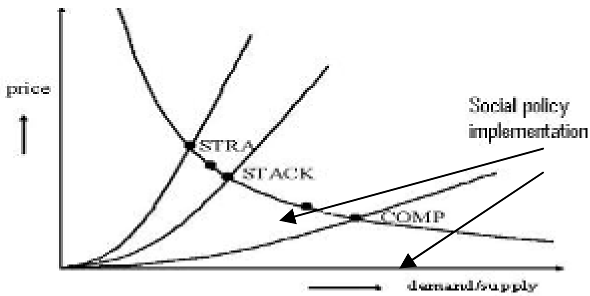

Fig. 2 Social policy: lower interest rates under same type of the model.

A mathematically extended version of applied model by Kalashnikov et al. (2007) was published in the Proceedings of the 2nd ICICI Conference.

As discussed in sections above, market with two firms offering a homogeneous product (credit) . Let G represent the total credit output, and $p(G)$ denote an inverse demand function, i.e. the interest rate consumer has to pay. Let $q_{i,t}$ ***i=1,2, t=1,…,n***, denote the output of firm ***i*** using the capital source ***t***. Same list of sources both private and public firms has been assumed. At last, let $c_{i,t}(q_{i,t})$ stand for the production cost by firm ***i*** for the technology ***t***. As G is the total output, one has

$$G = \sum_{t=1}^{n} (q_{1,t} + q_{2,t}) \quad . \tag{1}$$

Firm 1 is a foreign private firm, which maximizes its own profits, and firm 2 is a domestic public firm that maximizes domestic social surplus. Domestic social surplus S is the sum of consumer surplus and profits of firm 2, and is given by:

$$S(G, q_2) = \int_0^G p(x)dx - \sum_{t=1}^{n} \left[p(G) q_{1,t} + c_{2,t}(q_{2,t}) \right] \ ; \tag{2}$$

here, of course $\quad q_2 = \sum_{t=1}^{n} q_{2,t}, q_1 = \sum_{t=1}^{n} q_{1,t}, q_1 = G - q_2 .$

The profit of firm 1 is given by: $\Pi(G,q_1) = p(G)q_{1,t} - \sum_{t=1}^{n} c_{1,t}(q_{1,t})$. (3)

First consider the classical Cournot equilibrium, i.e., a vector $Z = (G, q_{1,1},...,q_{1,n}, q_{2,1},...,q_{2,n}) \in R_+^{2n+1}$, such that:

$$G = \sum_{t=1}^{n}\sum_{i=1}^{2} q_{i,t}\ , \tag{4}$$

$$\begin{aligned} &q_{1,t} \geq 0,\ \ \phi_{1,t} \equiv c_{1,t}^{'}(q_{1,t}) - q_{1,t}p^{'}(G) - p(G) \\ &\geq 0,\ q_{1,t}\phi_{1,t} = 0\ \forall t \end{aligned}\ ; \tag{5}$$

$$\begin{aligned} &q_{2,t} \geq 0,\ \ \phi_{2,t} \equiv c_{2,t}^{'}(q_{2,t}) + (G - q_{2,t})p^{'}(G) \\ &- p(G) \geq 0,\ \ q_{2,t}\phi_{2,t} = 0,\ \ \forall t \end{aligned}\ . \tag{6}$$

Problem (4) – (6) is a standard complementarity problem. Therefore, applying analogous methods as in Kalashnikov et al. (2007), one can obtain the existence and the uniqueness of the equilibrium.

2.2 Stackelberg Model with Leadership of Domestic (Public) Firm

First, in this Section, the game where firm 2 (public one) is the leader is examined.

Firm 2 chooses its output volume $q_{2,t},\ \ t = 1,...,n$, and firm 1 (private one) chooses $q_{1,t},\ \ t = 1,...,n$ after having observed $q_{2,t},\ \ t = 1,...,n$, so as to maximize its net profit

$$\Pi(G,q_1) = p(G)q_1 - \sum_{t=1}^{n} c_{1,t}(q_{1,t})\ , \tag{7}$$

where $q_2 = \sum_{t=1}^{n} q_{2,t}, q_1 = \sum_{t=1}^{n} q_{1,t}, q_1 = G - q_2$.

