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ALTERNATIVE CALCULATION OF THE PHYSICAL MASS OF THE ρ-MESON

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We have derived an expression for the physical mass and width of the ρ-meson in vacuum from its spectral function, calculated in the vector meson dominance model when a ρ^0 meson couples to two virtual pions π^+–π^-. The propagator is computed after evaluating the ρ-meson self-energy. The real part of the ρ-meson self-energy is given by a divergent integral and needs to be regularized; the regularization is done by using a double subtracted dispersion relation. The result leads to a closed analytical expression which allow us to evaluate the spectral function in a closed way. The physical mass, defined as the magnitude of the four-momentum $|k|$ for which the spectral function $S(k^2)$ attains its maximum value, is obtained, and it gives a value of 770 MeV, which is in total agreement with the reported experimental value of the ρ-meson mass.

Keywords: Mesons in vacuum; self-energy; spectral function.

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1. Introduction

The study of the ρ-meson properties in vacuum and in nuclear matter has recently attracted growing attention because of the strong evidence that the ρ-meson masses change in a nontrivial way in nuclear medium, compared with its properties in free space.[1] Also, the dilepton production rate in relativistic heavy-ion collision is strongly influenced by the ρ-meson properties in hot and dense nuclear matter.[2] Particularly, the mass of vector mesons is of wide interest due to the possibility of direct observation of the nuclear medium effects, associated with partial restoration of chiral symmetry in dense nuclear matter.[3]

Both mass and width are important properties of unstable particles. There is no unique way of defining and determining theoretically these two quantities. For some authors, as in Ref. 4, the mass of a particle is defined as the pole in its complete propagator; this definition is also used in Refs. 1, 5 and 6. On the other hand, authors in Refs. 7 and 8 make use of the spectral function to define the mass of a particle. In Refs. 9 and 10, the S-matrix formalism is used to determine the mass and width of the meson. In this work, the spectral function of a ρ-meson in vacuum is studied when the meson couples strongly to two virtual pions. The ρ–π interaction is introduced in the formalism by considering the meson as a gauge boson. Defining the physical mass of the ρ-meson as the magnitude $|k|$ of the four-momentum, for which the ρ-meson spectral function gets its maximum, we find a closed expression for the regularized self-energy function, and thus we obtain an exact analytical function for the physical mass of the ρ-meson. The ρ-meson self-energy is calculated in the one-loop level and the propagator is computed by summing over ring diagrams, in the so-called chain approximation,[11] i.e. we calculate the self-energy to one-loop order. To carry out the summation we use the Dyson equation. Since the real part of the self-energy is ultraviolet divergent, it is regularized by using a double subtracted dispersion relation which preserves the gauge invariance of the theory. This is the main point of this work, offering an approach different from other contributions where the ρ-meson physical mass is calculated. The use of a double subtracted dispersion relation allow us to get a closed expression for the self-energy function. Our result differs slightly from that obtained in Ref. 8, but the difference is due to the fact that they use dimensional regularization in their calculations.

2. Formalism

We will use the definition of the physical mass of a particle in terms of its spectral function. This method of calculation is used extensively in the literature,[7,12,13] and it is well established. To evaluate the physical mass of the ρ-meson it is necessary to calculate its dressed propagator $D_{\mu\nu}(k)$ in vacuum, where k is the four-momentum of the propagating ρ-meson. We need the expression for the dressed-meson propagator $D(k)$, which is obtained from the Dyson equation, $D(k) = D_0(k) + D_0(k)\Sigma(k)D(k)$, where

$$D_{0\mu\nu}(k) = \left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{(m_\rho^0)^2} \right] \frac{1}{k^2 - (m_\rho^0)^2 + i\varepsilon} \tag{1}$$

is the free propagator of the ρ-meson, and $\Sigma(k)$ is the self-energy of ρ. The self-energy $\Sigma(k)$ contains all the information about the interactions of the meson with the quantum vacuum. Thus, to calculate $\Sigma(k)$, we must first specify the dynamical content of our model.

