## Has Mexican growth been pro-poor?

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#### Abstract

This paper proposes techniques to test for whether growth has been propoor. We first review different definitions of pro-poorness and argue for the use of methods that can generate results that are robust over classes of pro-poor measures and ranges of poverty lines. We then provide statistical pro-cedures that rely on the use of sample data to infer whether growth has been pro-poor in a population. We apply these procedures to Mexican household surveys for the years of 1992, 1998 and 2004. We find strong statistical evidence that Mexican growth has been absolutely anti-poor between 1992 and 1998, absolutely pro-poor between 1998 and 2004 and between 1992 and 2004, and relatively pro-poor between 1992 and 2004 and between 1998 and 2004. The relative assessment of the period between 1992 and 1998 is statistically too weak to lead to a robust evaluation of that period.

#### Key words

Pro-poor growth, Poverty, Inequality.

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## Resumen

Este trabajo propone técnicas para comprobar si el crecimiento ha sido a favor de los pobres. Primero repasamos las diferentes definiciones a favor de la pobreza o "pro pobres" y argumentamos por el uso de métodos que puedan generar resultados que sean robustos eligiendo entre clases de mediciones pro pobreza y rangos de líneas de pobreza. Luego generamos procedimientos estadísticos que se basen en datos muestrales y que permitan inferir si el crecimiento ha sido pro pobreza en una población. Aplicamos esos procedimientos a muestras de hogares mexicanos para los años 1992, 1998 y 2004. Encontramos fuerte evidencia estadística de que el crecimiento mexicano fue absolutamente anti pro pobreza entre 1992 y 1998, absolutamente pro pobreza entre 1998 y 2004 y entre 1992 y 2004, y relativamente pro pobreza entre 1992 y 2004 y entre 1998 y 2004. La evaluación de la medida relativa en el período entre 1992 y 1998 es muy débil estadísticamente para que conduzca a una evaluación robusta del período.

## Palabras clave

Crecimiento pro pobres, pobreza, desigualdad.

# Introduction

It would seem relatively uncontroversial to conceive of the pro-poorness of growth as referring generally speaking to the impact of growth on the wellbeing of the poor and therefore to its impact on poverty. Like many distributive concepts, however, its precise meaning and its usefulness are essentially a matter of judgement<sup>1</sup>. There are at least three elements of contention in trying to make the assessment of pro-poorness operational. The first fundamental issue in the definition of pro-poorness is whether it should be absolute or relative. A second issue is what poverty line should be chosen to separate the poor from the non-poor. A final issue is how we should assess in the aggregate the heterogeneous impact of growth among a population of heterogeneous poor, an issue which also addresses what relative normative weights are to be attributed to the different poor individuals.

<sup>&</sup>lt;sup>1</sup> See, among many recent contributions to that debate, Bourguignon (2003), Bruno, Ravallion, and Squire (1999), Dollar and Kraay (2002), Eastwood and Lipton (2001), United-Nations (2000), and World-Bank (2000).

This paper attempts to address all these three issues and to make it conceptually operational and empirically feasible to test for the pro-poorness of distributive changes. To do this, we first rely on the definitional framework of Duclos and Wodon (2004). Roughly speaking, and according to that framework, a relative definition of pro-poorness judges a distributive change to be pro-poor if the proportional change in the incomes of the poor is no less than some norm, often set as the growth rate in mean income or in some quantile such as median income. For an absolute definition, the incomes of the poor need to grow by an absolute amount that is no less than some norm, this time often set as zero or as some proportion of the absolute change in mean or median incomes. These different definitions can also be linked to the usual concepts of absolute and relative poverty. With relative poverty, the poverty line is usually defined as a proportion of some central tendency of an income distribution; with absolute poverty, the real level of the poverty line normally remains the same even if the income distribution changes.

The framework of Duclos and Wodon (2004) also enables to get around the difficult of having to choose 1) a poverty line to separate the poor from the nonpoor, and 2) a set of normative weights to differentiate among the poor. The framework does this by investigating how pro-poor judgements can be made robust to wide classes of pro-poor evaluation functions and to ranges of poverty lines.<sup>2</sup>

This paper then makes it empirically feasible to test for pro-poorness of growth. To do this, we derive the sampling distribution of the various estimators that are needed to test for absolute and relative pro-poorness. This enables us inter alia to draw confidence intervals around the differences that must be signed in order to conclude that a change has been robustly pro-poor — or not. We implement these statistical techniques taking full account of the sampling design of the surveys we use. We apply the procedures to Mexico's National Income and Expenditure Surveys collected in 1992, 1998 and 2004. We find strong evidence that

<sup>&</sup>lt;sup>2</sup> Many different approaches have been proposed to separate the poor from the nonpoor and to compute and aggregate index of pro-poorness. See, for instance, McCulloch and Baulch (1999), Kakwani, Khandker, and Son (2003), Kakwani and Pernia (2000), Ravallion and Chen (2003), Klasen (2003), Essama-Nssah (2005), Ravallion and Datt (2002) and Son (2004).