Let $q_1 = q_1(q_2) \geq 0$ be the (optimal) reaction function of firm **1**; that is, the value that satisfies the equality:

$$\phi_1(q_1(q_2)) \equiv \frac{\partial}{\partial q_1}\Pi(G, q_1(q_2)) \leq 0, \text{ and} \qquad (8)$$
$$\phi_1(q_1(q_2)) \cdot q_1(q_2) = 0$$

Definition 2.2.1. A Stackelberg equilibrium (with the domestic firm as a leader and the foreign firm as a follower) is the vector $Z = (G^{F,L}, q_{1,1}^F(Q^L), ..., q_{1,n}^F(Q^L), Q^L) \in R_+^{n+2}$ such that

$$G^{F,L} = \sum_{t=1}^{n}\left\{q_{1,t}^F(Q^L)\right\} + Q^L, \qquad (9)$$

$$Q^L \in Arg\max\left\{S_1(Q) \;\middle|\; Q \geq 0\right\}, \qquad (10)$$

$$q_1^F\left(Q^L\right) = \arg\max\left\{\Pi\left(G^{F,L}, q_1\right) \;\middle|\; q_1 \geq 0\right\}. \qquad (11)$$

where $Q^L = \sum_{t=1}^{n} q_{2,t}^L$ - total output of the domestic company and $q_1^F = \sum_{t=1}^{n} q_{1,t}^F(Q^L)$ - optimal response of the private firm. The following three relationships

$$G = Q^L + q_1^F, \qquad (12)$$

$$q_{1,t}^F \geq 0, \quad \psi_{1,t} \equiv c_{1,t}^{'}(q_{1,t}^F) - q_{1,t}^F p^{'}(G) \qquad (13)$$
$$-p(G) \geq 0, \quad q_{1,t}^F \psi_{1,t} = 0 \; \forall \;;$$

$$q_{2,t}^L \geq 0, \quad \psi_{2,t} \equiv c_{2,t}^{'}(q_{2,t}^L) + (G - q_{2,t2,t}^L) p^{'}(G) \qquad (14)$$
$$-p(G) \geq 0, \quad q_{2,t}\psi_{2,t} = 0, \quad \forall t$$

define a complementarity problem, analogous to the one discussed in Kalashnikov et al. (2007).

2.3 Stackelberg Model with Leadership of Foreign (Private) Firm

Now consider the game where firm 1 (foreign private firm) is a leader. Firm 1 chooses it $q_{1,t}$, $t = 1,...,n$ and firm 2 (domestic or public supplier) chooses

$q_{2,t}$, $t = 1,...,n$ after having observed $q_{1,t}$, $t = 1,...,n$, so as to maximize domestic social surplus:

$$S(G, q_2) = \int_0^G p(x)dx - \sum_{t=1}^{n} \left[p(G) q_{1,t} + c_{2,t}(q_{2,t}) \right] \tag{15}$$

where $q_2 = \sum_{t=1}^{n} q_{2,t}, q_1 = \sum_{t=1}^{n} q_{1,t}, q_1 = G - q_2$.

Definition 2.3.1. A Stackelberg equilibrium (with the foreign firm as a leader and the domestic firm as a follower) is the vector $Z = (G^{L,F}, Q_1^L, q_{2,1}^F(Q_1^L), ..., q_{2,n}^F(Q_1^L)) \in R_+^{n+2}$ such that

$$G^{L,F} = Q_1^L + q_2^F\left(Q_1^L\right), \tag{16}$$

$$Q_1^L \in Arg \max \left\{ \Pi_1\left(Q_1\right) \middle| \; Q_1 \geq 0 \right\}, \tag{17}$$

$$q_2^F\left(Q^L\right) = \arg\max \left\{ S\left(G^{L,F}, q_2\right) \middle| \; q_2 \geq 0 \right\}. \tag{18}$$

where $Q^L = \sum_{t=1}^{n} q_{1,t}^L$ - total output of the private firm and $q_2^F = \sum_{t=1}^{n} q_{2,t}^F(Q^L)$ - optimal response of the domestic company. Following three sentences

$$G = Q^L + q_2^F, \tag{19}$$

$$\begin{aligned} & q_{1,t}^L \geq 0, \quad \varphi_{1,t} \equiv c_{1,t}^{'}(q_{1,t}^L) - q_{1,t}^L p^{'}(G) \\ & -p(G) \geq 0, \quad q_{1,t}^F \varphi_{1,\text{t}} = 0 \; \forall t \end{aligned} ; \tag{20}$$

$$\begin{aligned} & q_{2,t}^F \geq 0, \quad \varphi_{2,t} \equiv c_{2,t}^{'}(q_{2,t}^F) + (G - q_{2,t2,t}^F) p^{'}(G) \\ & -p(G) \geq 0, \quad q_{2,t}^F \varphi_{2,t} = 0, \quad \forall t \end{aligned} \tag{21}$$

define a complementarity problem, analogous with to the one discussed in Kalashnikov et al. (2007).