Our starting point is the Lagrangian density L that describes the π–ρ dynamics,[14]

$$L = (D_\mu \pi)^*(D^\mu \pi) - m_\pi^2 \pi^* \pi - \frac{1}{4}\rho_{\mu\nu}\rho^{\mu\nu} + \frac{1}{2}{m_\rho^0}^2 \rho_\mu \rho^\mu \, ; \tag{2}$$

here, π is the charged pseudoscalar meson field; ρ_μ is the neutral ρ-meson field; the tensor

$$\rho_{\mu\nu} \equiv \partial_\mu \rho_\nu - \partial_\nu \rho_\mu \tag{3}$$

is the field tensor for the ρ-meson, and

$$D_\mu \equiv \partial_\mu - ig_\rho \mathbf{T} \cdot \boldsymbol{\rho}_\mu \tag{4}$$

is the covariant derivative, with $2\mathbf{T} = \boldsymbol{\tau}$ as the Pauli matrices in the isospin space, and $\boldsymbol{\rho}$ as the meson field which is a vector in the isospin space. The quantities m_ρ^0 and m_π in Eq. (2) are the bare ρ-meson mass and the π-meson mass, respectively.

The equation of motion for the ρ-meson field is written as

$$\partial_\mu \rho^{\mu\nu} + {m_\rho^0}^2 \rho^\nu = g_\rho J^\nu \,, \tag{5}$$

where

$$J^\mu \equiv i\pi^* \partial^\mu \pi - i\pi \partial^\mu \pi^* - 2g_\rho \rho^\mu \pi^* \pi \,. \tag{6}$$

On the other hand, the equation of motion for the pion fields reads

$$\partial^\mu \pi + i\frac{g_\rho}{2}\rho^\mu \pi = 0 \tag{7}$$

and

$$\partial^\mu \pi^* - i\frac{g_\rho}{2}\rho^\mu \pi^* = 0 \,. \tag{8}$$

Combining Eqs. (5), (7) and (8), we arrive at

$$\partial_\mu J^\mu = 0 \,. \tag{9}$$

The current $J^\mu(x)$ is therefore conserved, and, according to Eq. (5), the ρ-meson couples to a conserved current. Furthermore, taking the divergence of Eq. (5) leads to $\partial_\mu \rho^\mu = 0$, which implies that the ρ-meson field is transverse.

The interaction between the pions and the mesons are introduced through the covariant derivative

$$(D_\mu \pi)^* (D^\mu \pi) = (\partial_\mu - ig_\rho \rho_\mu)\pi^* (\partial^\mu - ig_\rho \rho^\mu)\pi \,;$$

thus, the interaction Lagrangian is given by

$$L_{\rho\pi} = -g_\rho \rho_\mu J^\mu \,,$$

where the minus sign comes from the definition of J^μ. The explicit form of $L_{\rho\pi}$ is

$$L_{\rho\pi} = ig_\rho \rho^\mu \pi^* \partial_\mu \pi - ig_\rho \rho^\mu \pi \partial_\mu \pi^* + g_\rho^2 \rho_\mu \rho^\mu \pi^* \pi \,. \tag{10}$$

The propagation of a bare ρ-meson is represented by its free propagator

$$iD^0_{F\mu\nu}(k) = \left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{(m_\rho^0)^2}\right] \frac{i}{k^2 - (m_\rho^0)^2 + i\varepsilon} \,, \tag{11}$$

schematically represented as a wavy line.