Mexican growth has been absolutely anti-poor between 1992 and 1998, absolutely pro-poor between 1998 and 2004 and between 1992 and 2004, and relatively pro-poor between 1992 and 2004 and between 1998 and 2004. The relative pro-poor assessment of the period between 1992 and 1998 is, however, statistically too weak to lead to a robust pro-poor evaluation of that period.

#### **Theoretical framework**

The setting

Let  $\mathbf{y}_1 = (\mathbf{y}_1, \mathbf{y}_2, \dots, \mathbf{y}_n) \in \mathbb{R}^n$  be a vector of non-negative initial incomes<sup>3</sup> (at time 1) of size  $n_1$ , and let  $\mathbf{y}_2 = (\mathbf{y}_1^2, \mathbf{y}_2^2, \dots, \mathbf{y}_n^2)$  be an analogous vector of posterior incomes (at time 2) of size  $n_2$ .

The following draws extensively from Duclos and Wodon (2004). To determine whether the movement from  $y_1$  to  $y_2$  is pro-poor, we first need to define a standard with which this assessment can be made. First consider the case of a relative standard, which we will take in this paper as the growth in average incomes, denoted by g. Intuitively, for growth to be relatively pro-poor, we wish the poor's "representative" income to undergo a proportional change that is no less than 1+g. That growth g can be negative as well as positive. Relative pro-poorness is consistent, for instance, with the view of Kakwani and Pernia (2000) that "promoting pro-poor growth requires a strategy that is deliberately biased in favor of the poor so that the poor benefit proportionately more than the rich. (p.3)".

Then denote by z > 0 a poverty line, defined in real terms. Let W ( $\mathbf{y}_1, \mathbf{y}_2, \mathbf{g}, z$ ) be the pro-poor evaluation function that we want to use. It is defined as the difference between two evaluation functions  $\prod (\mathbf{y}_1, z)$  and  $\prod^* (\mathbf{y}_2, 1+g, z)$ , each for time 1 and time 2, respectively, and which are analogous to poverty indices for each of the two time periods:

$$W(\mathbf{y}_1, \mathbf{y}_2, g, z) \equiv (\mathbf{y}_2, 1+g, z) - \prod (\mathbf{y}_1, z).$$
 (1)

<sup>&</sup>lt;sup>3</sup> Or consumption, wealth, or any other welfare indicator of interest.

The change from y1 to  $\mathbf{y}_2$  will be deemed pro-poor if  $W(\mathbf{y}, \mathbf{y}_2, g, z) \leq 0$ .

Clearly, whether the distributional change will be deemed pro-poor will depend on the way in which z,  $\prod$ , and  $\prod^*$  will be chosen. To put some structure on the form of W in which we should be interested, we need to invoke a few axioms. The first one is a *focus axiom*: W is not sensitive to the values of y1 that exceed z. The distribution of min( $y_i$ , z) is thus sufficient for judgements of pro-poorness.

Second, we can postulate an axiom of *population invariance*. This says that adding a replication of a population to that same population has no impact on W. It is a common axiom in welfare economics that enables us to make pro-poor judgements even when the absolute population size varies across the distributions.

A third axiom is that of *anonymity:* this says permuting the incomes of any two persons in any given distribution should not affect pro-poor judgements. Note that this axiom will typically lead to violations of the well-known Pareto efficiency criterion; our framework does not not require that none of the poor be penalized by a distributional change for that change to have a chance to be declared pro-poor. Were we not to impose this axiom, it would be practically impossible to order the initial and posterior distributions in the presence of a large number of individuals, and we would also need panel data.

The next axiom is a *normalization* one: if there has been no distributional change, and thus also no change in the mean, then W = 0.

We may then also impose an axiom of *monotonicity:* for a given g, if anyone's posterior income increases, *W* should not increase, and may sometimes fall. Increasing posterior incomes make it more likely that the distributive change will be declared pro-poor.

#### Relative pro-poor judgements

Finally, we can also axiomatize our view of *relative* pro-poorness. Formally, suppose that  $\mathbf{y}/(1+g) = \mathbf{\dot{y}}/(1+\mathbf{\dot{g}})$ . Then, according to relative judgements of pro-poorness,  $\mathbf{y}$  and  $\mathbf{\dot{y}}$  should be judged equally pro-poor by W regardless of the initial distribution  $y_1$ .

Combined together, the axioms that we have invoked until now define what we can term to be a first-order class of relative pro-poor evaluation functions. Denote that class as  $\Omega^1(g, z +)$ . The class  $\Omega^1(g, z +)$  regroups all of the functions W that satisfy the focus, the population invariance, the anonymity, the monotonicity, the normalization and the relative axioms, and for which  $z \le z +$ .

Now let  $F_j(y)$  be the distribution function of distribution j. Also define as  $Q_j(p)$  the quantile function for distribution  $F_j$ . This is formally defined as  $Q_j(p) = \inf \{s \ge 0 | F^j(s) \ge p\}$  for  $p \in [0, 1]$ . With a continuous distribution and a strictly positive income density, Q(p) is simply the inverse of the distribution function, and it is the income of that individual who is at rank p in the distribution.

