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Abstract

We investigate Stackelberg mixed duopoly models where a state-owned public firm maximizing domestic social surplus, and a foreign firm compete. We examine a desirable role (either leader or follower) of both firms. Under these conditions, the firms may have two different types of optimal reaction at the Cournot equilibrium: weak or strong. We compare the profits and domestic social surplus and compare the volume of commodities for various combinations of weak and strong leaders and followers.

Keywords: Stackelberg equilibrium, mixed duopoly model

1.Introduction

Examinations of mixed oligopolies, in which social surplus-maximizing public firms compete against profit-maximizing private firms, have become increasingly popular in recent years. For pioneering works on mixed oligopolies, see [2], [5]. Excellent surveys can be found in [1], [6].

The interest in mixed oligopolies is high because of their importance in economies of Europe (Germany, England and others), Canada and Japan. There are examples of mixed oligopolies in United States such as the packaging and overnight-delivery industries. Mixed oligopolies are also common in the East European and former Soviet Union transitional economies, in which competition among public and private firms exists or still exists in many industries such as banking, house loan, airline, telecommunication, natural gas, electric power, hospital, health care, railways and others.

These situations have been investigated in different ways. Many works analyzed Cournot and Stackelberg models with the role of each firm given exogenously. However, it is more reasonable to assume that each firm decides what actions to take and when to take them.

DeFraja and Delbono [1] are pioneers in these kinds of works. They showed that in simultaneous-move games, privatization of the public firm may improve welfare. In the paper by Matsumura [4], the author investigates mixed duopoly and analyzes a desirable role (either leader or follower) of the public firm, when the inverse demand function $p = p(G)$ is concave. Under these conditions, Matsumura founds that the role of he public firm should be that of the leader. Matsumura also establishes the domestic social surplus in a mixed duopoly is greater than in a monopolistic market.

We want to examine the desirable role of the domestic public firm. In contrast to [4], here we consider an inverse demand function convex. In this case the role of the domestic public firm could be the leader or the follower, this role depends on the reaction function of private foreign firm.

In section 2, the model is described. We demonstrate theorems of existence and uniqueness in the Cournot problem. We define the concept of a *strong firm* and a *weak firm*. These definitions are based on the reaction function of every firm at the Cournot equilibrium.

Section 3 deals with the game where the domestic public firm is the leader and the private foreign firm is the follower. The private foreign firm will always be a weak firm.

Section 4 considers the game where the domestic public firm is the follower and the private foreign firm is the leader. The domestic public firm may have two different types of optimal reaction at the Cournot equilibrium, and as a consequence this firm could be weak or strong. The paper is finished with Conclusion.

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2. The Model Specification

Consider two firms producing a homogeneous product. Let G represent the total output, and $p(G)$ denote an inverse demand function, i.e. the price of a unit of the product. The goods produced by the two firms are sold at the domestic market. Let q_i, $i=1,2$, denote the output of firm i. At last, let $f_i(q_i)$ stand for the production cost by firm i. Let G be the total output; i. e.

$$G = q_1 + q_2 \ . \tag{1}$$

Firm 1 is a foreign private firm, which maximizes its own profits and firm 2 is a domestic public firm which maximizes domestic social surplus. Domestic social surplus S is the sum of consumer surplus and profits of firm 2:

$$S(q_1,q_2) = \int_0^{q_1+q_2} p(x)dx - (p(G)q_1 + f_2(q_2)) \ . \tag{2}$$

The profit of firm 1 is given by:

$$\Pi_1(q_1,q_2) = p(G)q_1 - f_1(q_1). \tag{3}$$

We accept the following assumptions concerning the price function and cost functions:

A1. Let $p(G) \ge 0$ be a twice continuously differentiable convex function defined over $G > 0$, with $p'(G) < 0$, and $p(G)G$ being a strictly concave function.

A2. We assume that $f_i(q_i)$, $i=1,2$, are functions continuously differentiable, non decreasing and convex, defined over $q_i \ge 0$.