The influence of the interaction of the ρ-mesons with virtual pions is introduced through the modification of the free propagator in the one-loop approximation;

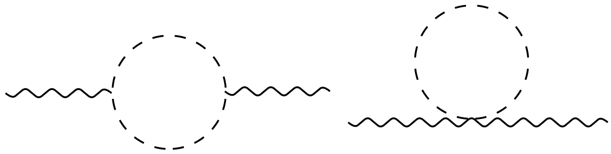

Fig. 1. Feynman diagrams associated with the two terms in Eq. (12), both contributing to the ρ-meson self-energy. The wavy line represents a neutral ρ-meson, and the dashed line represents a charged pion.

this is shown graphically in Fig. 1. These two Feynman diagrams contribute to the ρ-meson self-energy. The wavy line represents the ρ^0-meson propagator and the dashed lines represent the charged pions. The contribution of these diagrams to the modified propagator is written as

$$iD'_{F\mu\nu}(k) = iD^0_{F\mu\nu}(k) + iD^0_{F\mu\alpha}(k)(-i\Sigma^{\alpha\beta}(k))iD^0_{F\beta\nu}(k)\,,$$

where $iD'_{F\mu\nu}(k)$ and $iD^0_{F\mu\nu}(k)$ are the modified and the free Feynman propagators, respectively, and $-i\Sigma^{\alpha\beta}(k)$ is the ρ-meson self-energy tensor corresponding to the two diagrams in Fig. 1. The analytical expression for the self-energy is

$$-i\Sigma^{\mu\nu}(k) = g_\rho^2 \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \frac{(2q-k)^\mu(2q-k)^\nu}{[q^2-m_\pi^2+i\varepsilon][(q-k)^2-m_\pi^2+i\varepsilon]} - 2g_\rho^2 \int_{-\infty}^{\infty} \frac{d^4q}{(2\pi)^4} \frac{g^{\mu\nu}}{[q^2-m_\pi^2+i\varepsilon]}\,. \tag{12}$$

The first term on the right-hand side of this expression arises from the $\rho\pi\pi$-vertex, which is given by the first two terms in Eq. (10), and the last term in Eq. (12) comes from the vertex, which is given by the last term in Eq. (10).

The above expression for the modified propagator is calculated to second-order approximation, but going to farther orders, we can compute the full meson propagator. We will calculate the full propagator in the chain approximation, which consists of an infinite summation of the one-loop self-energy diagrams. The analytical expression for the full meson propagator $iD_{F\mu\nu}(k)$ is

$$iD_{F\mu\nu}(k) = iD^0_{F\mu\nu}(k) + iD^0_{F\mu\alpha}(k)(-i\Sigma^{\alpha\beta}(k))iD_{F\beta\nu}(k)\,.$$

This expression is known as Dyson equation. The solution of this equation, in matrix notation, is

$$D(k) = (D^0(k)^{-1} - \Sigma(k))^{-1} = \frac{1}{k^2-(m_\rho^0)^2-\Sigma(k)}\,. \tag{13}$$

We can see from the structure of $i\Sigma^{\mu\nu}(k)$ that it is symmetric, which is to say, $\Sigma^{\mu\nu}(k) = \Sigma^{\nu\mu}(k)$. After some algebraic work we can prove the transversality property of $\Sigma^{\mu\nu}(k)$, which is expressed as $k_\mu\Sigma^{\mu\nu}(k) = k_\nu\Sigma^{\mu\nu}(k) = 0$. This property, combined with the Lorentz invariance and the fact that the self-energy has tensorial structure, uniquely determines the form of the self-energy[8]:

$$\Sigma^{\mu\nu}(k) = \left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right]\Sigma(k)\,. \tag{14}$$

Multiplying Eq. (14) by $g_{\mu\nu}$ we obtain that $\Sigma(k) = -\frac{1}{3}g_{\mu\nu}\Sigma^{\mu\nu}(k)$.

Now, if we work in the ρ-meson rest frame, i.e. $k^\mu = (k^0, \mathbf{0})$, we can note that, as a consequence of the symmetry of the integrand, $\Sigma^{\mu\nu}(k)$ is diagonal, and that $\Sigma^{00}(k) = 0$. Thus, the three spatial terms $\Sigma^{ii}(k)$ are equal to each other.