The popular class of FGT indices is then given by:

$$P_{f}(z;\alpha) = \int_{0}^{P_{f}(\alpha)} (1 - Q_{f}(p)/z)^{\alpha} dp.$$
 (2)

 $P_j(z; a = 0)$  is the headcount index (and the distribution function) at *z*, and  $P_j(z; a = 1)$  is the average poverty gap. Duclos and Wodon (2004) show that a movement from  $y_1$  to  $y_2$  will be judged pro-poor by all propoor evaluation functions  $W(\cdot, \cdot, g, z)$  that are members of  $\Omega^1(g, z+)$  if and only if

$$P_2((1 + g)z; \alpha = 0) \le P_1(z; \alpha = 0)$$
 for all  $z \in [0, z^+]$ . (3)

A distributional change that satisfies (3) is called first-order relatively pro-poor since all pro-poor evaluation functions within  $\Omega 1$  (g, z +) will find that it is pro-poor, and this, for any choice of poverty line within [0, z +] and any W that obeys the above-defined axioms. Verifying (3) simply involves checking whether — over the range of poverty lines [0, z +] — the headcount index in the initial distribution is larger than the headcount index in the posterior distribution when that distribution is normalized by 1 + g.

An alternative and equivalent way of checking whether a distribu-

tional change can be declared first-order relatively pro-poor is to compare the ratio of the quantiles to the ratio of the mean, or again equivalently, to compare the growth in quantiles to the growth in the mean. That this, we check whether, for all  $p \in [0, F1 (z +)]$ ,

$$\frac{Q_2(p)}{Q_1(p)} \ge 1 + g \tag{4}$$

or whether

$$\frac{Q_2(p) - Q_1(p)}{Q_1(p)} \ge g.$$
(5)

Using (5) is equivalent to Ravallion and Chen (2003)'s suggestion to use "growth incidence curves" to check whether growth is pro-poor. These curves show the growth rates of living standards at different ranks in the population.

First-order pro-poor judgements can be demanding in expansion periods. They require *all* quantiles of the poor to undergo a rate of growth at least as large as the rate of growth in mean income. We may, however, be willing to relax this condition if the rate of growth for the poorer among the poor is sufficiently large to exceed g even though the rate of growth for the not-so-poor may be below g. An axiom that captures this is the *distribution sensitivity* axiom. It says that the evaluation functions  $\prod$  should give more weight to the poorer than to the not-so-poor among the poor. Distribution-sensitive pro-poor judgements imply that shifting incomes from the richer to the poorer is by itself a pro-poor distributional change. This axiom is known as the Pigou-Dalton principle in the welfare literature.

Adding the distribution-sensitive axiom to the earlier axioms defines a second-order class of relative pro-poor evaluation functions  $\Omega^2(g, z+)$ . Formally,  $\Omega^2(g, z+)$  is made of all functions  $W(\cdot, \cdot, g, z)$  that satisfy the focus, the population invariance, the anonymity, the monotonicity, the normalization, the distribution sensitivity and the relative axioms, and for which  $z \le z+$ .

It can then be shown that a movement from  $y^1$  to  $y^2$  will be judged propoor by all pro-poor evaluation functions W(, , g, z) that are members

of  $\Omega^2$  (*g*, *z* +) if and only if

$$P_2((1 + g)z; \alpha = 1) \le P_1(z; \alpha = 1)$$
 for all  $z \in [0, z^d]$ . (6)

As for (3), a distributional change that satisfies (6) is called second-order relatively pro-poor since all pro-poor evaluation functions will find that it is pro-poor, and this, for any choice of poverty line within [0, z +] and for any W that obeys the above-mentioned axioms for  $\Omega^2$  (g, z +). Verifying (6) simply involves checking whether the average poverty gap in the initial distribution is larger than that in the posterior distribution when that distribution is normalized by 1 + g and this, over the range of poverty lines [0, z +].

As for first-order pro-poor judgements, there are alternative ways of checking condition (6). The cumulative income up to rank p (the Generalized Lorenz curve at p) is given by

$$C_j(p) = \int_0^p Q_j(q) dq. \tag{7}$$

The use of the Generalized Lorenz curve provides an intuitive sufficient condition for checking second-order relative pro-poorness. A distributional change is indeed second-order relatively pro-poor if for all  $p \in [0, \overline{F_2}(z+)]$ ,

$$\lambda(p) \equiv \frac{C_2(p)}{C_1(p)} \ge 1 + g.$$
(8)

Expression (8) involves computing the growth rates in the cumulative incomes of proportions p of the poorest, and to compare those growth rates to g. For 1 + g equal to the ratio of mean income, condition (8) is equivalent to checking whether the Lorenz curve for  $\mathbf{y}_2$  is above that of  $\mathbf{y}_1$  for the range of  $p \in [0, F_2((1 + g)z^+)]$ .