A3. For $i=1$, there exists an $H_1 > 0$, such that:

$$f_1'(H_1) = p(H_1)$$

whereas for $i=2$, there exists an $H_2 > 0$, such that:

$$p(H_2) - H_1\left(1 + \frac{H_1}{H_2}\right)p'(H_2) = f'_2(H_2).$$

A4. Principle of Potential Participation.
For $i=1$ there exist $G_0 > 0$ and $q_1^0 > 0$ such that $G < G_0$ implies that $q_1 < q_1^0$:

$$p(G) + p'(G)G - f_1'(q_1) > 0 \ . \tag{4}$$

Remark 1. Examples of functions that satisfy **A1, A3** and **A4** are: $p(G) = AG^{-\gamma}$; with $0 < \gamma < 1$; among others. ■

Remark 2. As $p'(G) < 0$ and $\lim_{G\to\infty} p'(G) = 0$, the relationship

$$p(H_2) - H_1\left(1 + \frac{H_1}{H_2}\right)p'(H_2) = f'_2(H_2)$$

from assumption **A3** implies that there exists an $H_3 > 0$ such that

$$p(G) - H_1\left(1 + \frac{H_1}{H_2}\right)p'(G) - f'_2(q_2) < 0$$

for all $G \ge q_2 > H_3$. ■

Lemma 1. *Under assumptions A1 and A2, firm 1's profits function $\Pi_1(q_1,q_2)$ is concave with respect to q_1.* ■

Lemma 2. *Under assumptions A1 and A2, the domestic social surplus function $S(q_1,q_2)$ is concave with respect to q_2.* ■

We compare the equilibrium volumes for various scenarios. First we define the classical Cournot equilibrium, i.e., a vector $Z = (G, q_1, q_2) \in R_+^3$, such that

$$G = \sum_{i=1}^{2} q_i \ , \tag{5}$$

$$q_1 \ge 0, \ \varphi_1 \equiv f'_1(q_1) - q_1 p'(G) - p(G) \ge 0, \text{ and } q_1\varphi_1 = 0; \tag{6}$$

$$q_2 \ge 0, \ \varphi_2 \equiv f'_2(q_2) + (G - q_2)p'(G) - p(G) \ge 0, \text{ and } q_2\varphi_2 = 0. \tag{7}$$

Theorem 1. *Let assumptions A1 - A4 be valid. Then problem (5) – (7) has a (nontrivial) solution.* ■

Definition 1. Let $Z = [G, q_1, q_2]$ be an equilibrium, i. e. a solution of Cournot model (5) – (7). We say that this equilibrium is **non-monopolistic** if $q_i < G, \ i = 1,2$.

Theorem 2. *Under assumptions A1 – A4 the cleared market quantity G is the same at each non-monopolistic equilibrium.* ■

Definition 2. We say that firm i is ***strong*** if the right derivate of its reaction function is positive at the Cournot equilibrium: $q'_i(G^C + 0) > 0$. On the other hand, firm i is called ***weak*** if its reaction function's derivative has a non-positive value at the Cournot equilibrium: $q'_i(G^C + 0) \le 0$.

In other words, firm i is strong if it is ready to increase its production volume beyond its Cournot optimal response value even if the price is going down. Otherwise, firm i is a weak firm.

Now we accomplish a comparative analysis for various strategies of the firms. We are going to compare the volume of the Cournot equilibrium

G^C with the Stackelberg equilibriums when domestic public firm is leader $G^{F,L}$ and when private firm is leader $G^{L,F}$. Also we compare the domestic social surplus of domestic firm and the profits of private firm.

3. Leadership of the Domestic (Public) Firm

First, we consider the game where firm 2 (public one) is the leader. This corresponds to the agents' strategies selected as follows: $e_2 = 2, e_1 = 3$. In other words, firm **2** chooses q_2 and firm **1** (the private one) chooses q_1 after observing q_2, so as to maximize its net profit $\Pi(G, q_1) = p(G)q_1 - f_1(q_1)$.