Carrying out the integration of $\Sigma(k)$ in Eq. (12) with respect to q_0 by using the Cauchy residue theorem, and integrating in the q_0 complex plane, we obtain

$$\Sigma(k^2) = \frac{g_\rho^2}{6\pi^2}\int_0^\infty d|\mathbf{q}|\mathbf{q}^2 \frac{3k_0^2 - 8\mathbf{q}^2 - 12m_\pi^2}{\sqrt{\mathbf{q}^2 + m_\pi^2}[k_0^2 - 4(\mathbf{q}^2 + m_\pi^2) + i\varepsilon]}\,. \tag{15}$$

The real and imaginary parts of Eq. (15) can be separated with the use of the well known formula $\frac{1}{x+i\varepsilon} = P\frac{1}{x} - i\pi\delta(x)$. The integration for the imaginary part is readily obtained, and it gives

$$\operatorname{Im}\Sigma(k^2) = -\frac{g_\rho^2 k_0^2}{48\pi}\left(1 - \frac{4m_\pi^2}{k_0^2}\right)^{\frac{3}{2}} \tag{16}$$

for $k_0^2 > 4m_\pi^2$, and zero for $k_0^2 < 4m_\pi^2$. We note that in Eq. (16) the characteristic threshold value $k_0^2 > 4m_\pi^2$ for the production of real $\pi^+\pi^-$ pairs for the ρ-field.

On the other hand, the real part of $\Sigma(k^2)$ is ultraviolet divergent. We regularize the real part of $\Sigma(k^2)$, $\operatorname{Re}\Sigma(k^2)$, by following the standard procedure of subtracting infinites:

$$\operatorname{Re}\Sigma(k^2) = \operatorname{Re}\Sigma(k^2) - \operatorname{Re}\Sigma_0(k^2) + \operatorname{Re}\Sigma_0(k^2)\,, \tag{17}$$

where $\operatorname{Re}\Sigma_0(k^2)$ is an infinite quantity, chosen conveniently to cancel the infinite terms of $\operatorname{Re}\Sigma(k^2)$. Replacing Eq. (17) into Eq. (13) we obtain

$$D(k) = -\frac{1}{k^2 - m_\rho^2 - \operatorname{Re}\Sigma^R(k^2) - i\operatorname{Im}\Sigma(k^2)}\,, \tag{18}$$

where we have defined the finite difference $\operatorname{Re}\Sigma^R(k^2) = \operatorname{Re}\Sigma(k^2) - \operatorname{Re}\Sigma_0(k^2)$ as the regularized real part of the ρ-meson self-energy, and the renormalized mass m_ρ as $m_\rho^2 = (m_\rho^0)^2 + \operatorname{Re}\Sigma_0(k^2)$ (see Ref. 1). Thus, the spectral function $S(k^2)$ of the ρ-meson can be written as[17]

$$S(k^2) = -\frac{2\operatorname{Im}\Sigma(k^2)}{[k_0^2 - m_\rho^2 - \operatorname{Re}\Sigma^R(k^2)]^2 + [\operatorname{Im}\Sigma(k^2)]^2}\,. \tag{19}$$

We calculate $\operatorname{Re}\Sigma^R$ from the double subtraction relation[15]

$$\Sigma(t) = -\frac{t^2}{\pi}\int_0^\infty \frac{\operatorname{Im}\Sigma(t')}{t'^2(t'-t) - i\varepsilon}dt'\,, \tag{20}$$

which is a convergent integral. Again, by using $\frac{1}{x+i\varepsilon} = P\frac{1}{x} - i\pi\delta(x)$, we obtain that the regularized real part $\Sigma^R(k^2)$ of $\mathrm{Re}\,\Sigma(k^2)$ can be written as

$$\mathrm{Re}\,\Sigma^R(k^2) = -\frac{g_\rho^2 k_0^2}{48\pi^2} P \int_{4m_\pi^2}^{\infty} \frac{(1-\frac{4m_\pi^2}{x'})^{\frac{3}{2}}}{x'(x'-k_0^2)} dx' \,. \tag{21}$$