#### Absolute pro-poor judgements

Absolute pro-poor judgements are made by comparing the absolute change in the poor's incomes to some absolute pro-poor standard. Denote that standard as *a*. The axiom of *absolute* pro-poorness says

essentially that  $\prod^*$  should be "translation invariant" in **y** and *a*, or that the pro-poor judgement should be neutral whenever the poor gain in absolute terms the same as the standard *a*. Hence, this axiom demands that if  $\mathbf{y} + a = \dot{\mathbf{y}}$ , then  $W(\mathbf{y}, \dot{\mathbf{y}}, \mathbf{a}, \mathbf{z}) = 0$ . This allows us to define the class of first-order absolute pro-poor evaluation functions  $\tilde{\Omega}^1(a, z +)$ as made of all those functions  $W(\cdot, \cdot, a, z)$  which satisfy the focus, the population, the anonymity, the monotonicity, the normalization and the absoluteness axioms, and for which  $z \le z + .$  We will later set *a* to zero for the empirical illustration of this paper.

We can then show that a movement from y1 to y2 will be judged first-order absolutely pro-poor (that is, pro-poor by all evaluation functions W (,, a, z) that are members of  $\tilde{\Omega}1$  (a, z +)) if and only if

$$P_2(z + a; a = 0) \le P_1(z; a = 0)$$
 for all  $z \in [0, z^+]$ . (9)

An equivalent way of checking whether a distributional change can be declared first-order absolutely pro-poor is to compare the absolute change in the values of the quantiles for all  $p \in [0, F(z + )]$ :

$$Q_2(p) - Q_1(p) \ge a.$$
 (10)

An analogous result holds for absolute second-order pro-poor judgements. These judgements also obey the axiom of distribution sensitivity: they are made on the basis of the class of indices  $\Omega^2(a, z +)$ , which is defined as for  $\Omega^1(a, z +)$  but with the additional requirement of distribution sensitivity. We can then show that a movement from  $y_1$  to  $y_2$  will be judged second-order absolutely pro-poor if and only if

$$(z + a)P_2((z + a; \alpha = 1) \le zP_1(z; \alpha = 1) \text{ for all } z \in [0, z^+].$$
 (11)

A sufficient condition for condition (11) is then to verify whether, for all  $p \in [0, F2 (z^+ + a)]$ , the change in the average income of the bottom p proportion of the population is larger than a:

$$C_2(p) - C_1(p) \ge ap.$$
 (12)

## **Statistical inference**

In practice, household data surveys are needed to check if growth is pro-poor or not. This forces us to deal with issues of sampling variability and statistical inference. Indeed, a difference observed in a sample may not be empirically strong enough to be significant from a statistical point of view<sup>4</sup>.

Each of the conditions noted above takes the form of testing whether  $\Delta^{s}(z) \leq 0$  or  $\Delta^{s}(p) \geq 0$  over some *range* of *z* or *p*. This therefore involves testing *jointly* over a *set* of null hypotheses. For primal tests of pro-poorness, our formulation of our null hypothesis is thus that of a union of null hypotheses

$$H_0: \Delta^s(z) > 0 \text{ for some } z \in [0, z^+]$$

$$(13)$$

to be tested against an alternative hypothesis which is an intersection of alternative hypotheses

$$H_1 : \Delta^s(z) \le 0$$
 for all  $z \in [0, z^*]$ . (14)

For dual tests, we use a union of null hypotheses

$$H_0: \Delta^*(p) < 0$$
 for some  $p \in [0, 1]$  (15)

to be tested against an intersection of alternative hypotheses

$$H_1: \Delta^s(p) \ge 0 \text{ for all } p \in [0, 1].$$

$$(16)$$

Our decision rule will be to reject the union set of null hypotheses in favor of the intersection set of alternative hypotheses only if we can reject each of the hypotheses in the null set at a  $100 \cdot \theta$  % significance level. This can be conveniently carried out using a  $100 \cdot (1 - \theta)$ % one-sided confidence interval, a devise we use repeatedly in the empirical Mexican graphs below. To see how this can be done, denote by  $\hat{\Delta}^s(z)$  the sample estimator of  $\Delta^s(z)$ , by  $\Delta^s_0(z)$  its sample value, and by  $\sigma^2_{\Delta^*(z)}$  the sampling variance of  $\Delta^s(z)$ . Let  $\zeta(\theta)$  be the  $(1 - \theta)$ -quantile of the normal distribution. Given that all of the estimators used in this pa-

<sup>&</sup>lt;sup>4</sup> See Araar (2006).