Let $q_1(G)$ be the reaction function of firm **1**; that is, the value that satisfies:

$$q_1(G) = \arg\max_{q_1 \geq 0} \Pi(G, q_1). \qquad (8)$$

By the strict convexity of the cost function f_1 and by assumption **A3**, this reaction function is well-defined.

Lemma 3. *Under assumptions A1 – A4,*

||||$q'_1(G) < 1$ *for all* $G \neq \tilde{G}_1$. (9)

Moreover, $q'_1(\tilde{G}_1 - 0) < 0$, *and* $q'_1(\tilde{G}_1 + 0) = 0$.|||||■
||||||

Now firm 2 (domestic producer) chooses $q_2 \equiv Q \geq 0$ so as to maximize

$$S_2(Q) \equiv S(G(Q), Q) = \\ = \int_0^{G(Q)} p(q)dq - p(G(Q))q_1(G(Q)) - f_2(Q), \qquad (10)$$

where | $G(Q) = q_1(G(Q)) + Q$.

Definition 3 **.** Now we recall that the Stackelberg equilibrium (with the domestic firm as a leader and the foreign firm as a follower) is the vector $Z = \left(G^{F,L}, q_1^F(Q^L), Q^L\right) \in R_+^3$ such that

$$G^{F,L} = q_1^F(Q^L) + Q^L, \qquad (11)$$

$$Q^L \in Arg\max\{S_1(Q) \mid Q \geq 0\}, \qquad (12)$$

$$q_1^F(Q^L) = \arg\max\{\Pi(G^{F,L}, q_1) \mid q_1 \geq 0\}. \qquad (13)$$

Now we establish relationships to compare the production volumes of the firms at the Stackelberg equilibrium (11) – (13) with those at the Cournot equilibrium defined by the complementarity problem (5) – (7). Besides, it is interesting to compare the values Q^C and Q^L to the domestic firm's optimum output volume Q^P at the perfect competition equilibrium, that is, when the domestic producer ignores the price variation and solves the following complementarity problem: Find a $Q \geq 0$ such that

$\beta_2(Q) = f'_2(Q) - p(G(Q)) \geq 0,$ and $\beta_2(Q)Q = 0$. (14)

Theorem 3. *Under assumptions A1 – A4, the following estimates hold:*

$$0 < Q^P \leq \min\{Q^C, Q^L, H_3\}; \qquad (15)$$

here H_3 *is the parameter from* ***Remark 2****.* |■

Note that the estimates obtained in Theorem 3 involve the expressions $\min\{Q^C, Q^L\}$ and $\max\{Q^C, Q^L\}$, because assumptions **A1 – A4** in general do not imply the (strict) concavity of the leader's (domestic social surplus) function $S_2(Q)$ over all $Q \geq 0$. Now we introduce an additional assumption which allows one to establish this concavity of the domestic social surplus function and hence deduce more exact global comparative static results making use of some local information only.

A5. Assume that the foreign firm's cost function is linear:

$$f_1(q_1) = c_1 q_1, \text{ for all } q_1 \geq 0, \qquad (16a)$$

where $c_1 > 0$ is a constant, and the inverse demand function has the following property: the ratio $\dfrac{p''(G)}{p'(G)}$ is a differentiable function of $G > 0$, and the following estimate holds:

$$\frac{d}{dG}\left(\frac{p''}{p'}(G)\right) \geq -\frac{1}{G} \cdot \frac{p''(G)}{p'(G)}. \qquad (16b)$$

Lemma 4. *Under assumptions A1 – A5 and with the leadership of the domestic supplier, the domestic social surplus function* $S_2(Q)$ *is strictly convex over* $Q \geq 0$.■

Now we obtain the complete comparison static classification for the above particular case.

Theorem 4. *Under conditions of Lemma 4, the following global estimates based upon the local information are true.*

(i) *If* $G'(Q^C - 0) \leq 1$ *and* $G'(Q^C + 0) = 1$, *then* $Q^L = Q^C$, *hence* $G(Q^L) = G(Q^C)$. *This case can occur only at the point* $Q^L = Q^C = \tilde{G}_1$.