This integral is convergent[16] and can be solved analytically, leading to the expression

$$\mathrm{Re}\,\Sigma^R(k^2) = -\frac{g_\rho^2 k_0^2}{24\pi^2}\left\{\frac{1}{3} + \sqrt{1-\frac{4m_\pi^2}{k_0^2}} + \frac{1}{2}\left(1-\frac{4m_\pi^2}{k_0^2}\right)^{\frac{3}{2}} Ln\left|\frac{\sqrt{1-\frac{4m_\pi^2}{k_0^2}}-1}{\sqrt{1-\frac{4m_\pi^2}{k_0^2}}+1}\right|\right\}; \tag{22}$$

this is the main result of this work. Substituting Eqs. (16) and (22) into the expression for the spectral function $S(k^2)$ given by Eq. (19), we arrive at a closed expression for the spectral function. The parameters in Eq. (19) are the ρ-meson mass m_ρ which we take as 770 MeV, the experimental value of the ρ-meson mass, and the bare $\rho\pi\pi$ coupling constant $g_\rho = 6.05$[8]; both quantities were chosen in order to adjust the position and the height of the peak in the electromagnetic form factor $F(k^2)$. The plot of the spectral function given by Eq. (19) is shown in Fig. 2, where we have used Fermi units. As it can be noted, the maximum of $S(k^2)$ is in 3.9 fm^{-1}, which corresponds to 770 MeV, in accordance with the reported experimental value for the mass of the ρ-meson.[18] We also measured the width Γ of the peak of

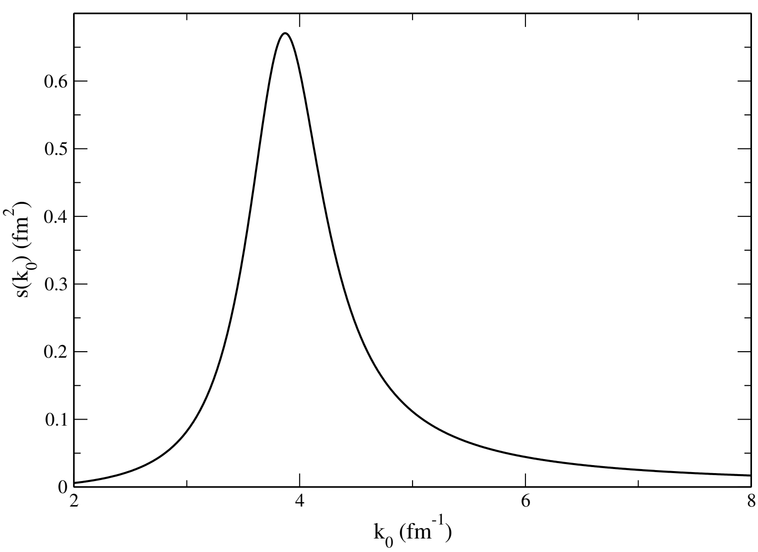

Fig. 2. Spectral function $s(k)$ for the ρ-meson in vacuum.

the spectral function at half of its height; we obtained a value of $\Gamma = 165.8$ MeV, a value somewhat wider than that of 149 MeV reported in Ref. 18.

3. Conclusion

We have studied the propagation of a ρ-meson in vacuum and calculated its physical mass and its width. From our calculations, we have succeeded in obtaining a value for the ρ-meson physical mass which is in full agreement with that measured experimentally. The use of a double subtraction relation has allowed us to evaluate the regularized part of the ρ-meson self energy $\mathrm{Re}\,\Sigma^R(k^2)$, giving a closed analytical expression. On the other hand, the result in the calculation of the width of ρ in free space from the spectral function is about ten percent larger than that reported elsewhere. The fact that we can obtain the physical mass of the ρ-meson by considering that it couples in vacuum to two virtual pions, is a strong evidence that the ρ-meson is a two-pion resonance.

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