per can be shown to be asymptotically normally distributed, we can use  $\Delta_{I}(z) + \sigma_{\Delta_{I}(z)}(z)$  as alternative lower and upper bounds for one-sided confidence intervals for  $\Delta^{s}(z)$ . For instance, an upper-bounded confidence interval  $\Delta_{I}(z) + \sigma_{\Delta_{I}(z)}(z)$  shows all of the values of  $\eta$  for which we could not reject a null hypothesis  $H_0 : \Delta^{s}(z) \leq \eta$  in favor of  $H^1 : \Delta^{s}(z) \leq \eta$ . Our decision rule is then to reject the set of null hypotheses (13) in favor of (14) if:

$$\Delta_{0}^{s}(z) + \sigma_{\bar{\Delta}^{s}(z)}\zeta(\theta) < 0 \quad \forall z \in [0, z^{+}].$$
 (17)

For dual tests, we proceed similarly, noting that the signs in (15) and (16) are inverted. We thus build a confidence interval  $\Delta_0(p) - \sigma_{\Delta^*(p)} \zeta(\theta)$  and reject (15) in favor of (16) if

$$\Delta_0^s(p) - \sigma_{\Delta^s(p)} \zeta(\theta) > 0 \ \forall p \in [0, 1],$$
(18)

There remains to define  $\Delta^{s}(z)$ ,  $\Delta^{*}(p)$ ,  $\sigma_{\Delta^{*}(p)}$  and  $\sigma_{\Delta^{*}(p)}$ . Let H be the number of sample observations of incomes from a distribution  $j, y_{j}^{1}, ..., y_{j}^{H}$ . Then, we have that

$$\tilde{P}_{j}(\hat{g}z;\alpha) = H^{-1} \sum_{h=1}^{H} \left(\frac{\hat{g}z - y_{j}^{h}}{\hat{g}z}\right)^{\alpha},$$
(19)

$$\tilde{Q}_j(p) = \min\left(y \left| \hat{F}_j(y) \ge p \right),$$
(20)

where the empirical distribution function is given by

$$\hat{F}_{j}(y) = \hat{P}_{j}(z;0),$$
 (21)

and that

$$\hat{g} = \hat{\mu}_2 / \hat{\mu}_1,$$
 (22)

and

$$\hat{\mu}_{j} = H^{-1} \sum_{h=1}^{H} y_{j}^{h}.$$
(23)

Using a first-order approximation, we find

$$\hat{P}_2(\hat{g}z;\alpha) - P_2(gz;\alpha) = \hat{P}_2(gz;\alpha) - P_2(gz;\alpha)$$
(24)

+ 
$$gzP_2^{gs}(gz;\alpha)\left(\frac{\bar{\mu}_2 - \mu_2}{\mu_2} - \frac{\bar{\mu}_1 - \mu_1}{\mu_2}\right)$$
 (25)

$$+ o(H^{-1/2})$$
 (26)

where  $P_3^{as}(gz; \alpha) = \alpha (gz)^{-1} (P_2(gz; \alpha - 1) - P_2(gz; \alpha)) > 0$  for  $\alpha > 0$  and  $P_2^{as}(gz; 0) = f_2(gz) > 0$  (the density at gz) for  $\alpha = 0$ . Hence, we have:

$$\hat{P}_{2}(\hat{g}z;\alpha) - P_{2}(gz;\alpha) = \hat{P}_{2}(gz;\alpha) - P_{2}(gz;\alpha)$$
(27)

+ 
$$\alpha \left( P_2(gz; \alpha - 1) - P_2(gz; \alpha) \right) \left[ \left( \frac{\hat{\mu}_2 - \mu_2}{\mu_2} \right) - \left( \frac{\hat{\mu}_1 - \mu_1}{\mu_1} \right) \right]$$
 (28)

$$+ o(H^{-1/2}).$$
 (29)

Therefore, for  $\alpha > 0$ , we can express  $\Delta^{*}(z) - \Delta^{*}(z)$  as

$$\hat{\Delta}^{*}(z) - \Delta^{*}(z) = \hat{A} - \hat{B} + o(H^{-1/2})$$
  
(30)

where

$$\hat{A} = \left(\hat{P}_2(gz;\alpha) - P_2(gz;\alpha)\right) + \alpha \left(P_2(gz;\alpha-1) - P_2(gz;\alpha)\right) \left(\frac{\hat{\mu}_2 - \mu_2}{\mu_2}\right)$$
(31)

$$\hat{B} = \left(\hat{P}_1(z;\alpha) - P_1(z;\alpha)\right) - \alpha \left(P_2(gz;\alpha-1) - P_2(gz;\alpha)\right) \left(\frac{\hat{\mu}_1 - \mu_1}{\mu_1}\right). \tag{32}$$

Suppose that the two empirical distributions come from independent samples, namely, the selection of the sampling units was made independently in each sample. We then have

$$\operatorname{var}\left(\hat{\Delta}^{s}(z) - \Delta^{s}(z)\right) = \operatorname{var}\left(\hat{A}\right) + \left(\hat{B}\right). \tag{33}$$

If, however, the two samples are dependent because they come, for instance, from the same panel data, then the variance must be estimated jointly over the two samples and we then have

$$\operatorname{var}\left(\check{\Delta}^{*}(z) - \check{\Delta}^{*}(z)\right) = \operatorname{var}\left(\check{A} - \check{B}\right). \tag{34}$$