2.4 Perfect Competition

Besides, it is instructive to compare the values of the produced electricity in cases of oligopolistic behaviors with the total value of the perfect competition equilibrium, that is, when the producers ignore variations in price and solve the following complementarity problem: Find a q_1, q_2 such that

$$\begin{aligned} &\beta(q_i) \equiv c_i'(q_i) - p(G(q_i)) \geq 0, \\ &\text{and } \beta(q_i) q_i = 0, \quad \forall i = 1,2, \end{aligned} \tag{22}$$

and the total value of the electricity produced in the perfect competition scenario is equal $G = q_1 + q_2$. The inverse demand is:

$$G = D_o \cdot \left(\frac{p(G)}{p_o} \right)^{\sigma} , \tag{23}$$

where D_o reference credit demand, p_o weighted average interested rate on the credit, σ - elasticity parameter.

3 Theoretical Results

In order to illustrate obtained theoretical results, numerical experiments for the classical Cournot-Nash model and the mixed Cournot-Nash one were performed. For the inverse demand function, the function of form (23) has been chosen. Elasticity parameter σ was assumed to be equal to 0.9., which refer to dependence of the credit volumes on the interest rate (Figure 3) .This figure represents the increase in time of the total credit volume (billions of pesos), the inflation rates, and the average interest rates respectively.

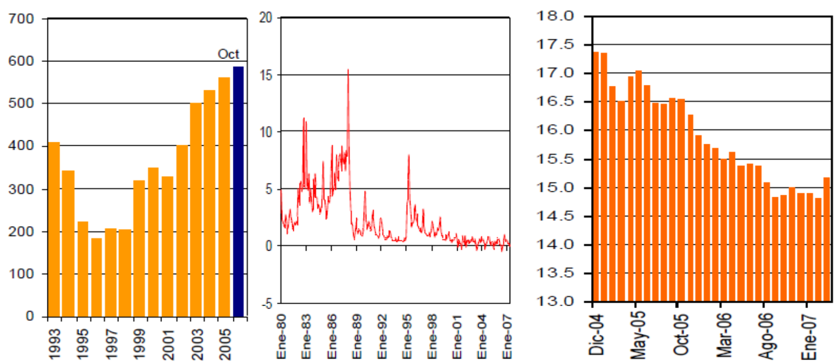

Fig. 3 Total credit volume, the inflation rate, average interest. (Source: Bank of Mexico)

The socially-oriented model offers the credit rates under the existing rates – around 6.7%. The classical model yields the equilibrium rate equal to 9.3%, which is still lower than actually offered rates. This difference can be explained by underestimation of non-payment risks, which is a goal of the further analysis.

4 Conclusion

In the above analysis, we investigated three different types of equilibrium in the duopoly with a private (foreign) agent aiming at maximization of its own profit, and a domestic firm maximizing domestic social surplus. After presenting the Cournot equilibrium in the above-described model, we examined two versions of Stackelberg game, with the private firm as a leader and domestic one as a follower, and vice versa.

In order to compare the equilibrium volumes in various scenarios we introduce the concepts of a weak and a strong firm, in dependence on the sign of the agent's optimal reaction function's derivative at the Cournot equilibrium. With such a characteristic, it turns out that if the inverse demand function is convex, then the state owned market agent is always weak, and vice versa: if the inverse demand function is concave, then the domestic agent is always strong.

For the Stackelberg equilibrium with the public owned agent as a leader, we obtain that the production volume by the leader (and hence, the total cleared market volume) is higher than that in the Cournot equilibrium, if the private firm (the follower) is strong. Otherwise, if the private agent is weak, then the total cleared market volume is lower with the domestic producer as a leader (Kalashnikov et al. (2007)).

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