(ii) *If* $G'(Q^C - 0) > 1$ *then* $Q^L > Q^C$, *hence* $G(Q^L) > G(Q^C)$.

(iii) *If* $G'(Q^C + 0) < 1$ *then* $Q^L < Q^C$, *hence* $G(Q^L) < G(Q^C)$. ■

4. Leadership of the Foreign (Private) Firm

Now consider the game where firm **1** (foreign private firm) is the leader. Firm **1** chooses q_1 and firm **2** (domestic or public supplier) chooses q_2 after observing q_1, so as to maximize the social surplus.

$$S(G,q_2)=\int_0^G p(q)dq-q_1p(G)-f_2(q_2); \qquad (21)$$

here, as previously, $G=q_1+q_2$.

Let $q_2(G)$ be the reaction function of firm **2**; that is, the quantity that satisfies the condition below:

$$q_2(G)=\arg\max_{q_2\geq 0} S(G,q_2). \qquad (22)$$

Definition 4. The Stackelberg equilibrium (with the foreign firm as a leader and the domestic firm as a follower) is the vector $Z=\left(G^{L,F},Q_1^L,q_2^F\left(Q_1^L\right)\right)\in R_+^3$ such that

$$G^{L,F}=Q_1^L+q_2^F\left(Q_1^L\right), \qquad (23)$$

$$Q_1^L\in Arg\max\left\{\Pi_1(Q_1)\middle| Q_1\geq 0\right\}, \qquad (24)$$

$$q_2^F\left(Q^L\right)=\arg\max\left\{S\left(G^{L,F},q_2\right)\middle| q_2\geq 0\right\}. \qquad (25)$$

In what follows we establish relationships which allow one to compare the production volumes of the market players at the Stackelberg equilibrium (23) – (25) to those at the Cournot equilibrium defined with the complementarity problem (5) – (7). Besides, it is instructive to compare the values Q_1^C and Q_1^L to the foreign firm's optimum output Q_1^P at the perfect competition equilibrium, that is, when the foreign (private) producer ignores variations in price and solves the following complementarity problem: Find a $Q_1\geq 0$ such that

$\beta_1(Q_1)\equiv f'_1(Q_1)-p(G(Q_1))\geq 0,$ and $\beta_1(Q_1)Q_1=0$.

Theorem 5. *Under assumptions A1 – A4, the following relationships are valid:*

$$0\leq Q_1^C\leq Q_1^L\leq Q_1^P\leq H_1. \qquad (26) \quad \blacksquare$$

Remark 3. Due to the property $G'(Q_1)>0$, inequalities (29) imply the relationships

$$G^C=G\left(Q_1^C\right)\leq G\left(Q_1^L\right)=G^{L,F}\leq G\left(Q_1^P\right). \qquad (27)$$

Therefore, the leadership of the private firm is better for the individual consumer than the Cournot competition between the private and public producers, as the former may bring to a lower retail price of the good at the market. ■

5. Conclusions

The paper extends the results of existence and uniqueness of the conjectural variations equilibrium at oligopolistic markets with homogeneous product and similar agents (using profit functions of the same structure with probably different parameters) to the duopoly of agents with essentially distinct object functions. One of the competitors is a foreign firm maximizing her expected profit, whereas the second agent is a domestic firm aiming at maximizing the social surplus. This diversity of the competitors' targets makes the investigation more complex as in the case of homogeneous agents, but it allows one to obtain new interesting results concerning the comparative static between the Cournot and Stackelberg formulations of the model. To do that, we introduce the concepts of weak and strong firms, in dependence of their reaction functions and show that the domestic social surplus is higher if the leader is strong than when the leader is weak.

In this paper we prove that, if the inverse demand function is convex, the domestic public firm is follower and private public firm could be strong or weak. These results complement those obtained by Matsumura [4] who demonstrate that if the demand function is concave the domestic public firm is strong and private firm is always weak.

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