For the dual or percentile approach, first-order approximations of the sampling distribution of the quantile estimator and of its cumulative function until percentile are given by:

$$\hat{Q}(p) - Q(p) = H^{-1} \frac{\sum \left( I[p^k < Q(p)] - p \right)}{f(Q(p))} + o(H^{-1/2})$$
(35)

and

$$C(p) - C(p) =$$

$$H^{-1} \sum \left( f(p) - C(p) \right) = 0 \quad \text{(36)}$$

$$\frac{1}{1 + o(H^{-1/4})} \sum_{i=1}^{n} (1/\mu < se(0)) - p (se(0)) + \mu + \mu + se(se(0)) - c (p))$$
(37)  
(37)

(38)

where  $I[y^h < Q(p)]$  is is an indicator function taking the value of 1 if its argument is true and 0 otherwise.

Supposing that the number of primary sampling units increases asymptotically to infinity, we can then estimate the sampling distribution of the above estimators in  $\Delta(a)$  and  $\Delta(b)$  taking full account of the survey design. This is done using the procedure described in Duclos and Araar (2006), pages 284–287, a procedure which takes into account the sampling weights, the sampling design and the number of statistical units (individuals) within each of the last sampling units (each household observation in the sample).

#### Has the Mexican economy been pro-poor?

We apply the above methodology using Mexican data spanning the last decade and a half. Mexico is a particularly interesting economy over which to test the pro-poorness of growth. Mexico has indeed undergone very significant economic changes since 1990. After the 1994-1995 economic crisis, which culminated in an important devaluation of its currency and was probably the most severe in the country's economic history, rapid growth in exports (facilitated by North American Free Trade and other trade agreements) as well as macroeconomic and public sector restructuring led to strong growth. Recent institutional changes have inter alia encouraged competition and growth in transportation, telecommunications, and power generation and distribution. An important issue is whether this relatively recent tidal growth has "lifted all boats".

The data used for our application come from the National Income

and Expenditure (ENIGH) Surveys collected in 1992, 1998 and 2004. These sample data are representative at the national level. The objective of the ENIGH surveys has been to collect information on incomes and expenditures, goods and services used for self-consumption, and socioeconomic characteristics and labor market activities of all household members. The sampling process was stratified and multi-staged, with the final sampling units being households and all their members.

As is common in Mexico, we use total income *per capita* as the measure of living standards for all members of a household. To adjust for temporal variation in prices, we express incomes in reference to the 2004 consumer price index. To correct for spatial variation in prices, we assess all incomes in reference to rural prices. This is done by multiplying urban household incomes by the ratio of the rural to the urban poverty line. The rural poverty line in 2004 is often estimated to be around 550 pesos per month *per capita*. We use the product of household size and household sampling weight as an expansion factor to ensure that our samples are representative of the national distribution of the living standards of individuals. For the estimation of standard errors and thus for statistical inference, we take into account the stratification and multi-stage structure of the survey design as explained at the end of Section 3.

We begin our investigation by considering the evolution of the density of *per capita* incomes in Figure 1. The distribution of per *capita income* has worsened until 1998 since the density curves have shifted to the left. It has however exhibited a strong and quick recovery between 1998 and 2004, as shown by the shift of the density curve to the right. The estimates of the Lorenz curves and Gini indices presented in Figure 2 and Table 5, respectively, suggest that inequality has decreased between 1992 and 2004. Figures 3 and 4 and the results of Table 5 suggest that absolute poverty, as measured by the headcount and poverty gap indices, has increased between 1992 and 1998 and decreased between 1998 and 2004.

Formal statistical testing for first-order absolute pro-poorness of Mexican growth can be done using the information presented in Figures 5 to 10. The top line of Figure 5 shows the sample estimates of

$$\Delta^{z}(z) = P_{1998}(z; \alpha = 0) - P_{1992}(z; \alpha = 0)$$
(39)

for the difference between 1998 and 1992, whereas the dotted bottom curve is the lower bound of the one-sided confidence interval,

$$\Delta_0^1(z) - \sigma_{\bar{\Delta}^*(z)} \zeta(\theta). \tag{40}$$

Since  $\Delta_0(z) - \sigma_{\Delta^*(z)}(\theta) > 0$  is verified on Figure 5 for all reasonable poverty lines, we can infer from our data that growth was absolutely anti-poor during the period 1992 and 1998. The same result obtains from Figure 6 using differences in quantiles. The sample estimates of the difference in quantiles between 1992 and 1998 is shown by the dashed curve, and the upper bound of a one-sided confidence interval is shown by the dotted curve. Since we can see on Figure 6 that  $\Delta_0(p) + \sigma_{\Delta^*(p)} \zeta(\theta) < 0$ for all percentiles *p* between 0 and 0.95, we can again conclude from our data that growth was absolutely anti-poor during the period 1992 and 1998.

Opposite results are obtained when comparing 1998 to 2004. Judging from Figures 7 and 8, the change in distribution was first-order absolutely pro-poor. The upper bound of the confidence interval for  $\Delta^{1}(z)$  is everywhere negative, whatever reasonable poverty line is selected, and the lower bound of the confidence interval for  $\Delta^{1}(p)$  is everywhere positive, whatever reasonable percentile is selected.

Given the conflicting results reported above, it would seem useful to check for pro-poorness over the entire period 1992 to 2004. This can be done using Figures 9 and 10. The distributive change was almost certainly first-order absoulutely pro-poor. The lower bound of the confidence interval for  $P_{\text{box}}(z;\alpha=0) - P_{\text{box}}(z;\alpha=0)$  is everywhere negative, again whatever reasonable poverty line is selected, and the lower bound of the confidence interval for the difference in quantiles,  $Q_{\text{box}}(p) - Q_{\text{box}}(p)$  is everywhere positive, until at least the 0.8 percentile. Thus, the anti-poor movement of 1992 to 1998 was outdone by the pro-poor movement of 1998 to 2004 so that the entire period of 1992 to 2004 can be inferred to be overall first-order absolutely pro-poor.

Given the robust results obtained for first-order pro-poorness, it is not

useful to test for second-order pro-poorness since first-order pro-poorness implies second order pro-poorness. This can be seen by noting that

$$P_{j}(z; \alpha = 1) = \int_{0}^{\pi} P_{j}(y; \alpha = 0) \, dy.$$
(41)

If first-order pro-poorness obtains at order 1, then by (41) second-order pro-poorness also obtains. The same relation is obtained by noting from equations (7), (10) and (12) that the Generalized Lorenz curve condition is implied by the quantile condition.

Testing for first-order relative pro-poorness can be done using Figures 11 to 20. Figure 11 shows why observing pro-poorness in samples does not mean that we can infer it in populations; to go from sample pro-poorness to population pro-poorness, we need to apply statistical inference methods. To see this, note that despite the fact that average income fell by about 30% between 1992 and 1998, the sample estimates of  $P_{1998}(qz; \alpha = 0) - P_{1992}(z; \alpha = 0)$  suggest that the distributive movement during that period is first-order relatively pro-poor since that difference is always negative in the samples observed. But drawing a confidence interval around the sample estimates make it clear on Figure 11 that the observed differences  $P_{1998}(gz; \alpha = 0) - P_{1992}(z; \alpha = 0)$  are not statistically significant over a wide range of bottom poverty lines — the upper bounds of the one-sided confidence intervals extend above the zero line for z up to around 600 pesos and p up to around 0.28. Hence, with a conventional level 95% of statistical, the first-order relative pro-poor condition is not satisfied. An analogous result is obtained on Figure 12 from comparing growth in quantiles to growth in average income. Again, for a substantial range of percentiles, the one-sided confidence interval overlaps with the zero line.

Moving to second-order relative pro-poorness does not help, as shown by Figures 13 and 14. The statistical insignificance now extends over a wider range or z (up to around 900 pesos) and p (up to around 0.4) values. This may seem counter-intuitive at first sight, but it follows from the fact that statistical uncertainty for first-order comparisons at the bottom of the distributions builds up at the second-order since second-order conditions are made of cumulatives of first-order statistics (as discussed above). There is therefore an important lesson to be drawn here. If one were to omit testing for statistical significance, it might seem that second-order relative pro-poorness over the 1992-1998 period certainly cannot be weaker than first-order relative pro-poorness over the same period. But if one takes into account the effect of sampling variability at the bottom of the distribution, than the evidence for second-order relative pro-poorness is statistically weaker than that for first-order relative pro-poorness.

Testing for relative pro-poorness between 1998 and 2004 is more conclusive, as shown on Figures 15 and 16. The confidence interval around the sample estimates of  $P_{\text{root}}(qz;\alpha=0) - P_{\text{root}}(z;\alpha=0)$  on Figure 15 is always below zero for z up to around 1200 pesos (as opposed to 1800 pesos for the sample estimates), which leads us to infer a robust first-order relative pro-poorness change in that period. A similar result is obtained on Figure 16 from comparing growth in quantiles to growth in average income. For a range of percentiles up to about 0.7, the lower bound of the confidence interval lies above the zero line.

Given the above results, it would seem interesting to test for second-order relative pro-poorness for the 1998-2004 period. The results are shown on Figures 17 and 18. We now obtain even stronger (and very strong) evidence of the relative pro-poorness of that period. The confidence interval is always below zero for differences  $P_{\text{Pool}(g2;\alpha=1)} - P_{\text{Pool}(2;\alpha=1)}$  and above zero for differences  $C_{\text{2004}}(p)/C_{\text{1005}}(p) - P_{\text{2004}/P_{1005}}$  2004 1998 2004 1998 This is not surprising given that, as discussed above, if first-order pro-poorness is verified statistically at order 1, then we can expect second-order pro-poorness also to be inferred statistically.

The results of the tests for relative pro-poorness over the period 1992 to 2004 are even stronger. These are shown on Figures 19 and 20. The confidence interval around the sample estimates of  $P_{\text{bost}(az; a = 0)} - P_{\text{bost}(z; a = 0)}$  on Figure 19 is always below zero even as we extend z beyond 3000 pesos. The same strong evidence is displayed on Figure 16 from the comparison of growth in quantiles to growth in average income between 1992 and 2004. For a range of percentiles up to about 0.9, the lower bound of the confidence interval is everywhere above the zero line. Hence, the period 1992-2004 shows a statistically and ethically very robust degree

of relative pro-poorness change in that period.

### Conclusion

This paper proposes techniques to check for whether growth has been pro-poor. It first reviews different definitions of pro-poorness and argues for the use of methods that can generate results that are robust over classes of pro-poor measures and ranges of poverty lines. It then makes it empirically feasible to test for pro-poorness of growth. To do this, it derives the sampling distribution of the various estimators that are needed to test for absolute and relative pro-poorness. This leads to the convenient use of confidence intervals around the curves that must be ranked in order to conclude that a change has been robustly pro-poor — or anti-poor.

These statistical techniques are then implemented using Mexico's National Income and Expenditure Surveys collected in 1992, 1998 and 2004 and taking full account of the sampling design of these surveys. We find strong evidence that Mexican growth has been absolutely anti-poor between 1992 and 1998, absolutely pro-poor between 1998 and 2004 and between 1992 and 2004, and relatively pro-poor between 1992 and 2004 and between 1998 and 2004. The assessment of the period between 1992 and 1998 is, however, statistically too weak to lead to a robust evaluation of this period, and this is true both for both first and second-order assessments of pro-poorness.

Statistics	1992	Year 1998	2004
Sample size (households)	10530	10952	20595
Gini index	0.622 (0.017)	0.577 (0.011)	0.524 (0.010)
Average încome per capita:	2115 (142) [6.52]	1492 (69) [6.83]	2430 (87) [10.88]
(#year/#1992	1.000 (0.000)	0.705 (0.057)	1,149 (0.087)
Headcount index	0.291 (0.014)	0.354 (0.022)	0.106 (0.009)
Average poverty gap	0.129 (0.009)	0.169 (0.015)	0.036 (0.004)

# Table 1: Descriptive statistics

- (...): Standard errors

- [...]: Sampling design effect

Figure 1: Density functions





Figure 2: Lorenz curves

Figure 3: Poverty headcount curves:  $P(z; \alpha = 0)$  for a range of z





Figure 4: Average poverty gap curves:  $P(z; \alpha = 1)$  for a range of z

Figure 5: 1992 to 1998 is first-order absolutely anti-poor:  $P_{1998}(z; \alpha = 0) - P_{1992}(z; \alpha = 0)$ 





Figure 6: 1992 to 1998 is first-order absolutely anti-poor:  $Q_{1998}(p) - Q_{1992}(p)$ 

Figure 7: 1998 to 2004 is first-order absolutely pro-poor:  $P_{2004}(z; \alpha = 0) - P_{1998}(z; \alpha = 0)$ 





Figure 8: 1998 to 2004 is first-order absolutely pro-poor:  $Q_{2004}(p) - Q_{1998}(p)$ 

Figure 9: 1992 to 2004 is first-order absolutely pro-poor:  $P_{2004}(z; \alpha = 0) - P_{1992}(z; \alpha = 0)$ 





Figure 10: 1992 to 2004 is first-order absolutely pro-poor:  $Q_{2004}(p) - Q_{1992}(p)$ 

Figure 11: 1992 to 1998 is not statistically first-order relatively pro-poor:  $P_{1998}(gz; \alpha = 0) - P_{1992}(z; \alpha = 0)$ 



Figure 12: 1992 to 1998 is not statistically first-order relatively pro-poor:  $Q_{1998}(p)/Q_{1992}(p) - \mu_{1998}/\mu_{1992}$ 



Figure 13: 1992 to 1998 is not statistically second-order relatively pro-poor:  $P_{1998}(gz; \alpha = 1) - P_{1992}(z; \alpha = 1)$ 



Figure 14: 1992 to 1998 is not statistically second-order relatively pro-poor:  $C_{1998}(p)/C_{1992}(p) - \mu_{1998}/\mu_{1992}$ 



Figure 15: 1998 to 2004 is first-order relatively pro-poor:  $P_{2004}(gz; \alpha = 0) - P_{1998}(z; \alpha = 0)$ 







Figure 17: 1998 to 2004 is strongly second-order relatively pro-poor:  $P_{2004}(gz; \alpha = 1) - P_{1998}(z; \alpha = 1)$ 





Figure 18: 1998 to 2004 is strongly second-order relatively pro-poor:  $C_{2004}(p)/C_{1998}(p) - \mu_{2004}/\mu_{1998}$ 

Figure 19: 1992 to 2004 is first-order relatively pro-poor:  $P_{2004}(gz; \alpha = 0) - P_{1992}(z; \alpha = 0)$ 







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